

**CORRELATION OF STATISTICAL
REPRESENTATIONS OF TRACK
GEOMETRY WITH PHYSICAL APPEARANCE**



Final Report

July 1978

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| 16. Abstract <p>Railway track has previously been represented as a random process with certain parameters that relate to the quality of the track. This report helps to make these parameters more meaningful by correlating the random process and its parameters with more familiar descriptions of the track. The correlation is done by presenting photographs and verbal descriptions of five selected types of track and then comparing this information with the statistical representations of the track.</p> <p>The recently developed method of statistically representing track is applied to the five track zones. Alignment and profile track geometry are processed to give space curves and mid-chord offsets for chords of 62 feet and one-half, one and two times the rail length. This information is presented for the five zones along with power spectral density representations that characterize both the stationary random track process and the random joint amplitude process. For the three zones with bolted rails, histograms of joint amplitudes are also provided.</p> <p>This report is a helpful companion to more analytical reports on the same subject because it correlates these mathematical treatments with more traditional track descriptions.</p> | | | | | |
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PREFACE

The Federal Railroad Administration (FRA) through the Office of Rail Safety Research, is investigating the impact of track geometry irregularities on the integrity of the rail transportation system. These deviations from design directly impact vehicle dynamic response, including derailment tendency, vehicle-induced degradation of the track, track maintenance procedures and the setting of meaningful track accuracy specifications. The goal of the FRA research is improved safety, performance and reliability of rail transportation consistent with reduced life cycle costs in maintaining the permanent way.

The ultimate success of this effort requires the cooperation of many diverse elements that contribute to rail transportation technology. This includes industry suppliers, designers of permanent way fixtures, track maintenance departments, and research organizations involved with track-vehicle dynamics studies. Each perceives the track differently: the track supervisor's immediate concern is with point defects in the track and with simple statistical indices that permit maintenance-of-way planning; the dynamicist desires detailed statistical descriptions that envelop all of the salient features involved in a generic description of the track.

The Transportation Systems Center (TSC) has been supporting the FRA research in relating track geometry to wheel-rail forces and derailment tendencies of vehicles. In particular, through a contract with ENSCO, Inc., DOT-TSC-1211, definitive models of track geometry power spectral densities (PSD's) were developed. By processing data from the FRA track survey cars, the PSD models are applied to real geometry data to produce parametric descriptions of track. These descriptions then form a powerful tool for studying all of the key items of vehicle dynamic response and track maintenance. In this report the parametric descriptions are correlated with more traditional photographic and verbal descriptions of the track.

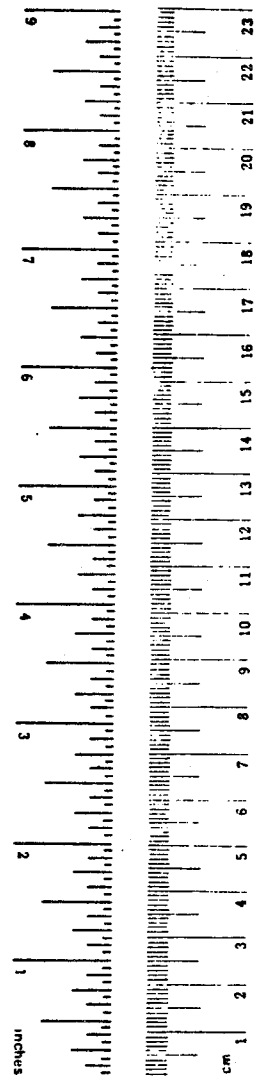
The author wishes to thank Messrs. William B. O'Sullivan and Jerry H. Sullivan of FRA for their comments which have contributed greatly to the clarity of the material contained herein. Appreciation is also extended to ENSCO coworkers Mr. Richard Sutermeister, Ms. Laura Simkins, and Dr. Preston Smith who assisted with computer programming, data processing and presentation of results.

