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A REVIEW OF RAIL-WHEEL CONTACT STRESS PROBLEMS

BURTON PAUL

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16. Abstract

From its earliest days, railroad technology has been limited by an inadequate understanding of the mechanics of load transfer between wheel and rail. It is the purpose of this paper to indicate the major problems in this area, and to review the progress made to date in the solution thereof. Attention is focussed upon investigations of the stresses (normal pressure and tangential shear) on the contact patch, rather than upon studies of bending stresses in the rail. The physical basis of Hertz's widely used analysis is outlined, and the assumptions and limitations of that analysis are indicated. The need is shown for the development of solutions to important non-Hertzian problems such as: conformal contact (e.g. between worn wheels and track), contact of rough bodies, rolling friction, adhesion, and creep. The literature on these problems, as well as work in progress, is reviewed. A detailed mathematical treatment is avoided, but the principal results of much of the theory are illustrated through geometrical and physical descriptions. Recent works on the effects of surface waviness, plastic deformation, and residual stresses in rail, are reviewed.

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A Review of Rail-Wheel Contract Stress Problems

by B. Paul

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1. Introduction

The earliest self-propelled rail vehicle was built by Richard Trevithick in 1804. It successfully pulled 25 tons, at four miles per hour, along the tramroad of an English Ironworks; but it came to grief because the cast iron tram rails* broke under the weight [Wailes, 1963]. Trevithick's (and the world's) second locomotive, called the "Catch-me-who-can", suffered a similar fate in 1808, after several weeks of running in a circle, at 12 miles per hour, for the amusement of passengers who cared to pay a shilling for the ride. Unfortunately, for Trevithick, the number of shillings collected, prior to the break in the rail, (and subsequent derailment) were insufficient to pay for putting the wreckage back together again.

The first commercially successful self-propelled rail vehicle (see Fig. 1.1) was designed and built in 1812, by Matthew Murray, for John Blenkinsop (an inspector at a Colliery near Leeds). This vehicle was driven by two large cogwheels which engaged the teeth of a cast iron "rack rail". Despite (or perhaps because of) Trevithick's experience with "adhesion drive" there was so little faith in the traction capability of a metal wheel on a metal rail that cogwheels and rack rails were felt to be necessary and were indeed successfully employed in a total of four locomotives built for Blenkinsop in 1812 and 1813.

Apparently, however, William Hedley recognized that the defects in Trevithick's design lay elsewhere than in the adhesion drive, and he successfully demonstrated this in his adhesion

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*A photograph of the rails appears in Sinclair [1907 ,p. 22]. They have an angle cross-section, with upright guiding flanges on the inside.

locomotive "Puffing Billy" of 1813. From that point onward faith in smooth wheels and rails was almost, but not quite fully, vindicated. Inadequate knowledge of track construction resulted in so many broken rails that locomotives had to be replaced by horses at an alarming rate. Ironically, it was Blenkinsop, the champion of rack rails, who rescued the smooth wheel and track by the introduction, in 1829, of fish-bellied rolled iron edge rails which had sufficient strength to successfully carry contemporary locomotives, such as Stephenson's prize-winning Rocket [Skeat, 1963].

This capsule history illustrates that - from the beginning - the progress of railroad technology has been hampered by inadequate understanding of rail design, and uncertainty about the mechanics of stress transmission (pressure and traction) at the rail-wheel interface. Indeed, the stresses at the interface of wheels and rails have a profound effect on a number of crucial problems, e.g.: wear of wheel and rail, limiting drawbar pull, braking, acceleration and headway limits, rolling friction, wheel screech, hunting and other oscillations, etc. In fact, the economic capacity of the line, may be said to depend upon what is happening in the small contact patch under each wheel of a train. However, a satisfactory understanding of the many complex phenomena associated with contact stresses still eludes us.

It is convenient to subdivide the problem of rail stresses into two categories, viz: contact stresses at the rail-wheel interface, and stresses due to beam action of the rail and cross-tie system. In this paper, we will concentrate on problems of the first category. Surveys of work in the second category will be found in Kerr [1975], Eisenmann [1969,1971], and Hanna [1967]. Of course, the two categories are not completely decoupled. For example, residual stresses (produced by contact stresses which exceed the elastic limit) interact with bending stresses in a complex fashion, which depends upon the history of loading, and determine the probability of fatigue failures [Johnson and Jefferis, 1963], [Martin and Hay, 1972],[Eisenmann, 1927], [ORE, 1970,1972]. Also, the stresses depend directly upon dynamic track loading,which has been studied intensively by Birmann [1965, 1966], Birmann and Eisenmann [1966], Koci [1972].

A description of eleven different types of characteristic rail failures will be found in [Prause, Meacham, et.al. 1974]. It should also be mentioned that several extensive Bibliographies exist which describe all manner of studies on rail failure; e.g. FRA [1973,1974], [Prause, Pestel, Melvin, 1974].

If wheels and rails were made out of perfectly rigid materials they would contact at a mathematical point (or possibly a line), rather than over a finite area. Therefore, any force applied to these hypothetical rigid bodies would create infinite stress on the vanishingly small contact region. Fortunately, wheels and rails are never perfectly rigid; they possess the property of elasticity,which permits the initial point of contact to spread into a finite area as loading progresses, thereby limiting stresses to finite values. Nevertheless, these stresses can easily become excessive,and it is essential to have quantitative information on the size and shape of the contact patch, and on the distribution of the stresses in its immediate neighborhood.

In order to illustrate the practical significance of such "contact stresses", it should be noted [Stampfle, 1963] that fully 60% of 8,703 rail failures, surveyed by AREA, were associated with large contact stresses.

The earliest attempts to solve the difficult problem of contact stresses resulted in semi-empirical formulas of Winkler [1867], Grashof [1878], and an extensive set of test results on the size and shape of the contact patch under locomotive wheels [Johnson, 1894]. However, the first dependable mathematical solution was produced by Hertz [1881] in a milestone paper that still represents the point of departure for most current research. One remarkable aspect of Hertz's work is that he drew upon the fundamental solution, that had just been published by Boussinesq [1878], for a concentrated force acting normal to the plane boundary of the infinite elastic "half-space" occupying one side of a plane (Fig. 1.2). Because he used Boussinesq's influence function (load displacement relation) for a flat surface, Hertz's solution is valid only for frictionless surfaces which contact over a patch whose dimensions are small compared to the radii of curvature of the undeformed surface. Thus, the theory is valid for "counterformal" contact, such as that between balls or rollers in a bearing, but not for closely fitting or "conformal" contact as in the case of a pin in a closely fitting hole (Fig. 1.3). When Hertz's theory is applicable (see Sec. 2 for specific criteria), the contacting surfaces can be modelled as ellipsoids, and the contact patch will be elliptical.

In the neighborhood of a small contact patch, a railhead can be modelled as a cylinder with axis parallel to the track direction (see Fig. 1.4), and the wheel tread can be thought of as a cylinder with its axis at right angles to that of the track -

provided that the contact patch is small and is well removed from the wheel flange. For such crossed cylinders, Hertz's assumptions are fulfilled and his theory may be applied. Unfortunately, Hertz's form of solution involves elliptic functions in some rather unwieldy transcendental algebraic equations. The first extensive numerical evaluation of Hertz's theory with application to the rail wheel problem (modelled as crossed cylinders) was that of Belajef [1917]. More extensive stress calculations were done by Thomas and Hoersch [1930], and Lundberg and Odqvist [1932]; these last two references provide convenient tables, diagrams, and examples for the various cases that arise within the Hertz framework. Some of this material is described in Timoshenko and Goodier [1951]. More recently, Luré [1964] has presented the Hertz theory in a convenient format for numerical applications. A great deal of literature exists on contact stresses in the theory of elasticity. Fortunately, a number of good bibliographies on this subject have been compiled, and may be found, for example, in the surveys of Kalker [1975], Johnson [1975], and Lubkin [1962].

A concise but clear survey of the contact problem with particular reference to adhesion and frictional phenomena in rolling contact has been given by Ollerton [1963-64], whereas more detailed surveys on the same subject have been given by Johnson [1966] and Kalker [1967].

The texts by Muskhelishvili [1949], Gălin [1961] and Luré [1964] contain literature reviews of theoretical work in the field, especially of that done by Soviet workers, including Shtaerman, Belajef, and Savin.

Because of the smallness and inaccessability of the contact patch, it is difficult to make direct measurements of contact stresses. However, experimental determinations of the size and shape of the contact patch have been made by Johnson [1894], Fowler [1907], and Andrews [1958-1959]. The more difficult problem of stress determination has been attacked by way of photoelastic studies of plastic models. Examples of such studies are given by Frocht [1956], Fessler and Ollerton [1957], Frocht and Wang [1962], Haines and Ollerton [1963],and Haines[1964-65] .

Subsequent sections of this paper are arranged as follows: In Sec. 2 we will outline the basic ideas and results associated with Hertz's treatment of counterformal contact. Then, we shall show how certain problems beyond the scope of Hertz's analysis may be treated, starting in Sec. 3 with a discussion of conformal contact, such as at the interface of worn wheels and rails. Some of the basic physical concepts underlying phenomena related to rolling contact, e.g. rolling friction, creep and adhesion, are reviewed in Sec. 4. In Sec. 5, additional non-Hertzian contact stress phenomena due to surface waviness and molecular adhesion are described. A brief introduction to recent investigations dealing with plastic deformation and residual stresses in rails will be found in Sec. 6.

2. Theory of Contact of Frictionless Counterformal, Elastic Bodies (Hertzian Formulation)

Hertz's paper of 1881 represents the starting point for all subsequent investigations of the deformations and stresses experienced by two elastic bodies in contact. Hertz's solution is valid provided that the following restrictions are satisfied:

1. The bodies are homogeneous, isotropic, obey Hooke's Law, and experience small strains and rotations (i.e. the linear theory of elasticity applies).
2. The contacting surfaces are frictionless.
3. The dimensions of the deformed contact patch remain small compared to the principal radii of curvature of the undeformed surfaces. This will be the case for counterformal surfaces (e.g. two spheres, or a small sphere in a large bowl); but may not be true, even at low strain levels, for conformal surfaces (e.g. a ball in a closely fitting spherical socket).
4. The deformations are related to the stresses in the contact zones as predicted by the linear theory of elasticity for half-spaces (Boussinesq's influence functions are valid).
5. The contacting surfaces are continuous, and may be represented by second degree polynomials (quadric surfaces) prior to deformation.

We shall assume throughout this paper that restriction (1) holds, and we shall indicate briefly the main arguments in Hertz's development, so that we can better understand the condition under which some of his other restrictions may be removed.

Let the upper and lower bodies, "1" and "2", be bounded by two surfaces in the undeformed state, described by the respective equations

$$z_1 = f_1(x,y)$$
$$z_2 = f_2(x,y) \tag{2.1}$$

where the orthogonal axes(x,y) lie in the common tangent plane at the point O of mutual contact prior to deformation. Let $z_1(z_2)$ be the axis, perpendicular to the x, y axes, which points into body 1(2) at the origin O, as indicated in Fig.2.1-a, which shows a section through the x-z plane. When equal and opposite thrusts P are applied to the two bodies along their negative z axes (i.e. P presses the bodies together), the contact point O, spreads into a contact region Ω, and particles in the body 1(2) far removed from Ω experience a rigid body translation of amount δ_1 (δ_2) in the direction opposite to the axis of $z_1(z_2)$. In other terms, distant points in body 1 approach distant points in body 2 by the relative approach

$$\delta \equiv \delta_1 + \delta_2 \tag{2.2}$$

If no elastic deformations occur, the rigid body motions would require that the initial contact point A_1 on the upper body penetrate into the lower body (and similarly for point A_2 on the lower body) as indicated in Fig.2.1-b. Since the bodies are impenetrable, it is necessary that a mutual contact stress p(x,y) acts between the bodies in such a way as to push up the lower part of body 1 through an elastic displacement $w_1(x,y)$; similarly, body 2 experiences an elastic displacement $w_2(x,y)$. If the bodies are frictionless (restriction 2), $p(x,y)$ is a normal stress (pressure). The net result is that the deformed bodies just touch within a region (contact patch) denoted by Ω, and are separated by a positive distance outside of Ω. From Fig. 2. 1-b it may be seen that

$$(w_1 + z_1) + (w_2 + z_2) = \delta_1 + \delta_2 \equiv \delta \text{ (inside } \Omega \text{)} \qquad (2.3\text{-a})$$

$$w_1 + z_1 + w_2 + z_2 > \delta \quad \text{(outside } \Omega \text{)} \qquad (2.3\text{-b})$$

Now let us impose restriction 3, and note that for counterformal surfaces, the contact region is, by definition, so small that no distinction need be made between the displacement normal to the undeformed surfaces and the displacement w parallel to the z axis. Therefore it is consistent to impose restriction 4 and assume that deformed surfaces respond to the pressure $p(x,y)$ in the manner of elastic halfspaces.

For the halfspace occupying the region $z \geq 0$, Boussinesq showed* that a pressure $p(x',y')$ acting on a small patch of surface area $dx'dy'$, at point (x',y') will produce at any surface point (x,y) a deflection dw, in the z direction, given by

$$dw = k\, p\, \frac{dA}{r} \qquad (2.4)$$

*See Timoshenko & Goodier [1951, p. 365].

where

$$dA = dx'dy'$$

$$r = [(x-x')^2+(y-y')^2]^{1/2} \qquad (2.5\text{-a})$$

$$k = (1-\nu^2)/(\pi E) \text{ (elastic parameter)} \qquad (2.5\text{-b})$$

ν = Poisson's ratio

E = Young's Modulus

Therefore, the normal displacement at surface point (x,y) on body i, due to a distribution of pressure over a region Ω, is given by the summation

$$w_i(x,y) = \int_\Omega dw_i = k_i \int_\Omega \frac{pdA}{r} \; ; \; (i = 1,2) \qquad (2.6)$$

where $k_i = (1-\nu_i{}^2)/(\pi E_i)$ (2.7)

When Eqs. (2.6) are substituted into conditions (2.3-a) we find

$$K \int_\Omega \frac{p(x',y')dx'dy'}{[(x-x')^2+(y-y')^2]^{1/2}} \geq \delta - F(x,y) \qquad (2.8)$$

where $K = k_1 + k_2 = \dfrac{1-\nu_1{}^2}{\pi E_1} + \dfrac{1-\nu_2{}^2}{\pi E_1}$ (2.9)

and $F(x,y) = z_1(x,y) + z_2(x,y)$ (2.10)

is the profile function, defined by the known surface shape functions (2.1).

When the equality sign is used in (2.8) we have an integral equation of the first kind which governs the entire theory of contact stresses. For arbitrary profiles, Eq. (2.8) defies solution in analytical terms. However, Hertz, the great master of classical mechanics[1] and electromagnetic theory, recognized the analogy between Eq. (2.8) and the equation governing the potential of a surface distribution of electric charge or gravitational mass. In particular, he saw that a complete analytic solution could be developed in terms of elliptic integrals when the profile function F(x,y) was a second degree polynomial. Thus he was motivated to introduce restriction (5) which is equivalent to the assumption that the bodies in contact either are, or may be approximated by ellipsoids with a profile function of the form

$$F(x,y) = Ax^2 + By^2 \qquad (2.11)$$

For such surfaces he showed that the contact region is bounded by an ellipse

$$(x/a)^2 + (y/b)^2 = 1 \qquad (2.12)$$

where the semidiameters a and b are given* by

$$a = m' \, [KP/(A + B)]^{1/3} \qquad (2.12\text{-a})$$

$$b = n' \, [KP/(A + B)]^{1/3} \qquad (2.12\text{-b})$$

and m', n' are tabulated functions of the ratio (B/A).

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[1]Although most famous for his experimental confirmation of the existence of the electromagnetic waves predicted by the theoretical work of Maxwell, Hertz was profoundly interested in the logical foundations of Mechanics as may be seen from his axiomatic treatment in Hertz [1900].

*Equations (2.12) through (2.16) may be found in Timoshenko and Goodier [1951, pp. 377-382]; along with a tabulation of m' and n'.

The contact pressure associated with the ellipse (2.12) is itself of ellipsoidal form described by

$$p = p_o[1-(x/a)^2-(y/b)^2]^{1/2} \qquad (2.13)$$

where the peak contact pressure is given by

$$p_o = \frac{3}{2}\frac{P}{\pi ab} \qquad (2.14)$$

Expressions for all stress components ($\sigma_x, \sigma_y, \sigma_z, \tau_{xy}, \tau_{yx}, \tau_{zx}$) have been derived, but they are extremely complex in form and requre a good deal of numerical work in order to interpret. Examples of such calculations are given in Thomas and Hoersch [1930]. For present purposes, it suffices to note that the lateral normal stresses at the surface are given by

$$\sigma_x = -2\nu p_o - (1-2\nu)p_o b/(a+b)$$
$$\sigma_y = -2\nu p_o - (1-2\nu)p_o a/(a+b) \qquad (2.15)$$

where the appropriate value of ν (ν_1 or ν_2) is to be used; and the maximum shear stress τ_{max} occurs at a distance h beneath the surface. Typical values of h and τ_{max} are given below for steel ($\nu = .3$) for two different aspect ratios (b/a) of the contact patch:

$$(b/a) = 1.00: \quad h = 0.47a; \quad \tau_{max} = 0.31\ p_o$$
$$(b/a) = 0.34: \quad h = 0.24a; \quad \tau_{max} = 0.32\ p_o \qquad (2.16)$$

These are the principal results of Hertz's analysis. To illustrate their use, consider a cylindrical wheel of radius R_1 = 15.8 in., pressed by a force of P = 20,000 lb.* against a railhead with a radius of curvature of R_2 = 12 in. at the initial point of contact. For this case, the above formulae predict that

a = 0.256 in; b = 0.215 in.

p_o = 173,500 psi;

$\tau_{max} \doteq 53{,}784$ psi at a depth of $h \doteq 0.0882$ in.

Ordinary rail steels (not heat treated) have a yield stress in shear of about 37,500 psi [Stampfle, 1963]. Therefore at the given wheel load, plastic flow will occur.

It should also be noted that, for ductile metals, the maximum shear stress is a much better index of plastic flow and fatigue than the maximum normal stress [cf. Paul, 1968]. Thus the fact that the maximum shear stress can occur underneath the surface has often been pointed to as a contributing factor involved in the rail-head spalling type of failure often described as rail shelling. It should be mentioned though that Fromm [1929] and Smith and Liu [1953] have shown that the peak shear stress can shift to the top of the rail when tangential (frictional) stresses are taken into account, and that Wandrisco and Dewez [1960] report that their examinations of shelled wheels failed to reveal any indications that shelling cracks originate beneath the tread surface. A discussion of the mechanics of rail shelling is given in Sec. 6.

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*This load is near but below the limit permitted by the Joint Committee of AREA and AAR [Stampfle, 1963].

3. Conformal Contact Problems

Hertz was able to find solutions of the governing integral equation (2.8) in terms of elliptic integrals precisely because he restricted the profile function $F(x,y)$ to a quadratic expression of the type associated with contacting ellipsoids. If the contacting bodies cannot be modelled as ellipsoids it is not possible to utilize the Hertz solutions. Since the strains must be small (of the order of 10^{-3}) for metals in the elastic range, one is tempted to think that the displacements w_1 and w_2 must be small, and hence the initial separations which are identical to $F(x,y)$ must be small and therefore may be represented by a Taylor series whose leading terms are $Ax^2 + By^2$; i.e. of the type postulated by Hertz. This is permissible for a new wheel in contact with the railhead at a point well removed from the flange, as shown in Fig.(3.1-a). In this case, the profile function is well approximated by the quadratic expression

$$F(x,y) = z_1 + z_2 = \frac{x^2}{2r} + \frac{y^2}{2R} \qquad (3.1)$$

where R is the wheel radius (neglecting conicity of the tread) and r is the radius of curvature of the rail head. However, suppose that the wheel tread is concave, due to wear*, or by design, as in a Heumann "preworn" profile [Koffman and Bartlett, 1965]; or that the railhead contacts the wheel near the flange. In such cases the contact region can extend over a considerable portion of the railhead, as illustrated in Fig.3.1-b the contact is of the conformal type, and Hertz's assumption 3 (dimensions of contact patch small compared to radius of curvature)

*Typical worn wheel profiles are shown in Jennings [1961], and in Fowler [1907]. Experience with "preworn" profiles is described by Koffman [1965] and King [1968].

is invalid. When assumption 3 is invalidated, we are no longer justified in using either assumption (4) (pressure displacement relations are those of a half plane) or assumption (5) (profile function is quadratic). Thus we are motivated to examine the class of frictionless non-Hertzian problems where restrictions 3, 4 and 5 are removed.