METRIC CONVERSION FACTORS

Approximate Conversions to Metric Measures

| Symbol | When You Know | Multiply by | To Find | Symbol |
|----------------------------|---------------------------|----------------------------------|------------------------|-----------------|
| LENGTH | | | | |
| in | inches | *2.5 | centimeters | cm |
| ft | feet | 30 | centimeters | cm |
| yd | yards | 0.9 | meters | m |
| mi | miles | 1.6 | kilometers | km |
| AREA | | | | |
| in ² | square inches | 6.5 | square centimeters | cm ² |
| ft ² | square feet | 0.09 | square meters | m ² |
| yd ² | square yards | 0.8 | square meters | m ² |
| mi ² | square miles | 2.6 | square kilometers | km ² |
| | acres | 0.4 | hectares | ha |
| MASS (weight) | | | | |
| oz | ounces | 28 | grams | g |
| lb | pounds | 0.45 | kilograms | kg |
| | short tons (2000 lb) | 0.9 | tonnes | t |
| VOLUME | | | | |
| tsp | teaspoons | 5 | milliliters | ml |
| Tbsp | tablespoons | 15 | milliliters | ml |
| fl oz | fluid ounces | 30 | milliliters | ml |
| c | cups | 0.24 | liters | l |
| pt | pints | 0.47 | liters | l |
| qt | quarts | 0.95 | liters | l |
| gal | gallons | 3.8 | liters | l |
| ft ³ | cubic feet | 0.03 | cubic meters | m ³ |
| yd ³ | cubic yards | 0.76 | cubic meters | m ³ |
| TEMPERATURE (exact) | | | | |
| °F | Fahrenheit temperature | 5/9 (after subtracting 32) | Celsius temperature | °C |

*1 in = 2.54 (exact) (1). For other exact conversions and more detailed tables, see NBS Misc. Publ. 286, Units of Weights and Measures, Price \$2.25, SO Catalog No. C 13.10.286.



Approximate Conversions from Metric Measures

| Symbol | When You Know | Multiply by | To Find | Symbol |
|----------------------------|-----------------------------------|----------------------|---------------------------|-----------------|
| LENGTH | | | | |
| mm | millimeters | 0.04 | inches | in |
| cm | centimeters | 0.4 | inches | in |
| m | meters | 3.3 | feet | ft |
| m | meters | 1.1 | yards | yd |
| km | kilometers | 0.6 | miles | mi |
| AREA | | | | |
| cm ² | square centimeters | 0.16 | square inches | in ² |
| m ² | square meters | 1.2 | square yards | yd ² |
| km ² | square kilometers | 0.4 | square miles | mi ² |
| ha | hectares (10,000 m ²) | 2.5 | acres | |
| MASS (weight) | | | | |
| g | grams | 0.035 | ounces | oz |
| kg | kilograms | 2.2 | pounds | lb |
| t | tonnes (1000 kg) | 1.1 | short tons | |
| VOLUME | | | | |
| ml | milliliters | 0.03 | fluid ounces | fl oz |
| l | liters | 2.1 | pints | pt |
| l | liters | 1.06 | quarts | qt |
| l | liters | 0.26 | gallons | gal |
| m ³ | cubic meters | 35 | cubic feet | ft ³ |
| m ³ | cubic meters | 1.3 | cubic yards | yd ³ |
| TEMPERATURE (exact) | | | | |
| °C | Celsius temperature | 9/5 (then add 32) | Fahrenheit temperature | °F |

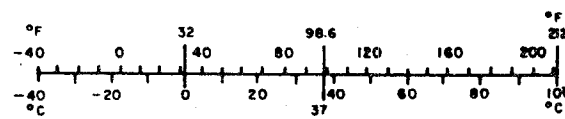


TABLE FOR METRIC CONVERSION OF PSD LEVELS

| <u>Given:</u> | To find: | |
|-------------------------|-------------------------|-------------------------|
| | in ² /cy/ft | cm ² /cy/m |
| | <u>Multiply by:</u> | |
| ft ² /cy/ft | 144. | 2.83 × 10 ² |
| in ² /cy/ft | 1.00 | 1.97 |
| in ² /cy/in | 8.33 × 10 ⁻² | 0.164 |
| ft ² /rad/ft | 9.05 × 10 ² | 1.78 × 10 ³ |
| in ² /rad/ft | 6.28 | 12.4 |
| in ² /rad/in | 0.524 | 1.03 |
| m ² /cy/m | 5.09 × 10 ³ | 1.00 × 10 ⁴ |
| cm ² /cy/m | 0.509 | 1.00 |
| cm ² /cy/cm | 5.09 × 10 ⁻³ | 1.00 × 10 ⁻² |
| m ² /rad/m | 3.20 × 10 ⁴ | 6.28 × 10 ⁴ |
| cm ² /rad/m | 3.20 | 6.28 |
| cm ² /rad/cm | 3.20 × 10 ⁻² | 6.28 × 10 ⁻² |

TABLE FOR METRIC CONVERSION OF SPATIAL FREQUENCY

| <u>Given:</u> | To find: | |
|---------------|-------------------------|-------------------------|
| | cy/ft | cy/m |
| | <u>Multiply by:</u> | |
| cy/ft | 1.00 | 3.28 |
| cy/in | 12.0 | 39.4 |
| rad/ft | 0.159 | 4.85 × 10 ⁻² |
| rad/in | 1.91 | 4.04 × 10 ⁻³ |
| cy/m | 0.305 | 1.00 |
| cy/cm | 30.5 | 1.00 × 10 ² |
| rad/m | 4.85 × 10 ⁻² | 0.159 |
| rad/cm | 4.85 | 15.9 |