Since it will be necessary to relax restriction 5, it is worthwhile to consider the class of conformal problems where only restriction 5 is relaxed. If we permit the possibility of arbitrary surfaces rather than ellipsoids, we should not expect to find solutions in analytic form, but should strive to develop numerical methods which are well adapted to infinitely variable surface profiles. Such numerical methods have been developed by Singh and Paul [1974], Conry and Seirig [1971], and Kalker and Van Randen [1972]. The method of Singh and Paul is, in brief, as follows: An initial guess is made of a representative contact region Ω for some unknown load P. The candidate region is divided by a network of mesh lines into n cells numbered 1, 2,....n. Within each cell Ω_j the contact pressure p_i (i = 1,2,...n) is assumed to be constant. For a fixed field point (x_i, y_i) a numerical quadrature is performed to evaluate $\int_{\Omega_j} dA/r$ for the cell Ω_j (with proper care for the singularity in cell Ω_i). Thus the integral in Expression (2.8) can be expressed as a linear combination of the unknown pressures in the form

$$K \sum_{j=1}^{n} b_j \, p_j \overset{>}{=} \delta - F(x_i, y_i) \qquad (3.2)$$

where b_j represents a known constant arising from the numerical quadrature. By repeating this procedure for m different field points (i = 1,2,...m) in the interior of Ω we obtain a set of m linear equations in the unknowns $(p_1, p_2, \ldots p_n, \delta)$, which may be solved for so long as $m \geq n + 1$. If all p_j are positive, and the inequality form of (3.2) is satisfied for (x_i, y_i) outside of Ω, the true solution has been found; otherwise an adjustment is made on the boundaries of the latest assumed region and the procedure repeated until Inequality (3.2) is satisfied outside of Ω, with $p_j > 0$ inside of Ω. Because of notorious difficulties in the solution of integral equations of the first kind (which may have no solution at all if the given data of the problem are perturbed by extremely small amounts) it has been necessary to devise special numerical procedures [Singh and Paul 1973,1974] to overcome the inherently ill-posed nature of the problem.* The success of these methods has been illustrated by the solution of several non-Hertzian counterformal contact problems such as the following: indentation of a cubic punch [Singh and Paul, 1974]; a contact region with a hole, e.g. a corrosion pit [Singh, Paul and Woodward, 1975]; a nonelliptic contact region due to crowning of the ends of needle bearings [Singh and Paul, 1975].

Having found a way to deal with general profile functions (i.e. having dropped restriction 5) we may now attempt to drop restriction (3) and (4) in order to solve conformal problems such as that of the worn wheel on a closely fitting railhead (Fig.3.1-b). Very little work has been published in this area. Among the few published papers dealing with conformal contact of two <u>elastic</u> bodies are those of Goodman and Keer [1965], which treats a sphere in a spherical socket, and Persson [1964],

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*I. L. Paul and P. R.Nayak [1966] and Nayak and Paul [1968] attempted a numerical solution of the integral equations governing contact problems with friction. Although they could not explain why their numerical method failed to converge, it is quite likely that their difficulties arose from the ill-posed nature of the problem; similar difficulties were overcome in the papers by K. P. Singh and B. Paul [1973, 1974].

which considers a pin in a cylindric bearing. Both of these papers introduce certain simplifying assumptions whose implications remain to be fully explored. Shtaerman [1949] has also discussed the problem of the closely fitted pin but assumes that a fictitious concentrated force exists on pin and bearing in the clearance space diametrically opposite the initial contact point.

In order to treat general geometries, such as the worn wheel-railhead combination, it is necessary to use numerical methods. The author is currently working, with W. Woodward, to extend the non-Hertzian analysis of Singh and Paul to include general conformal geometries typical of worn wheel and rail combinations.

4. Rolling Contact, Adhesion, and Creep

So far we have (in common with Hertz) considered only the normal pressures exerted between the wheel and the rail. In practice, rather large tangential (shear) stresses are also exerted on the track by the wheel. The presence of these shear stresses during braking, or accelerating is easy to accept. What may not be so obvious is that such stresses occur even under conditions of steady rolling. This effect is the result of "microslip" or "creepage" which is a frictional effect associated with the compliance of the wheels and rails; perfectly rigid wheels and rails would not experience such effects.

Although it is useful to the track designer to know what magnitudes of the applied shear stress may be expected, it should also be borne in mind that much of the dynamic loading on tracks due to truck hunting, and other modes of oscillation, is determined by the creepage, damping, and other phenomena which are traceable to the interaction of shearing stress, induced by rolling action.

The mathematical theory of rolling contact, including the effects of friction and adhesion, is extremely intricate. It is not our intention to review the development of the mathematical theory in this paper. Instead we will attempt by means of physical and geometrical arguments, to illustrate the principal results of the theory. Those who desire a more rigorous treatment of the subject are urged to consult Bidwell [1962], the several works of Kalker or Johnson, and the numerous pertinent references mentioned in their recent survey papers: Kalker [1975] and Johnson [1975]. An earlier survey by Hobbs [1967] is also noteworthy, as is the frequently cited note of Vermeulen and Johnson [1964].

When a perfectly rigid wheel of radius r rolls over a rigid plane surfaces (the track) the center of the wheel advances, in one revolution, through a distance equal to its circumference. For steady rolling the ideal speed v_o of the center is

$$v_o = r\omega$$

where ω is the angular speed of the wheel. However, due to the elasticity of the wheel and the track, the center of the wheel will not travel at its ideal speed $r\omega$, but will appear to be retarded (i.e. to slip).

This phenomenon of apparent slip (now called "microslip", "creep", or "creepage") was first pointed out by Osborne Reynolds [1876], on the basis of physical reasoning and experiments with rollers and tracks made of rubber, cast iron, glass, brass and wood. Reynolds did not attempt a quantitative analysis of the problem (in fact there isn't a single equation in his twenty page paper) but his reasoning can be described somewhat as follows.* Suppose that a rigid wheel with equidistant marks a,b,c,.. engraved along its circumference (e.g. spaced at 0.1 inch) rolls along a rigid plane having equally spaced marks A, B, C, etc., as shown in Fig. 4.1a.

If point a on the wheel initially contacts point A on the plane, the wheel will have rolled through the angle $\Delta\phi$ by the time that point b contacts point B; i.e.

$$\Delta\phi = ab/r = AB/r \qquad (4.1)$$

Now let us imagine that the plane is elastic (e.g. made of rubber) so that a pressure of the rigid wheel against the plane produces a lateral expansion of the plane surface under the

*This is not intended to be a complete or literal interpretation of Reynold's ideas.

wheel. That is, the plane undergoes a surface extensional strain ε which enlarges the distance AB to the stretched distance

$$A'B' = (1+\varepsilon)\,AB \qquad (4.2)$$

as shown in Fig. 4.1-b. Of course, after the wheel is well past B' that point will revert to its initial location B. In order for the wheel to contact the point B' it must roll through the angle

$$\Delta\phi' = A'B'/r = (1+\varepsilon)\,AB/r = (1+\varepsilon)\,\Delta\phi \qquad (4.3)$$

Thus, for the same forward motion of its center, the wheel on the elastic track must roll through $(1+\varepsilon)$ times the number of revolutions as the wheel on the rigid track. In other terms, for each revolution of the wheel on the elastic track, it advances through a distance

$$x' = \frac{2\pi r}{1+\varepsilon} \qquad (4.4)$$

whereas the wheel on the rigid track advances through the ideal distance

$$x = 2\pi r \qquad (4.5)$$

The ratio of the apparent slippage $(x-x')$ to the ideal distance traversed is called the "longitudinal slip" or "creepage" defined by

$$c_x = \frac{x-x'}{x} \qquad (4.6)$$

Upon using Eqs. (4.4) and (4.5) in Eq. (4.6), we find that

$$c_x = \frac{\varepsilon}{1+\varepsilon} \doteq \varepsilon \qquad (4.7)$$

where the last equation is valid whenever $\varepsilon \ll 1$, as it is for structural materials in the elastic range. Equation (4.7) tells us that the longitudinal creep is equal to the surface strain in the track of our highly idealized model.

If we suppose that the track is rigid and that the roller undergoes a circumferential surface strain ε at the contact point, then the above argument may be reversed to show that the creepage is negative; i.e. the elastic roller actually overtakes the rigid roller, when both execute the same number of revolutions. In fact, it follows that if both wheel and track are elastic and undergo respective surface strains of ε_{wheel} and ε_{track}, the longitudinal creepage is given by the relative strain

$$c_x = \varepsilon_{track} - \varepsilon_{wheel} \tag{4.8}$$

Now let us consider the influence of the wheel radius on the creepage phenomenon. If an elastic wheel indents an elastic plane, made of identical material, as shown in Fig. 4.2-a, a contact pressure p will arise at the interface (mn, on the roller, MN on the track). Due to the effect of Poisson's ratio, the contact pressure will cause an extension of the surfaces mn and MN. However, in the absence of friction, the radially converging pressure on mn (illustrated in Fig. 4.2-b) will have a tendency to squeeze mn inward (counteracting to some extent the expansion). Similarly, the radially diverging pressure on the concave curve MN will tend to augment the extension of MN. Thus the relative extension of MN exceeds[1] that of mn and the creep coefficient predicted by Eq. (4.8) will be positive. In other words, a steel wheel on a steel rail will experience a relative slippage due to elastic effects.

[1]Reynolds [1876] comes to this same conclusion by an argument that is unclear to me.

By the arguments just given, we see that if two identical wheels roll on each other, the relative slip vanishes. But, if difference size wheels, of the same material roll together, the wheel of smaller radius will slip relative to the larger wheel. By this reasoning, large driving wheels on locomotives will show less apparent slip than smaller wheels.

Whenever relative slip occurs between surfaces in contact, energy will be dissipated in friction. It is precisely this energy dissipation which accounts for much of the so-called rolling resistance which was empirically determined by Coulomb, Navier, and Morin.* According to the foregoing analysis we should expect the resisting force, in rolling, to decrease with increasing wheel radius, as indeed it does** according to Coulomb's rules for rolling friction [see Housner and Hudson, 1949, p. 126]. We should also expect to see the rolling friction and the creepage decrease for stiffer materials, and thus the wear of a stiff track should be considerably less than that of a more compliant track. Thus was Reynolds able to explain the "matter of much surprise" that steel rails showed considerably less wear than the more compliant iron rails which they supplanted.

From Fig. 4.1-b we conclude that due to symmetry about point A', the lateral displacement of the surface increases monotonically with the distance from point A'. Furthermore, the normal pressure will be a maximum at A', and will gradually taper off to zero at the boundaries of contact (see Fig. 4.2-b).

*Another factor which contributes to rolling resistance is the internal hysteresis loss in materials under cyclic stress [Tabor, 1955]. Johnson [1972] has also considered the influence of plastic deformation on rolling resistance.

**Coulomb and Morin believed that the rolling friction should vary inversely with wheel radius $\underline{r}$. However, Dupuit, believed that rolling friction varied inversely with $\underline{r}^{1/2}$. The ensuing controversy between Morin and Dupuit has been described by Tabor [1962].