TABLE FOR METRIC CONVERSION OF
ROUGHNESS PARAMETER UNITS

| <u>Given:</u> | To find: | |
|-------------------------|-------------------------|-------------------------|
| | in ² /cy/ft | cm ² -cy/m |
| ft ² -cy/ft | 144. | 3.05 × 10 ⁻³ |
| in ² -cy/ft | 1.00 | 21.2 |
| in ² -cy/in | 12.0 | 254. |
| ft ² -rad/ft | 22.9 | 485. |
| in ² -rad/ft | 0.159 | 3.37 |
| in ² -rad/in | 1.91 | 40.4 |
| m ² -cy/m | 472. | 1.00 × 10 ⁴ |
| cm ² -cy/m | 4.72 × 10 ⁻² | 1.00 |
| cm ² -cy/cm | 4.72 | 100. |
| m ² -rad/m | 75.2 | 1.59 × 10 ³ |
| cm ² -rad/m | 7.52 × 10 ⁻³ | 0.159 |
| cm ² -rad/cm | 0.752 | 15.9 |

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1. OBJECTIVE

As a part of contract DOT-TSC-1211 analytical models of track geometry were developed using power spectral densities (PSD's). The PSD's can be characterized by a few parameters, which provides a parametric representation of the track. The PSD models and the parameter lists are relatively new technologies. Accordingly, the recitation of these numerical values does not generate an instant image of the associated track structure in the mind's eye of the users. Nor is the user in a position to look at the track and develop a feeling for the parameter list that such track would produce.

The purpose of this report is to bridge this gap in understanding by displaying typical track structures and associated PSD's and parameter lists, so that these new analytical tools can receive the widest utilization possible.

2. TECHNICAL APPROACH

This report presents a composite evaluation of five carefully chosen track segments or zones. For each zone it includes: 1) a verbal description and tabular summary of the track, 2) a photograph of the geometric variations as seen through a telephoto lens, 3) track geometry representations that include zero phase shift space curves and multiple mid-chord offsets (MCO's), 4) model PSD's, and 5) the associated parameter lists.

The five zones were chosen according to the following criteria:

- 1) they should be within driving distance of Washington, D.C.;
- 2) they should be located on a cooperating railroad property;
- 3) there should exist recent track survey data for the zones, preferably collected by FRA track inspection car, T6; and
- 4) the data should be compatible with the statistical processing requirement.

An additional feature of this project was field visits to four of the sites by representatives of FRA, TSC, their contractors and other users. These visits were escorted by knowledgeable maintenance of way personnel from the participating railroad property.

2.1 VERBAL TRACK DESCRIPTIONS

The verbal commentary was derived by compiling personal observations, information obtained from representatives of the cooperating railroad properties, and notes on the reactions and interests of those who participated in the site visits.

2.2 PHOTOGRAPHIC TECHNIQUES

Long wavelength irregularities in surface and line are best displayed using the longest available focal length consistent with the method of camera support, available light, and depth of field considerations. For the photographic representations contained herein, a focal length of 300 mm was used on a 35mm camera.

A monopod support was used since there was the need to remove equipment quickly in the event of approaching traffic. Hence, the maximum exposure time was limited to 1/60 second. High Speed Ektachrome color film was used having an ASA rating of 125. This necessitated an aperture of between f/8 for hazy conditions and f/16 for sunny conditions. Consequently, the depth of field for good resolution is relatively shallow.

At 300 mm it was determined that surface variations are best captured from a position 4-8 feet on either side of the track. The position should be as low as possible consistent with the terrain, approximately 1-2 feet above the rail head.

Deviations in line on tangent track are best captured by positioning the camera directly over the rail head. For flat terrain and a 300 mm focal length, a height of 3-6 feet is recommended. Variations in line on curved track are very difficult to capture since surface variations produce apparent line variations. Over the short distance where there is near tangency between the line of sight and the curved rail, line detail is revealed well. Such photographs are useful for displaying alignment kinks at welds and in rigid track structures.

The 300 mm focal length was chosen because it accentuates rail deviations so that they can be associated with the deviations appearing on the track geometry plots. At 300 mm the deviations are accentuated well beyond what would be seen with the unaided eye, and the track appears to be in much poorer condition than it actually is.

The photographs in Figure 1 illustrate how much the deviations are accentuated. Figures 1a and 1b are for welded rail at 105 mm and 300 mm, respectively, and Figures 1c and 1d are the corresponding photographs for bolted rail. The 105 mm photographs show how the track actually appears. This focal length was chosen by taking a series of pictures at various focal lengths, printing them at the size they appear in this report, and visually comparing the prints with the track itself.

105 mm was found to give the most realistic portrayal of the track deviations.

Generally, short wavelength detail cannot be viewed by photographic techniques. However, on rare occasions, sunlight hits the polished surface of the railhead in a manner that reveals short wavelength profile variations. The right conditions were encountered on one field expedition enabling a photographic display of the pronounced short wavelength upward cusps that exist at welds. (See Figure 3c.)

An experiment was conducted to determine whether the presence of snow enhanced the photographic representation of track geometry but this technique proved to have no particular value. It was also observed that overcast days minimize the effects of heat shimmer and interference from shadows.



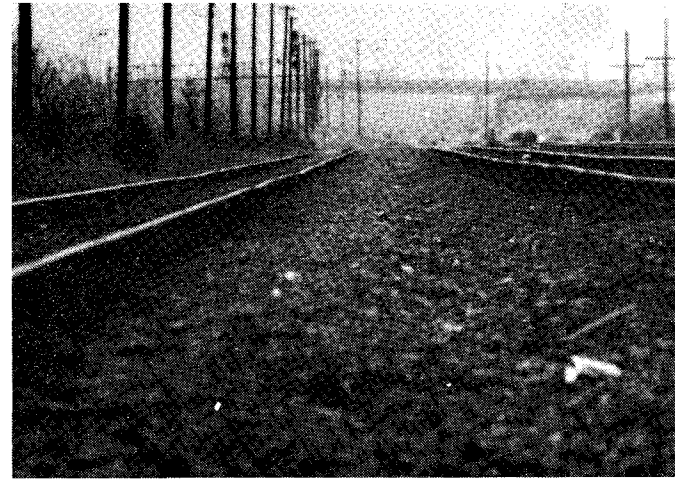
(a) Welded Rail at 105 mm



(b) Welded Rail at 300 mm



(c) Bolted Rail at 105 mm



(d) Bolted Rail at 300 mm

FIGURE 1. COMPARISON OF TRACK PHOTOGRAPHS AT DIFFERENT FOCAL LENGTHS

2.3 TRACK GEOMETRY REPRESENTATIONS

Processing of track geometry is performed to yield the following output in the form of distance records:

- Profile/Alignment (P/A) space curves
- P/A 62 foot MCO's*
- P/A $\left\{ \begin{array}{l} 2 \times \text{Rail length} \\ \text{Rail length} \\ \frac{1}{2} \times \text{Rail length} \end{array} \right\}$ MCO's*
- Crosslevel (Deviation from design)
- Superelevation
- Gage
- Curvature

Data flow for geometry and statistical processing is shown in Figure 2.

Geometry processing for this effort includes for the first time alignment space curve and simultaneous presentation of multiple chord alignment data.

Space curve is a relatively recent development by ENSCO, Inc., that yields phase distortionless and relatively speed independent representations of P/A from data collected by track inspection car T-6. Unlike chordal measurements, space curve is directly relatable to what is observed by a short absolute survey (100 feet), by an observer inspecting the track in the field, or by a photographic representation of the track.

*Recent data for one of the zones was available from a survey by FRA track inspection car T-2. T-2 data is not completely free of phase distortion, nor is alignment data available from this car.