Thus, the maximum available friction stress (μp) exists at A' and falls off with distance from A'. Therefore, Reynolds concluded that friction would inhibit slip in a central core or "locked" region surrounding the symmetry point A', but that relative slip takes place in regions adjacent to the contact boundaries. This conclusion was mathematically confirmed for elliptic and circular contact regions by Cattaneo [1938], and Mindlin [1949], respectively.

So far, we have only considered the case of "free rolling", i.e. under the influence of normal forces only. If a tangential force (e.g. a drawbar pull, or wind resistance, or gravity loading due to a grade) is applied to the center of a wheel, equilibrium cannot subsist unless an equal and opposite frictional force is generated at the track surface. If the magnitude P of the applied tangential force is less than the limiting friction μN, (where N is the force applied normal to the contact patch, and μ is the coefficient of static friction), equilibrium is possible; otherwise, the wheel will skid and accelerate in the direction of the excess force ($P-\mu N$). In the equilibrium state, there will be a region of the contact patch in which no relative slip occurs (i.e. a locked core where adhesion occurs), and a region where relative slip occurs. For a given value of normal force N, and for sufficiently small values of tangential force P, the relative surface strain increment ε is proportional to P and the creepage predicted by Eq. (4.8) is also proportional to P, as shown in Fig. 4.3 .

by the dotted line. As the force P increases, the slipped region occupies an increasingly larger portion of the contact patch, and the creepage c_x defined by Eq. (4.6) begins to increase at a faster rate, causing the curve of P vs. c_x to bend away from the dotted straight line as shown. When P reaches the limiting friction μN, the entire contact patch is undergoing sliding, with little or no rotation of the wheel. This condition is referred to as "skidding" if the wheel is locked (as in "panic" braking), or as "slipping" (as in starting under excessive torque) if the wheel is turning.*

Carter [1926] was the first to arrive at a mathematical relationship between the tangential force and the creepage. He calculated the longitudinal tractive force per unit longitudinal creepage, for an elastic cylinder rolling on a track of the same material. In the contact region, symmetry considerations show that the surface strain ε_x in the wheel (due to shear stress) is equal but opposite to that in the track, and hence the relative surface strain is $2\varepsilon_x$.

At this point in his development, Carter casually stated that a locked region with no relative slip (adhesion) exists between wheel and rail, starting at the leading edge and extending back a distance which depends upon the magnitude of the longitudinal force P (see Fig. 4.4). Because no change in the relative slip ($2\varepsilon_x$) can occur in the locked region, it follows that the strain ε_x is constant in the region of adhesion.

*Nayak et al [1970, p. 16] report that the ratio P/N for the case of slipping can exceed that for the case of skidding by as much as 50%. A similar observation was reported by Fowler [1907].

By finding an expression for the surface strain ε_x in terms of the normal load N and the longitudinal force P, Carter was able to utilize Eq. (4.8) to find the creepage $c_x = 2\varepsilon_x$.

Carter's statement concerning the location of the locked region was so laconic that it either was forgotten or not believed until Cain [1950] pointed out its physical basis, in a discussion of a paper by Poritsky [1950]. Perhaps the clearest physical explanation (an elaboration of Cain's) of why the locked region must adjoin the leading edge of contact regions of arbitrary elliptic shape (see Fig. 4.4-c) is given by Ollerton [1963-64], and is confirmed by experimental results of Haines and Ollerton [1963].

Before leaving the topic of rolling contact, it should be mentioned that a lateral force on the wheel (parallel to its axle) will cause the wheel to creep in the direction of the axle, even though the axle of the wheel remains perpendicular to the track axis. This "crablike" motion is due to the elastic deformations of the wheel and rail in the lateral direction, and the ratio of lateral displacement per wheel revolution to the circumference of the wheel is called the "lateral creep." This phenomenon is well-known in pneumatic automobile tires, and can easily be explained in physical terms [see Rocard, 1960]. The lateral creep is related to the lateral force in much the same way as shown in Fig. 4.3 for the longitudinal force and creep.

It should also be noted that if a rolling body has a component of angular velocity about the normal to the contact patch, this so called "spin" velocity will induce a lateral creep; cf. Johnson [1958-a], Kalker [1966], de Pater [1962].

So far the word "adhesion" has been used to describe the absence of relative slip in the locked region of the contact patch. The same word is often used in a somewhat different sense; i.e. to describe the maximum friction force, and hence the limiting draw-bar pull of locomotive wheels. A large body of literature exists on this topic which can be traced through the review papers of Marta and Mels [1969], Collins [1972], Verbeeck [1973], and the Symposium Proceedings of the Institution of Mechanical Engineers [1963-64]. Attempts have been made to improve traction by physical or chemical treatment of the contact surfaces, for example, high temperature plasma cleaning; however, according to Gifford et al, [1971] this approach is not currently feasible under practical operating conditions.

5. Additional Non-Hertzian Effects

Departures from the Hertzian formulation may arise for reasons other than the presence of friction or conformal contact. Some of these effects, which have been described briefly by Johnson [1975], include molecular adhesion, and microscopic surface waviness.

The former effect is important only for ultraclean microscopically smooth surfaces, and is not directly significant for the usual contaminated conditions which exist at the rail-wheel interface. However, a better understanding of this phenomenon may ultimately improve our basic understanding of friction-related phenomena and the possibilities for adhesion augmentation procedures. Although the classical theory of elasticity (and consequently Hertz's analysis) is not capable of treating problems involving molecular adhesion, Johnson [1975] and his colleagues have introduced adhesive effects into an "extension of the Hertz theory" by considering "surface energy" terms in the spirit of Griffith's [1921] well known analysis of crack propagation. It should be mentioned that much of the recent work in the theory of crack propagation represents an attempt to include such "surface energy" or "cohesion" effects in the theory of elasticity [see Goodier, 1968].

Surface Waviness Effects

With the possible exception of cleavage planes in crystals, real material surfaces are never ideally smooth. Contact between surfaces actually occurs at the crests of the numerous miniature mountains (asperities) which define the real surfaces. Therefore the contact pressure is distributed over the nominal contact area in the form of numerous pressure peaks surrounded by regions of zero pressure between the asperities. The smooth pressure distribution of Hertz can only represent the actual pressure distribution in an average sense. In order to better understand the effects of the surface waviness, it is necessary to utilize a statistical model of the wavy surface. Such models have been studied by Ling [1958], Greenwood and Williamson [1966], Whitehouse and Archard [1970], and Nayak [1971]. Additional references may be found in Nayak, et al [1970].

In qualitative terms, the surface asperities create a much more compliant subbase and tend to spread the load over a wider radius than that predicted by Hertz. However, as the load increases, the most heavily loaded asperities will be loaded into the "hardening regions" of Hertz's load-displacement curves and many asperities will experience plastic deformations which tend to spread the load rapidly and close up the gaps between the loaded regions. Therefore, we should expect that surface waviness will cause a great deviation from Hertz's predictions for light loads, but will have decreasing influence for heavier loads. This trend is illustrated in Fig. 5.1 from Greenwood and Trip [1967].

For the light load (Fig. 5.1-a), the peak pressure is only a third of that predicted by Hertz, and the effective area of contact has spread to about ten times that predicted by Hertz. For the heavier load (Fig.5.1-b) the Hertzian values of both pressure and contact area are essentially the same as for the rough surface-model.

According to the statistical model of Greenwood and Tripp the maximum shear stress for rough spherical surfaces still occurs at a depth of 35% to 47% the effective radius of the contact area (compare Eq. 2.16). However, because the effective radius is larger than that predicted by Hertz, the actual depth of the peak shear stresses will be greater than predicted by Hertz. Thus the microroughness of the contacting surfaces may contribute to the explanation of the observed fact that rail "shells" seem to originate at greater depths than predicted by Hertz.

Theories which account for the influence of vibrations on the rolling contact phenomena have been reviewed and further developed in the works of Nayak et al [1970], and in Nayak [1973].

6. Plastic Deformations and Residual Stresses

Because of the high stress levels accompanying the contact of counterformal bodies, it is likely that plastic flow will occur sometime in the history of track loading. When the external loads are removed from a body that has undergone plastic flow, a certain amount of permanent strain (or "set") remains along with an associated self-balanced set of "residual" or "locked in" stresses. When a load is reapplied to a body with residual stresses, further plastic flow may or may not occur depending upon the history of the loading process and the strain-hardening characteristics of the material. It is possible that loading states would not cause a massive failure if applied once, but can cause a monotonic increment in deformation in each cycle of loading, and lead to a so-called "incremental collapse" [Hodge, 1959, p. 127]. This may well be the mode of failure which results in the gross plastic deformation often observed at the edge of a rail. Under other circumstances, a cyclic loading produces a limited amount of displacement, but the material undergoes plastic flow in alternating directions during each cycle of loading. This condition of "alternating plasticity" can lead to "low-cycle fatigue." If, under cyclic loading, the amount of energy dissipated in plastic flow never exceeds a finite limit, the material is said to have "shaken down" and purely elastic behavior will occur after a fixed number of load cycles. Although theoretical rules exist [Hodge, 1959, p. 129] to determine whether or not a structure will "shakedown", we cannot infer that a structure is safe just because it can be shown to shakedown under the program of imposed loads.

It may well be that the structure will have undergone a low-cycle fatigue failure long before it reaches its shakedown state. Johnson [1962] has shown how the shakedown state can be found in problems of rolling contact.

Although residual stresses can be deleterious, they can also be helpful, if they occur in the proper direction with proper magnitude. Indeed much of the benefit of surface treatments (e.g. ball peening and thread rolling) may be attributed to the introduction, at potential fatigue sites, of residual stresses (primarily compressive).

The determination of residual stresses, in tracks, and their effects on rail life, has been the subject of a continuing series of researches sponsored by the Office for Research and Experiments of the International Union of Railways (O.R.E.) These investigations are directed towards "Question C 53, Behavior of the metal of rails and wheels in the contact zone." On the basis of experiments, it has been found in [O.R.E.,1970] that continued rolling (simulating axle loads of 13-21 tons at speeds up to 70 km/hr) caused work hardening to penetrate to a depth of 3-4 mm, and to change the residual stresses in a new rail from tension in the top 30 mm to compression in the top 5-10 mm. It was suggested that the most likely zone for the initiation of kidney-shaped fatigue flaws (shelling) is at the depth of 15 mm where a high hydrostatic residual tension exists.