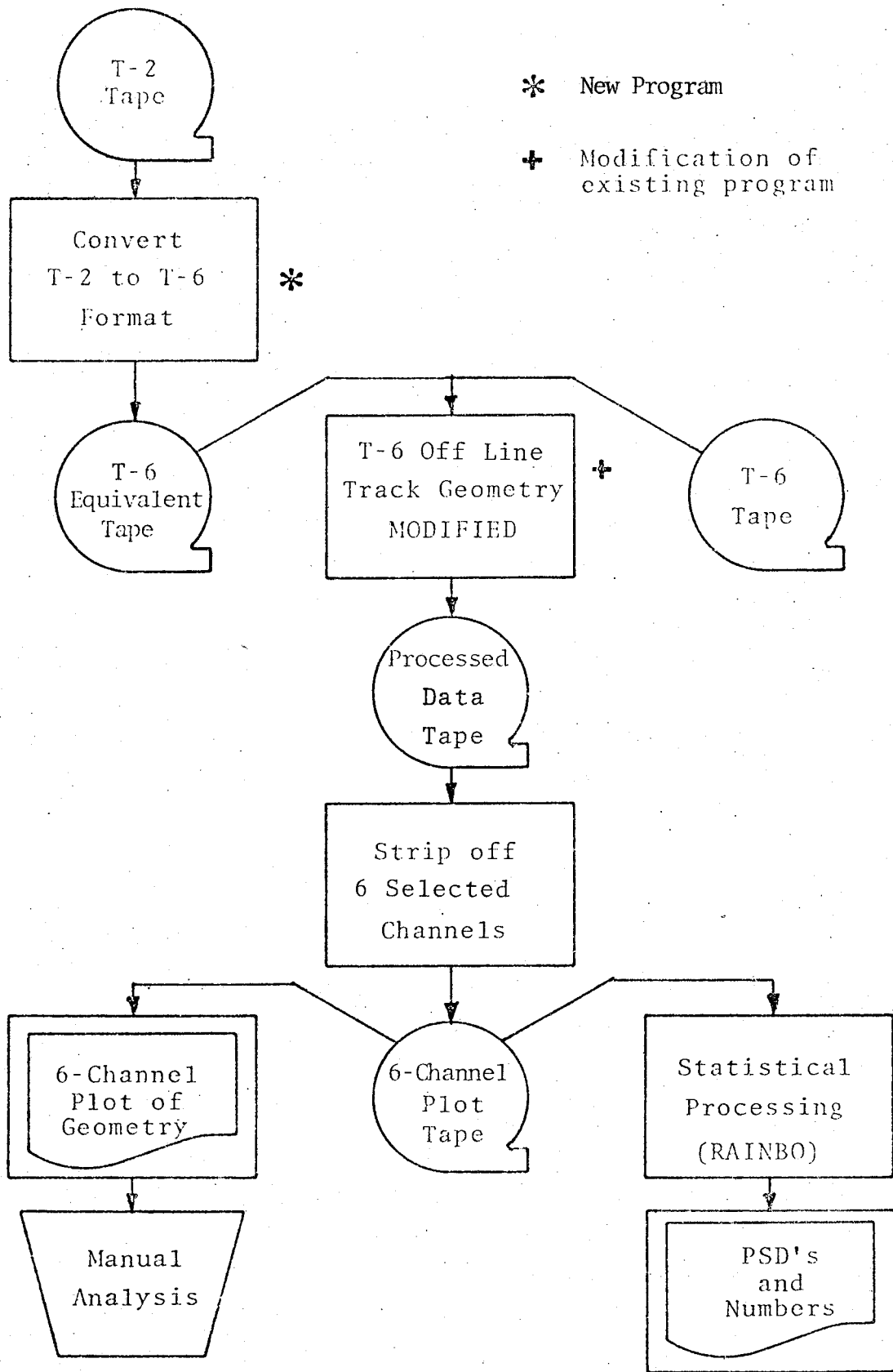


FIGURE 2. DATA PROCESSING FLOW

Space curve is itself a new concept that can be described in several different ways: 1) it represents the short wavelength deviations in geometry with the effects of terrain removed; 2) it is the deviation from a locally established mean line or curve; 3) it is the weighted sum of many individual MCO's centered at the measurement point where the longest of these MCO's is on the order of the longest wavelength of interest; or 4) it is implemented mathematically by means of the wavelength response as provided in Appendix A.

Track class was ascertained by determining the most severe geometric variation in the zone and comparing it with the current FRA Track Safety Standards¹. Linear interpolations are used to obtain fractional classifications of the track. *In all five zones track class was determined by geometry extrema at or near such anomalous events as grade crossings and turnouts.*

2.4 MODELING OF PSD's

PSD's were computed using geometry data from the homogeneous or anomaly-free sections of each zone. These were generated in the data processing stream as shown in Figure 2. PSD's were developed for the following geometry parameters: individual rail profile, mean profile, crosslevel, individual rail alignment, mean alignment, and gage.

Modeling of the PSD's follows procedures outlined in Corbin², and earlier in Corbin and Kaufman³. First the spectra are

¹Title 49, Code of Federal Regulations, Part 213.

²Corbin, J., "Statistical Representations of Track Geometry," Vol. I, Final Report, Contract DOT-TSC-1211, April 1979.

³Corbin, J. C. and Kaufman, W. M., "Classifying Track by Power Spectral Density," in "Mechanics of Transportation Suspension Systems," Winter Annual Meeting, ASME, Houston, December 1975.

separated into continuum and discrete portions where the latter are represented by prominent spectral peaks occurring at harmonically related frequencies. The continuum is then modeled by the spectral forms given in Reference 2, Tables 1 and 2 and illustrated in Reference 2, Figure 2. Spectral peaks lying near the continuum are corrected by the continuum PSD levels at the corresponding frequency.

2.5 DEVELOPING THE PARAMETER LIST

Once the PSD models are developed, most of the elements in the parameter list fall out directly. These include the overall PSD level of the continuum, and the continuum break frequencies.

In terms of statistical processes that have been identified in anomaly-free track, the continuum includes:

- The Stationary Random Process (SRP) which represents random irregularities that are uniformly present in the rail, and
- A periodic process that accounts for deviations that occur at joints, welds or rail straightening points. The non-zero mean of this process is called the Periodic Deterministic Process (PDP).

Because both processes produce a continuum, there is no reliable method to separate these processes using PSD's or other homogeneous processing techniques.