In [O.R.E. 1972], an attempt has been made to follow the stress history within a track as the result of the cyclic passage of a wheel. Upon a standard state of residual stress, the Hertz stress and bending stresses were superposed. The Hertz stresses were calculated from the formulas of Thomas and Hoersch [1930]. The stress histories were obtained at several depths on planes where high shear stresses occurred, and were plotted as trajectories in a plane of shear stress T, versus mean normal stress P. Unfortunately, these trajectories are difficult to interpret from the point of view of fatigue failure because of the lack of a reliable criterion of fatigue damage under multiaxial states of stress. However, by assuming that there exists a damage line, in the T-P plane, which divides the safe stress states from unsafe states* it was possible to draw some qualitative conclusions from the data. For example: the transverse curvature of the tires has a considerable effect at the surface, and at depth, suggesting that a "preworn" tire profile would be of great benefit; but an enlargement of the rail cross-section and a stiffening of its support creates a limited and "rather unfavourable effect on the formation of fatigue cracks." Other references on residual stresses and fatigue in rails include Johnson and Jefferis [1963], Martin and Hay [1972], Konyukhov, et. al. [1973].

*This hypothesis is based on a Doctoral thesis of K. Dang Van [1971].

7. Conclusions

Even though wheel-rail contact stresses play a central role in many significant railroad problems (wear, traction, guidance, braking, headway, etc.), an adequate understanding of these stresses still eludes us. The complexity of the problem has been illustrated, and the current status of our knowledge has been reviewed in such areas as: conformal contact (worn or "profiled" wheels), rolling contact, adhesion, creep, plastic flow, residual stresses, and surface roughness effects. Sources of information on these and related areas have been identified, and physical and geometrical plausibility arguments have been used to describe a number of major results which have been arrived at by complex mathematical procedures.

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REFERENCES

H.I. Andrews, "The Contact Between A Locomotive Driving Wheel and the Rail," WEAR, Vol.2, p. 468 (1958-59).

N.M. Belajef, "Application of the Theory of Hertz to the Calculation of the Local Stresses at the Point of Contact of a Wheel and a Rail," Vertnik inzhenerov i tekhikov. n2, (1917) in Russian.

J.B. Bidwell, Proc. Symposium on Rolling Contact Phenomena, Elsevier Publ. Co., N.Y., (1962).

F. Birmann, "Gleisbeanspruchung und Fahrzeuglauf bei Schnellfahrten" ("Track Stresses and Vehicle Motion at High Running Speeds") Eisenbahntechnische Rundschau (ETR), 14, Nr.8, 335-351 (1965).

F. Birmann, et al., "Serial Measurements on Rails to Determine Wheel Lateral Forces, Railway Technical Review, pp.155-164, May (1966).

F. Birmann and J. Eisenmann, "Reihenmessungen an der Schiene zur Ermittlung der Fuhrungskräfte" ("Series Measurement of Rail Guide Forces") Eisenbahntechnische Rundschau (ETR), 15, Nr.5, 155-164 (1966).

J. Boussinesq, "Application des potentials a l'etude de equilibre et du movement des solides elastiques, Gauthier-Villars, Paris (1885) (also see Comptes Rendus, Vol.86, (1878), p. 1260).

B.S. Cain, Discussion of paper by Poritsky [1950], Journal of Applied Mechanics, Trans. ASME, 72, p. 465, (1950).

F.W. Carter, "On the Action of a Locomotive Driving Wheel," Proc.Roy.Soc., A 112, p.151-157 (1926).

C. Cattaneo, "Rendiconti", Accademia dei Lincei, Series 6, Vol. 27, pp.342-348, 434-436, 474-478, (1938).

A.H. Collins, "Recent Research on Adhesion," Railway Engineering Journal (Institution of Mechanical Engineers, London) Vol.1, No.5, pp.19-28 (1972).

T.F. Conry and A. Seirig, "A Mathematical Programming Method for Design of Elastic Bodies in Contact," Trans. ASME, SER.E, Journal of Applied Mechanics, Vol. 38, pp.387-392 (1971).

K. Dang Van, Doctoral Thesis presented to Faculty of Sciences of Paris (1971) Ref. from O.R.E. [1972].

J. Eisenmann, "Stresses Acting on Rail--Recent Findings," International Railway Congress Association--Monthly Bulletin, Vol.44, No.7, pp.537-550, July (1967).

J. Eisenmann, "Scheinenkopfbeanspruchung-Vergleich Zwischen Theorie und Praxis," Eisenbahntechnische Rundschau (ETR), 355-361, Oct.(1967).

J. Eisenmann, "Stress Distribution in the Permanent Way Due to Heavy Axle Loads and High Speeds," AREA paper 71-622-3, pp.24-59 (1951). (American Railroad Engineering Association).

J. Eisenmann, "Loading of the Rail Regarded as a Beam," J. Railway Technical Review, pp.306-312, Aug. (1969).

F.R.A. "Railroad Research Bulletin," U.S. Dept. of Transp., Fed.Rail.Admin., Wash., D.C., Rept.No. FRA-ORD & D -74-34, Vol.1, No.1, Spring (1974).

F.R.A. "Published Research Reports Federal Railroad Administration," Rept. No.FRA-ORD & D-74-23, F.R.A. Office of Research, Development and Demonstrations, Washington, D.C., Oct.(1973).

H. Fessler and E. Ollerton, "Contact Stresses in Toroids Under Radial Loads," Brit.J. of Applied Physics, Vol.8, pp.387-393 (1957).

G.L. Fowler, The Car Wheel, Schoen Steel Wheel Co., Pittsburgh, Pa. (1907)

M.M. Frocht, "A Three Dimensional Photoelastic Study of Contact Stresses in the Head of a Model of Railroad Rail," Proc. of Society for Experimental Stress Analysis, Vol.14.N.1, (1956).

M.M. Frocht, and B.C. Wang, "A Three Dimensional Photoelastic Study of Interior Stresses in the Head of a Railroad Rail in the Region under a Wheel," ASME Proc. 4th U.S. Cong. of Appl. Mech., pp.603-609 (1962).

H. Fromm, "Zulässege Belastung von Reibungsgetrieben mit zylindrischen oder kegligen Rädern," Z.V.D.I. Vol.73, p.1031 (1929).

L.A. Galin, Contact Problems in the Theory of Elasticity, Gostekhizdat (1953) Transl. from Russian by H. Moss, Ed. by I. N. Sneddon, Dept. of Math., North Carolina State College (1961).

F.E. Gifford and R.T. Yoshino, "Plasma Treatment of Railway Rails," Trans. ASME, Vol.93, Ser.B, J. of Engineering for Industry, 867-874 (1971).

J.N. Goodier, "Mathematical Theory of Equilibrium Cracks," Fracture, An Advanced Treatise, Vol.II, Ed. by H. Liebowitz, Academic Press, N.Y., 1-66 (1968).

L.E. Goodman and L.M. Keer, "The Contact Stress Problem for an Elastic Sphere Indenting an Elastic Cavity," Int.J.Solids and Structures, 1, 407-415 (1965).

F. Grashof, Theorie der Elasticitat und Festigkeit, Berlin, pp.49-54 (1878).

A.A. Griffith, "The Phenomena of Rupture and Flow in Solids," Phil.Trans. Roy. Soc.London, Ser.A 221, 163-198 (1921).

J.A. Greenwood and J.H. Tripp, "The Elastic Contact of Rough Spheres," J.Appl. Mechanics, Trans.ASME, 89, 153-159 (1967).

J.A. Greenwood and J.B.P. Williamson, "The Contact of Nominally Flat Surfaces," Proc.Roy.Soc. (London), Vol.295, Ser. A, p.300 (1966).

S.G. Guins and C.E. Tack, Anthology of Rail Vehicle Dynamics, Vol.III - Axles, Wheels, and Rail-Wheel Interactions, A.S.M.E., N.Y. (1973).

D.J. Haines, "Contact Stresses in Flat Elliptical Contact Surfaces which Support Radial and Shearing Forces during Rolling," Proc.Lubrication Convention, Proc. Inst. Mech.Engrs. (Lond.), Vol.179, Pt.3F, (1964-65).

D.J. Haines and E. Ollerton, "Contact Stress Distributions on Elliptical Contact Surfaces Subject to Radial and Tangential Forces," Proc.Inst.Mech. Engrs. (Lond.), 177, 95-114 (1963).

A.N. Hanna, "Theoretische und experimentelle Untersuchung der Spannungen im Innern des Schienenkopfes," Mitt. des Inst. für Eisenbahnbau und Strassenbau der Technischen Hochschule München, Heft 10, (146 pp.) (1967).

H. Hertz, "On the Contact of Elastic Solids," Transl. from J. für die reine und angewandte Mathematik, 92, 156-171 (1881), in H. Hertz Miscellaneous Papers, MacMillan, pp.146-162 (1896).

H. Hertz, The Principles of Mechanics, Presented in a New Form, MacMillan (1900), reprinted by Dover Publ., N.Y. (1956).

A.E.W. Hobbs, "A Survey of Creep," B.R. Research Dept. Report, DYN/52, B.R. Research Dept., London Road, Derby, England (1967).

P.G. Hodge, Jr., Plastic Analysis of Structure, McGraw-Hill, N.Y. (1959).

G.W. Housner and D.E. Hudson, Applied Mechanics Statics, D.Van Nostrand, Princeton, N.J. (1949).

Institution of Mechanical Engineers, ADHESION, Proc.Inst.Mech.Engrs. Vol.178, Part 3 E, 1963-64.

J.R. Jennings, "Relation of Wheel Tread Wear and Brake Shoe Wear," ASME Paper 61-WA-217 (1961), Reprinted in Guins and Tack, pp.91-101 [1973].

K.L. Johnson, "Effect of Spin upon the Rolling Motion of an Elastic Sphere on a Plane," Journal of Applied Mechanics, Vol.25, Trans. ASME, Vol.80, Ser. E, pp.332-338 (1958-a).

K.L. Johnson, "A Shakedown Limit in Rolling Contact," Proc. of U.S. National Cong. of Appl.Mech., ASME, N.Y. (1962).

K.L. Johnson, "A Review of the Theory of Rolling Contact Stresses," WEAR 9, 4-19 (1966).

K.L. Johnson, "Non-Hertzian Contact of Elastic Spheres," Proceedings of IUTAM Symposium on Contact Between Deformable Bodies, (To be published, 1975) Delft, Holland.

K.L. Johnson, "Rolling Resistance of a Rigid Cylinder on an Elastic, Plastic Surface," Int. J. Mech. Sci., 14, 145-148 (1972).

K.L. Johnson and J.A. Jefferis, "Plastic Flow and Residual Stress in Rolling and Sliding Contact," Proc. of the Inst. of Mechanical Engineers, Symposium on Fatigue (1963).

J.B. Johnson, "Discussion on Paper of C.L. Cradall on Friction Rollers," Trans. A.S.C.E., Vol.32, 270-277 (1894).