Using formulae described in Corbin² continuum parameters are processed into the variances of the measures currently defined

²Ibid.

by the FRA Track Safety Standards¹. These formulae are listed in Appendix B.

The spectral peaks are harmonically related and they are directly associated with the rail length PDP. Using peaks from individual rail P/A PSD's, and a curve fitting procedure described in Paragraph 2.3.2 of Corbin², a corner frequency and a PSD level is identified which characterizes the PDP.

Using formulae described in the reference, the PDP parameters are converted into joint-related constants and into the peak 62-foot MCO that results from the PDP. These formulae are listed in Appendix B.

¹Ibid.

²Ibid.

3. RESULTS

Five zones were used to correlate the physical characteristics of track with statistical representations of geometry. These zones satisfy all of the prerequisites set forth in Chapter 2. The physical characteristics of the zones are summarized in Table 1.

3.1 VERBAL DESCRIPTION OF THE TRACK ZONES

All five zones share the common element of 132 lb rail. Otherwise, they vary among themselves in application, construction and maintenance practice. Fortunately, their variations are sufficiently controlled that meaningful cross comparisons are possible. These controls are cited in Table 2.

3.1.1 ZONE 1

Zone 1 represents main line continuous welded rail (CWR) track whose traffic load is on the order of 13½ million gross tons (MGT) per year. This includes long (100-car-plus) mixed consists, unit coal trains, passenger trains, local way-freights, and branch line generated traffic. It is rated at 60 mph by the railroad property, a restriction imposed by limiting speed on curves. On the basis of peak geometry variations, its rating is Class 4.5.

The rail was rolled in 1973 and was installed in 1973-74. This permanent way is the object of orderly, programmed maintenance. This is evidenced by a clean, well-formed bed of #4 granite ballast having 18 inch full shoulders and full cribs. There is an excellent drainage pattern throughout the cut portions of the zone. There are an average of 22 ties per

TABLE 1. PRELIMINARY DESCRIPTION OF ZONES

| <u>Zone #</u> | <u>General Description</u> | <u>Length Miles*</u> | <u>Class by Geometry†</u> | <u>Tonnage MGT/year</u> | <u>Rail Length</u> | <u>Rail Marking</u> |
|---------------|---|----------------------|---------------------------|-------------------------|--------------------|-----------------------------|
| 1 | Main line, CWR, Tangent, Passenger and Freight | 1.00 | 4.5,P,At | 13.5 | 39' | 132 RE CC 1973 Tenneco |
| 2 | As above but five part Compound Curve | 0.64 | 4.5,P,At | 13.5 | 39' | 132 RE CC 1973 Tenneco |
| 3 | Main Line, Bolted, Tangent, Passenger and Freight | 0.90 | 3.5,P,W | 8.0 | 39' | 132 RE OH 1944 Steelton |
| 4 | Branch Line, Bolted, Tangent, Freight | 0.75 | 2.5,P,W | 1.2 | 28-36' | 132 RE CC 1957 Steelton |
| 5 | Main Line, Bolted, Tangent, Freight | 2.00 | 3.0, P | See Text | 39' | 132 RE OH 1943 Bethlehem |

*Length of the anomaly-free sections of the zones.

†Most severe deviations classifying zone are indicated as follows:
P - Profile, W - Warp, At - Alignment in tangent track.

TABLE 2. COMPARISON CONTROLS FOR THE FIVE ZONES

| <u>Difference Element</u> | <u>Constant Element</u> | <u>Zones to Compare</u> |
|---------------------------|--|-------------------------|
| Tangent vs. Curved | CWR, Main Line, Tonnage, Maintenance | 1 vs. 2 |
| CWR vs. Bolted | Tangent, Main Line, Tonnage, Maintenance | 1 vs. 3 |
| Main Line vs. Branch Line | Tangent, Bolted, Maintenance | 3 vs. 4 |
| Maintenance Practice | Tangent, Bolted, Main Line | 3 vs. 5 |

rail length, all in excellent condition with occasionally irregular and wide spacings between the ties.

The CWR is meticulously boxed on alternate ties using Woodings type anchors. Within 100 feet of a 575-foot multiple-span deck girder bridge in the zone, all ties are boxed. A Conley expansion joint at one end of the bridge protects against the buildup of longitudinal thermal stresses.

The rail rests on 14" x 7 $\frac{3}{4}$ " tie plates. Fastening to tie is accomplished using two rail-holding spikes per plate.

3.1.2 ZONE 2

Zone 2 is identical to Zone 1 in terms of construction, traffic and maintenance recited in Paragraph 3.1.1. Zone 2 differs

from Zone 1 in that it includes a multiple compound curve. It has a 75-foot single-span deck girder bridge at the east end of the curve and the spiral terminates at a dragging equipment detector. The west end spiral terminates under an interstate highway overpass followed by a #10 spring frog turnout leading to an industrial siding.