J.J. Kalker, "Rolling with Slip and Spin in the Presence of Dry Friction," WEAR 9, 20, pp.20-38 (1966).

J.J. Kalker, "On the Rolling of Two Elastic Bodies in the Presence of Dry Friction," Doctoral Thesis, Delft Technological Univ. (1967).

J.J. Kalker, and Y. Van Randen, "A Minimum Principle for Frictionless Elastic Contact with Application to the Non-Hertzian Half-Space Contact Problems," J. of Eng. Math., Vol.6, 193-206 (1972).

J.J. Kalker, "Aspects of Contact Mechanics," Proc. of IUTAM Symposium on Contact Between Deformable Bodies, (To be published, 1975) Delft, Holland.

A.D. Kerr, "The Stress and Stability Analyses of Railroad Tracks," Trans. ASME Series E, Journal of Applied Mechanics, 41, pp.841-848 (1974).

B.L. King, "New Type Profiles for British Railways, Railway Gazette, London, England, pp.60-64, Jan.19 (1968).

L.F. Koci, "Mechanical Design and Maintenance Affect Wheel/Rail Loading," Railway Locomotives and Cars, Simmons-Boardman Publishing Corporation, P.O. Box 350, Bristol, Connecticut, 06010, Vol.146, No.3, p.22, March (1972).

J.L. Koffman, "Heumann Type Profile Tests on British Railways," Railway Gazette, Vol. 121, pp.279-283, Apr.2 (1965).

J.L. Koffman, and D.L. Bartlett, "An Appreciation of the Practical Problems: A Survey of the Problems and Their Importance," Proc.Inst'n of Mechanical Engineers, Vol.180, Part 3 F, pp.1-14 (1965-66).

A.D. Konyukhov, V.A. Reikhart, and V.N. Kaportsev, "Comparison of Two Methods for Assessing Residual Stresses in Rails," Industrial Lab. Vol.39, No.1 (1973).

F.F. Ling, "On Asperity Distributions of Metallic Surfaces," Journal of Applied Physics, 29, 1168 (1958).

J.L. Lubkin, "Contact Problems," Handbook of Engineering Mechanics, Ed. by W. Flügge, McGraw-Hill, N.Y., 42.1-42.12 (1962).

G. Lundberg and F. K.G. Odqvist, "Studies on Stress Distributions in the Vicinity of the Contact Region between Bodies, with Applications" (in Swedish with English Summary), Ing. Vetenskaps Akad. Handl., 116, 64 pp., Stockholm (1932).

A.I. Lur'e, Three-Dimensional Problems of the Theory of Elasticity, Interscience Publishers, N.Y. (1964). (Translated from Russian text of 1955).

Marta, H.A. and K.D. Mels, "Wheel-Rail Adhesion," Trans. ASME, Ser.B, Journal of Engineering for Industry, Vol.91, pp.839-846, Discussion pp.839-854 (1969).

G.C. Martin and W.W. Hay, "The Influence of Wheel-Rail Contact Forces on the Formation of Rail Shells," ASME Paper 72-WA/RT-8, (1972).

R.D. Mindlin, "Compliance of Elastic Bodies in Contact", J. of Applied Mechanics Trans. ASME, 71, 259-268 (1949).

N.I. Muskhelishvili, Some Basic Problems of the Mathematical Theory of Elasticity, Transl. from Russian, Ed. by J.R.M. Radok, Noordhoff, Groningen-Holland (1953).

P.R. Nayak, "Random Process Model of Rough Surfaces," J. Lub. Tech., Trans. ASME, 93, Ser.F, 398 (1971).

R.P. Nayak, "Contact Vibrations of a Wheel on a Rail," Journal of Sound and Vibration, Vol.28, No.2, pp.277-293 (1973).

P.R. Nayak, S.Hariharan, R. Stern, R. Abilock, P.A. March, and P.K. Gupta, "Friction and Creep in Rolling Contact," (Bolt, Beranek and Newman, Inc.) U.S. Dept. of Transportation Rept.FRA-RT-71-64, 255 pp.(1970).

P.R. Nayak and I.L. Paul, "A New Theory of Rolling Contact," M.I.T. Cambridge, DOT Rept. PB179433 (1968).

E. Ollerton, "Stresses in the Contact Zone," Paper 10, Proc. Instn.Mech.Engr. Vol.178, Part 3E, pp.161-171 (1963-64).

O.R.E., "Residual Stresses in the Rail (Continued) Study of the Work-Hardened Zone," Rept., No.6, Office for Research and Expts. of the Int. Union of Railways, Utrecht, Oct. 1970 (Question C3, "Behaviour of the Metal of Rails and Wheels in the Contact Zone").

O.R.E., "Composition of the Different Stress Conditions in the Wheel/Rail Contact Zone - Development of a New Fatigue Criterion," Rept. No.7, Office for Research and Experiments of the International Union of Railways, Utrecht, April, (1972).

A.D. de Pater, "On the Reciprocal Pressure Between Two Elastic Bodies," Proc.Symp. Rolling Contact Phenomena, Ed. J.B. Bidwell, Elsevier Publ., Amsterdam, 132-136 (1962).

B. Paul, "Macroscopic Criteria for Plastic Flow and Brittle Fracture," in Fracture an Advanced Treatise, Vol.II, Ed. by H. Liebowitz, Academic press, N.Y., pp.315-496 (1968).

I.L. Paul, and P.R. Nayak, "Stress and Strain in Rolling Bodies in Contact," M.I.T. Report PB 173 651, Dept. of Mech.Eng., Mass.Inst. of Tech., November (1966).

A. Persson, "On the Stress Distribution of Cylindrical Bodies in Contact," (1964), Thesis for doctor's degree presented to Chalmers Institute of Technology, Gothenberg, Sweden.

H. Poritsky, "Stresses and Deflections of Cylindrical Bodies in Contact with Application to Contact of Gears and of Locomotive Wheels," Journal of Applied Mechanics, Trans. ASME, 72, 191-201 (1950).

R.H. Prause, H.C. Meacham, H.D. Harrison, T.G. Johns, and W.A. Gloeser, "Assessment of Design Tools and Criteria for Urban Rail Track Structures, Vol.I," Rept. No. UMTA-MA-06-0025-74-3, Dept. of Transportation, U.M.T.A., Office of R & D, Washington, D.C. July (1974).

R.H. Prause, H.C. Pestel and R.H. Melvin, "A Bibliography on the Design and Performance of Rail Track Structures," U.S. Dept. of Transportation, Rept. No. UMTA-MA-06-0025-74-7, D.O.T. Office of R & D, Washington, D.C., Sept. (1974).

O. Reynolds, "On Rolling Friction," Philosophical Transactions of the Royal Society, London, Vol.166, 155-174 (1876).

Y. Rocard, General Dynamics of Vibration, F. Ungar Publ. Co., N.Y., pp.287-290 (1960).

I. Ya. Shtaerman, "Local Deformation Resulting from the Pressure of Elastic Circular Cylinders, the Radii of Which Are Almost Equal," Dokl.Akad.Nauk. S.S.S.R., 29 (1949).

A. Sinclair, Development of the Locomotive Engine, Reprint of original edition of [1907], annotated by J.H. White, Jr., The M.I.T. Press, Cambridge, Mass. (1970).

K.P. Singh and B. Paul, "A Method for Solving Ill-Posed Integral Equations of the First Kind," Computer Methods in Applied Mechanics and Engineering, Vol.2, 339-348 (1973).

K.P. Singh and B. Paul, "Numerical Solution of Non-Hertzian Elastic Contact Problems," Trans. ASME, Ser.E, 41, Journal of Applied Mechanics, 96 484-490 (1974).

K.P. Singh and B. Paul, "Stress Concentration in Crowned Rollers," Transactions of ASME, Ser.B, Journal of Engineering for Industry. (To be published (c.1975).

K.P. Singh, B. Paul, and W. Woodward, "Contact Stresses for Multiply-Connected Regions-The Case of Pitted Spheres," Proc. of I.U.T.A.M. Symposium on Mechanics of Contact of Deformable Bodies, To be published (c.1975), Delft University Press, Netherlands.

W.O. Skeat, "Landmarks in Steam Locomotive Design," in Engineering Heritage, Vol.I, Inst'n. of Mech'l. Engineers, London, 80-88 (1963).

J.O. Smith and C.K. Liu, " Stresses Due to Tangential and Normal Loads on An Elastic Solid with Applications to Some Contact Stress Problems," ASME Journal of Applied Mechanics, Trans. ASME, 75, 157-165 (1953).

R.F. Stampfle, "Wheel, Axle, and Rail Stress Problems Related to Higher Capacity Cars-Effect on Rail," Presented at ASME Winter Annual Meeting (1963) reprinted in Guins and Tack, pp.273-278 [1973].

D. Tabor, "The Mechanism of Rolling Friction, II-The Elastic Range," Proc. Royal Soc., A, Vol.229, p.198 (1955).

D. Tabor, "Introductory Remarks," Symposium on Rolling Contact Phenomena, Ed. by J.B. Bidwell, Elsevier Publishing Co., N.Y. (1962).

H.R. Thomas and V.A. Hoersch, "Stresses Due to the Pressure of One Elastic Solid Upon Another," Eng.Exp.Sta.Univ.Ill., Bull. 212 (1930).

S. Timoshenko and J.N. Goodier, Theory of Elasticity, McGraw-Hill, N.Y. (1951).

H. Verbeeck, "Present Knowledge of Adhesion and Its Utilisation," Rail International (International Railway Congress Association, 17-21 rue de Louvrain, 1000 Brussels, Belgium) No.6, 34pp, (1973).

P.J. Vermeulen and K.L. Johnson, "Contact of Non-Spherical Elastic Bodies Transmitting Tangential Forces," Journal of Applied Mechanics, Vol. 31, p.338 (1964).

R. Wailes, "Richard Trevithick - Engineer and Adventurer," Engineering Heritage, Vol.I, Inst'n. of Mech'l. Engineers, London, 70-74 (1963).

J.M. Wandrisco and F.J. Dewez, Jr., "Study of the Defects that Originate and Develop in the Treads of Railroad Wheels During Service," ASME Paper 60-RR-1 (1960). Reprinted in Guins and Tack, pp.79-90 [1973].

D.J. Whitehouse and J.F. Archard, "The Properties of Random Surfaces of Significance in Their Contact," Proc. Roy.Soc.A, 316, 97 (1970).

E. Winkler, Die Lehre von der Elasticitat und Festigkeit, Vol.i, p.43, Prague (1867).

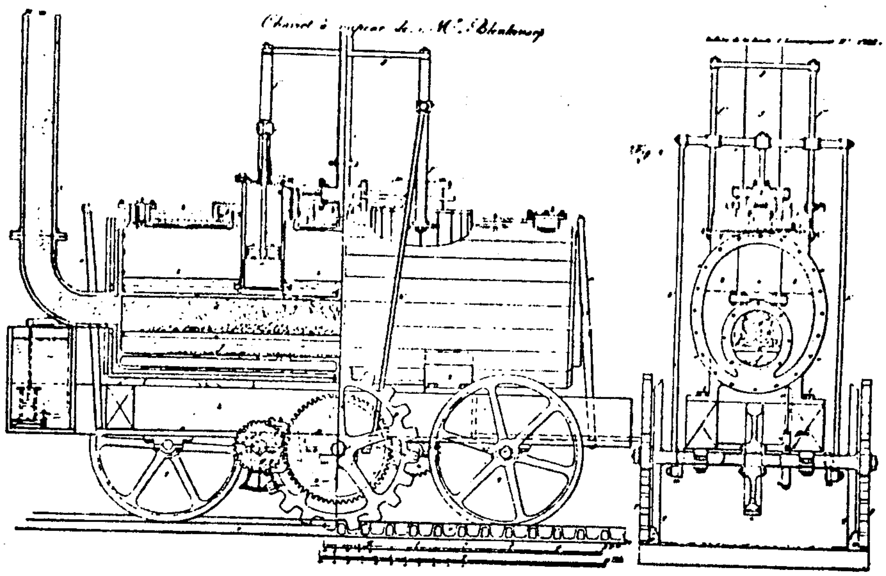

Fig. 1.1 Rack-railway locomotive designed by Murray for Blenkinsop in 1812. From Skeat [1963].

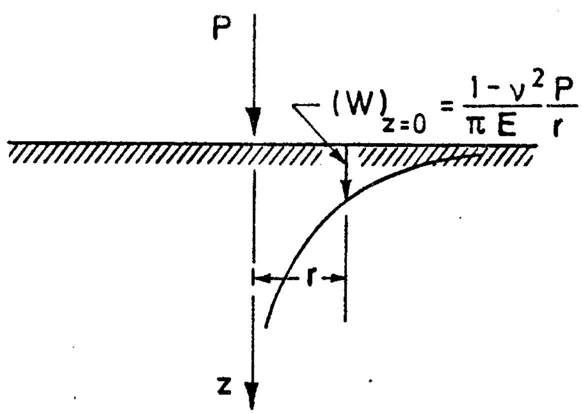

Fig. 1.2 Boussinesq's problem. A concentrated force P acting normal to the surface of an elastic halfspace

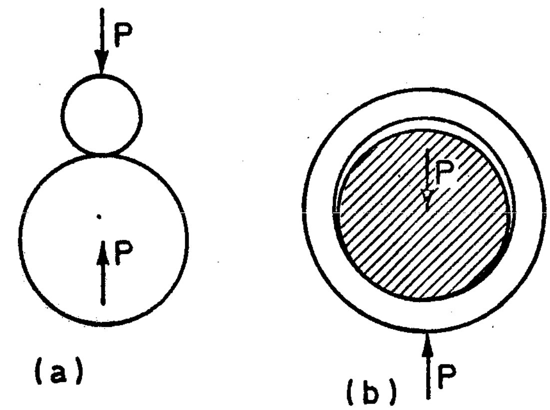

Fig. 1.3 Types of Contact Problems
(a) Counterformal (e.g. roller or ball bearings)
(b) Conformal (e.g. pin in a closely fitted hole)

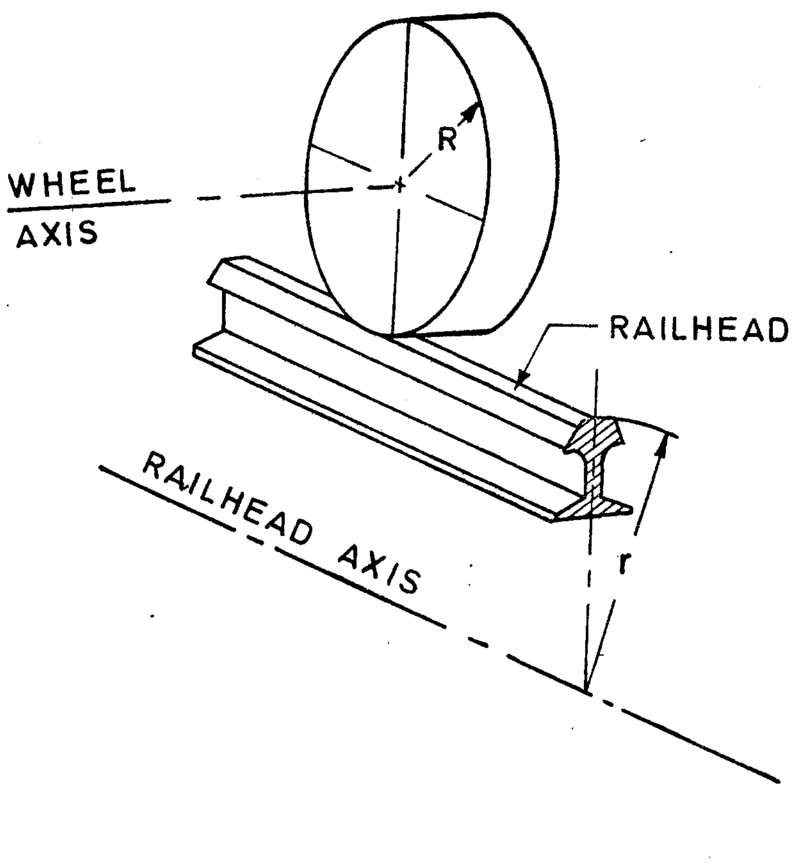

Fig. 1.4 Wheel and rail idealized as crossed cylinders

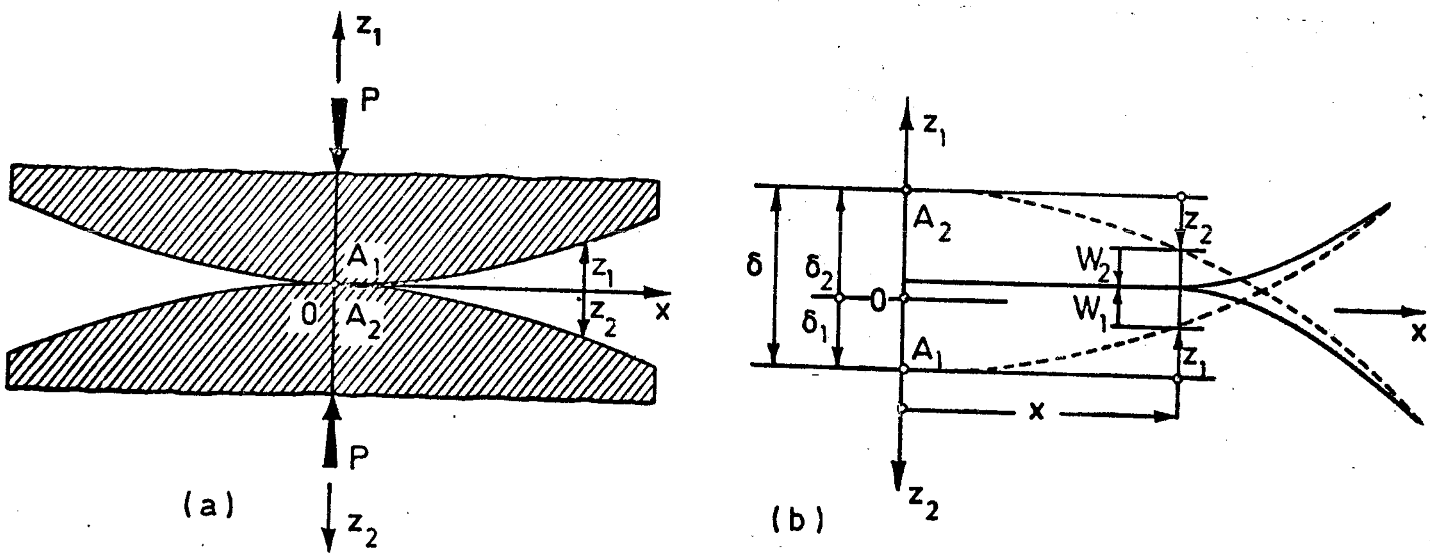

Fig. 2.1 Geometry of Contact
(a) Prior to deformation
(b) After deformation. Dotted curves show interpenetration without deformation of surfaces. Elastic displacements w_1 and w_2 restore compatibility as shown by solid curves.

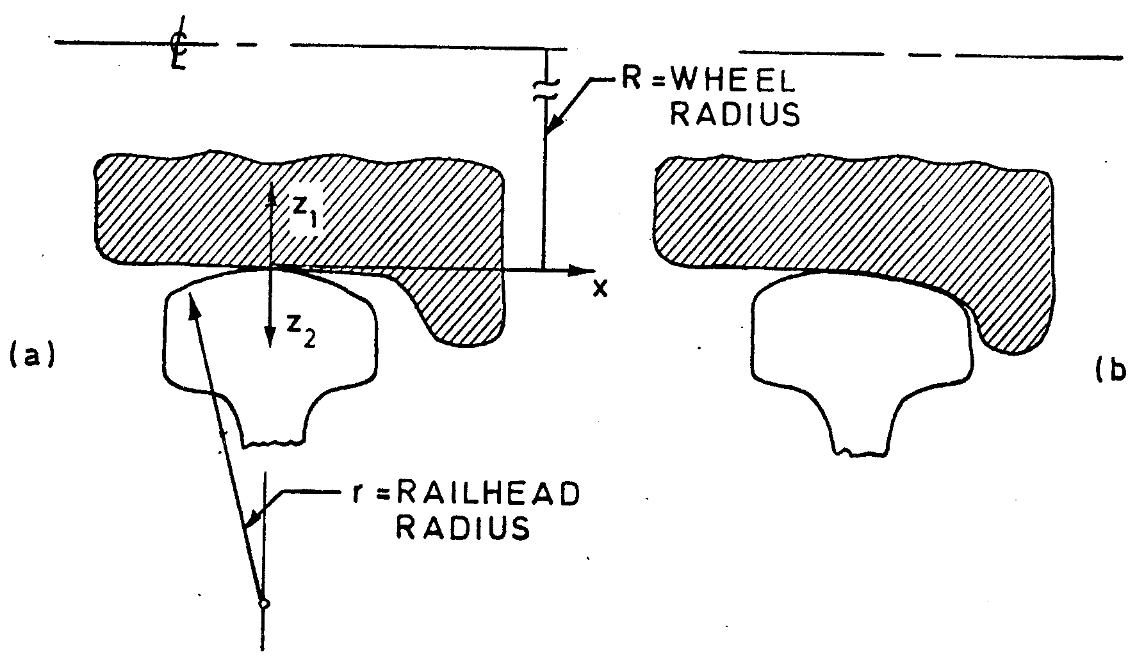

Fig. 3.1 Types of wheel-rail contact
(a) New wheel-counterformal contact
(b) Worn (or "preworn") wheel-conformal contact