Through most of the zone, the ballast and ties evidence the same intensive attention as found in Zone 1. Drainage is quite good everywhere except under the overpass and around the turnout where runoff is very restricted. Here the ballast is fouled and there is evidence of mud pumping.

As in Zone 1, every tie within 100 feet of the deck girder bridge is anchored. Throughout the curve, there are four spikes per tie plate, two rail holding and two plate holding.

Based on the railroad property track charts, the curve in this zone consists of five segments as follows:

1. 2°, 475 feet (east end);
2. 1°, next 900 feet;
3. 2°, next 1050 feet;
4. 3°, next 275 feet; and
5. 2°, 600 feet (west end).

According to this data, the total curve length is 3300 feet, the included angle is $59\frac{3}{4}^{\circ}$ and the average curvature is 1.8° .

Curvature track survey data indicates that there are some significant kinks in the curve. Also, the curve length is closer to 3450 feet long. If the average 1.8° feet curvature

is extrapolated over this length, the included angle becomes $62\frac{1}{2}^{\circ}$. This result is in very good agreement with Coast and Geodetic Survey maps of the curve.

Measured spiral rates in the curve are typically $1.1^{\circ}/100$ ft except the last part of the west and spiral, which tapers off at $2.2^{\circ}/100$ ft. This high spiral rate suggests the need to straighten the curve quickly to accommodate the overpass and a turnout in the adjoining tangent.

3.1.3 ZONE 3

Zone 3 is typical of low-intermediate speed main line bolted track. Traffic is on the order of 8 MGT/year which includes a mixture similar to that found on Zones 1 and 2. On the basis of geometry, it rates as Class 3.5.

The rail was rolled in 1944 and it was installed in this zone new. Maintenance intensity is comparable to Zones 1 and 2. The last out-of-face was in 1973-4 when new ties were installed at the rate of 900/mile. There are an average of 24 ties per rail length.

The track is supported by #4 granite ballast. The cribs are full and there are well-formed 18 inch shoulders. Drainage in the area is excellent and the ballast is exceptionally clean and well compacted.

The rail is joined using six-hole joint bars. It rests on $13" \times 8"$ tie plates and is longitudinally restrained using six Stead-type anchors per rail length. There are two spikes per tie plate, both rail holding.

In view of the bolted construction and the vintage of the rail, the existence of some rail end batter, chip-out, crushed heads and broken rails is not surprising. One particularly severe battered joint with chip-out was measured with an 18-inch straightedge. Results of this measurement indicated the following depressions: 0.104" away from the chip-out, and 0.30" in the chip-out. One of the most severe low joints in the zone was a broken rail repaired with joint bars whose bolts had worked loose.

3.1.4 ZONE 4

This zone is typical of a bolted rail branch line track. It is constructed of relay rail that was originally rolled in 1956-8 and installed as bolted rail on a heavy-tonnage main line. During the transfer, ends were cropped that had been bent or battered in service resulting in rail that is typically 28 to 36 feet long. Six-hole joint bars are used and only occasional loose bolts are to be found.

Current traffic is on the order of 1.2 MGT/year, up from 0.8 MGT/year two years ago. This tonnage consists of six trains per day, four of which are heavy loads of crushed stone. Since the prospects for increased traffic are quite good, the railroad property is considering the installation of relay CWR.

The zone exhibits a full ballast section with moderate to full cribs and 18-inch shoulders. There is evidence of mud pumping at some joints. Indeed, drainage is a problem on this line since it adheres closely to the original hill-and-dale terrain.

The rail is supported on 14" x 7 $\frac{3}{4}$ " tie plates. There are 20 ties per rail length, most of which are in good condition. The exceptions are crushed ties near pumping joints. Longitudinal restraint is provided by ten Gautier-type anchors per rail length.

Maintenance is quite intensive for a branch line. Recently, the high rail was turned to equalize the wear on a curve.

3.1.5 ZONE 5

This last zone of the survey is typical of a heavy-tonnage bolted main line track that has been the victim of deferred maintenance. This deferral was encouraged by the existence of two parallel CWR tracks whose quality is quite good and which were used for alternate routing of traffic. Therefore, this zone has experienced a gradual lightening of load over recent years. Its principal traffic now consists of way-freights and commuter trains which must use an outside track to discharge cars on sidings or passengers on platforms. Through freight trains use this track as a last resort when the density of high speed traffic using the parallel tracks demands it.

Speed restrictions in the zone are 10 mph for freight and 15 mph for passenger--a restriction imposed by a disturbed ballast condition. By geometry, the zone is Class 3.0. Extremes in deviations from zero crosslevel and in the profile MCO approach the Class 3 limits rather consistently.