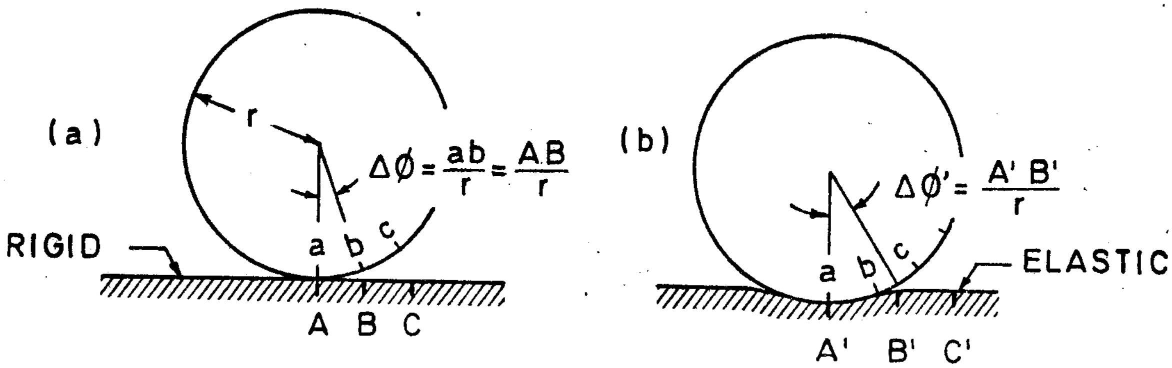

Fig. 4.1 Reynolds explanation of creepage for a rigid wheel on an elastic track
(a) rigid track
(b) elastic track

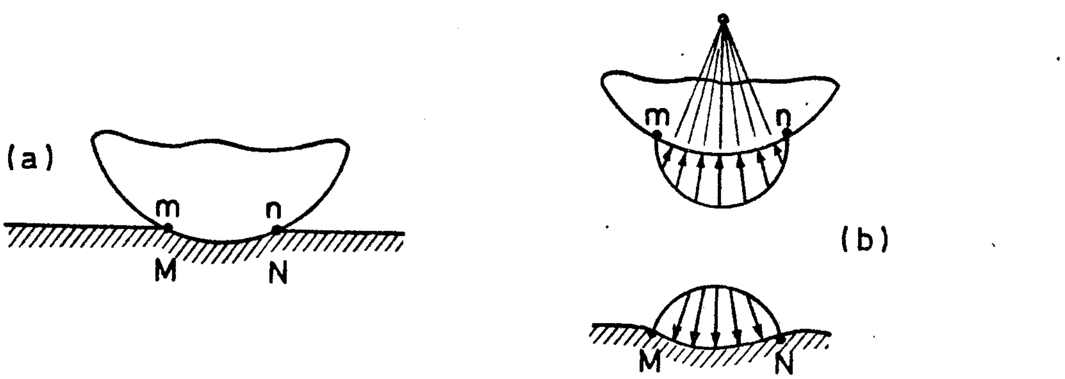

Fig. 4.2 Effect of wheel radius on creepage
(a) Wheel indentation creates concavity in track
(b) Showing contact pressure acting on convex wheel and concave portion of track

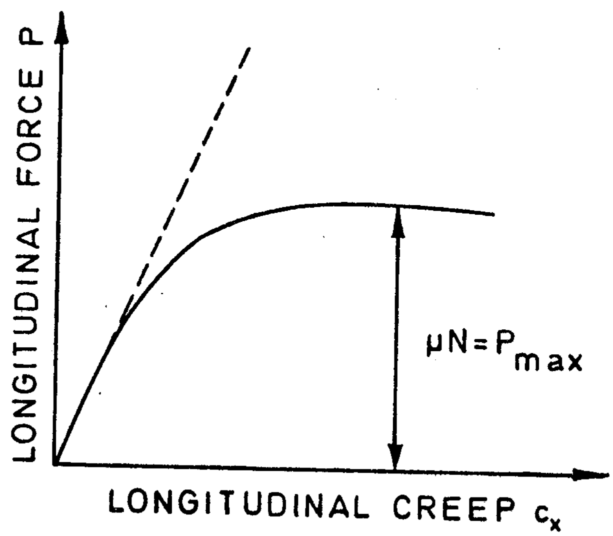

Fig. 4.3 Longitudinal force P vs. longitudinal creep c_x

(a)

(b)

(c)

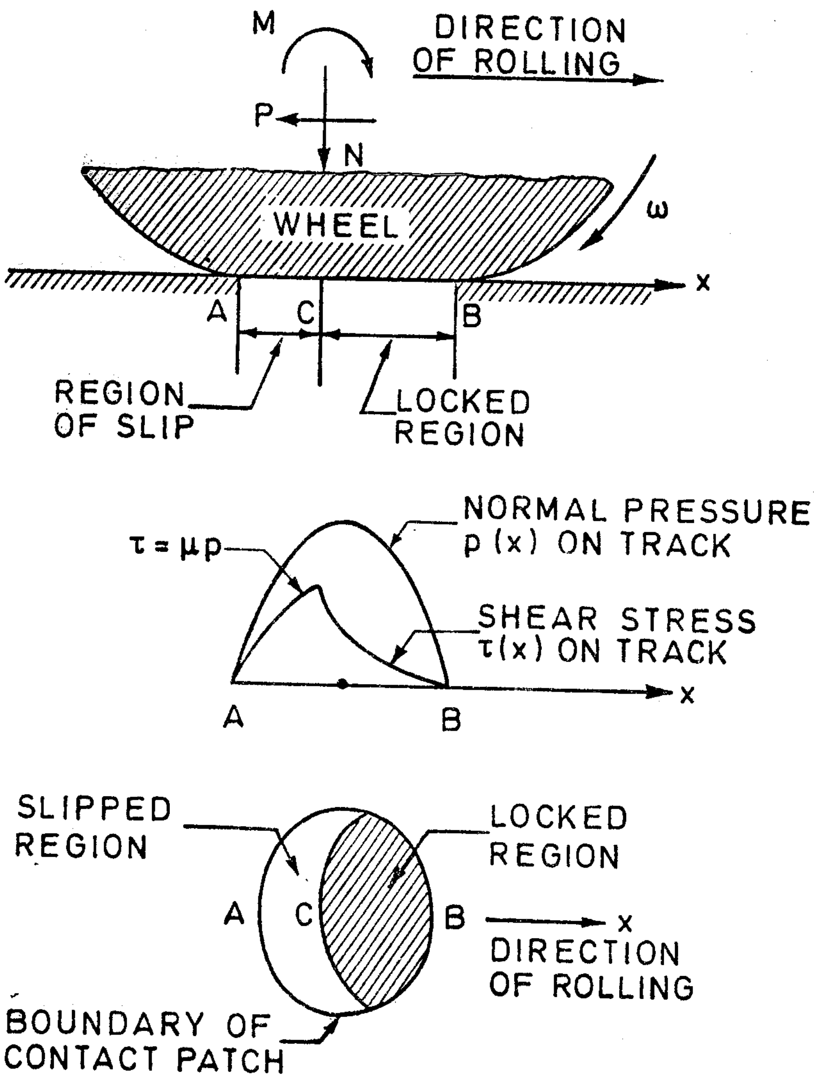

Fig. 4.4 Rolling with longitudinal force

(a) Showing normal force, N, torque M, longitudinal force P. Locked (adhesion) region CB adjoins leading edge of contact patch. Slipped region AC adjoins trailing edge.

(b) Hertzian contact pressure p(x), contact shear stress $\tau(x)$. In slipped region $|\tau| = \mu p$; in locked region $|\tau| < \mu p$; μ = coefficient of friction.

(c) Plan view of contact patch showing locked and sli-ped regions.

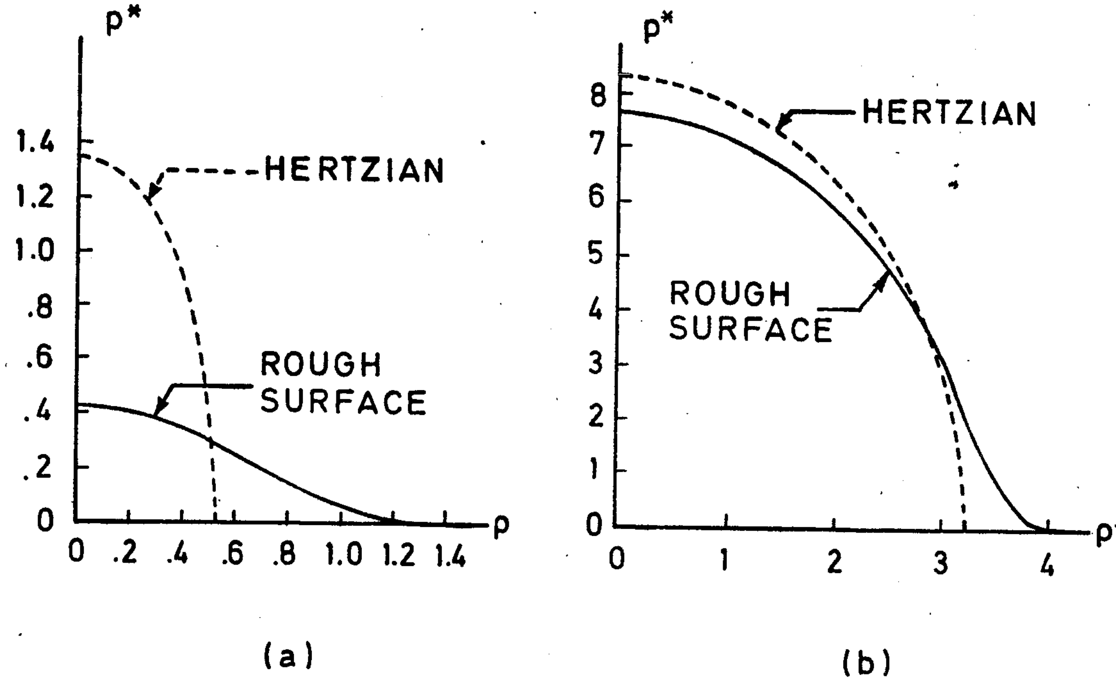

Fig. 5.1 Nondimensional contact pressure p* versus nondimensional radial coordinate ρ
After Greenwood and Tripp [1967]
(a) Light load
(b) Heavy load (note change in scales)