The disturbed ballast condition resulted from a recent undercutting operation conducted prior to the track inspection by T-2. At the time of photographing of the zone, compaction had not taken place. The undercutting equipment was not able

to approach grade crossings any closer than 120 feet. There is a sudden profile jump and high ballast at these points.

The rail was rolled in 1943, whereupon it was installed in the zone. The rail is joined by six-hole head contact joint bars and rests on $14 \times 7\frac{3}{4}$ tie plates and 22 ties per rail length. It is restrained longitudinally by ten Improved Fair Rail anchors per rail length.

The track rests on non-compacted ballast with half to quarter full cribs. The ballast is muddy along the shoulders where it has not been undercut. The left or field rail is consistently 0.5" low throughout the zone.

The profile geometry is typical of an extreme service bent condition. Therefore, the zone can be expected to have a high level of PDP. Examination through high-power binoculars reveals little randomness of the SRP or of the random joint amplitude variety. Because of the adjacent high-speed track, very long wavelength terrain undulations are not present.

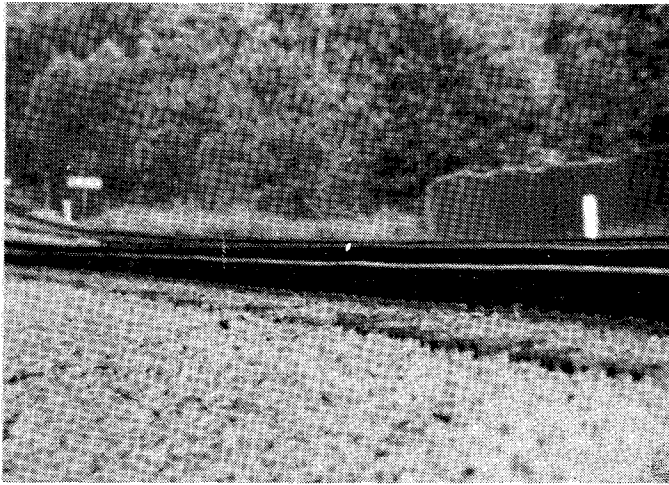
3.2 PHOTOGRAPHS AND GEOMETRY

Photographic and fully annotated geometric representations of the five zones are shown in Figures 3 through 35.

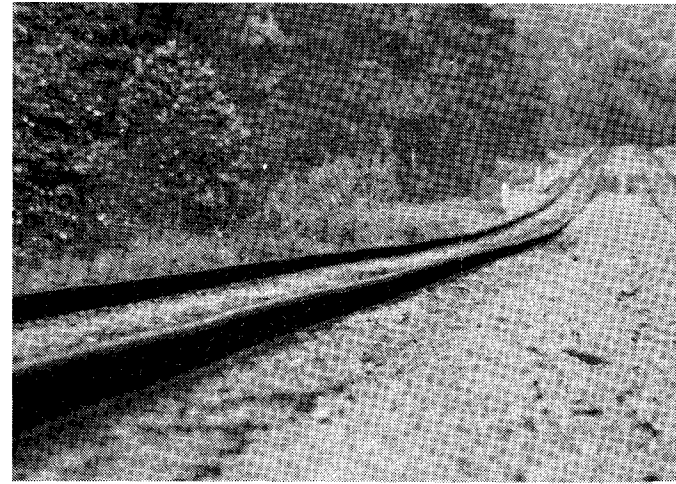
3.3 PSD MODELS

PSD models of surface for all five zones are shown in Figures 36, 37 and 38. In general, the roughness increased from Zone 1 through Zone 5.

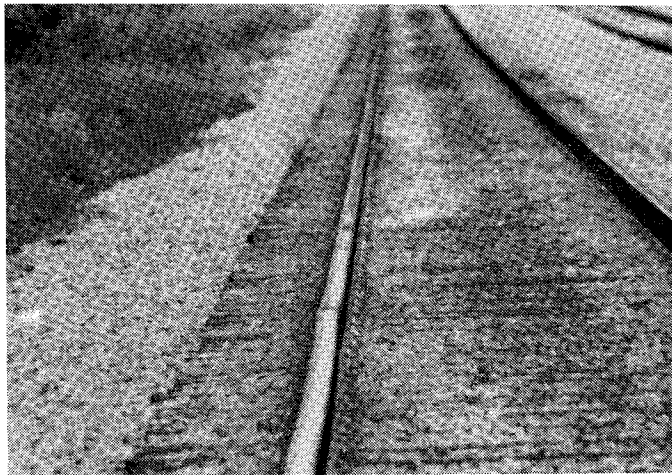
As mentioned in Paragraph 3.1.5, Zone 5 was expected to have a strong PDP and relatively little random content. This is



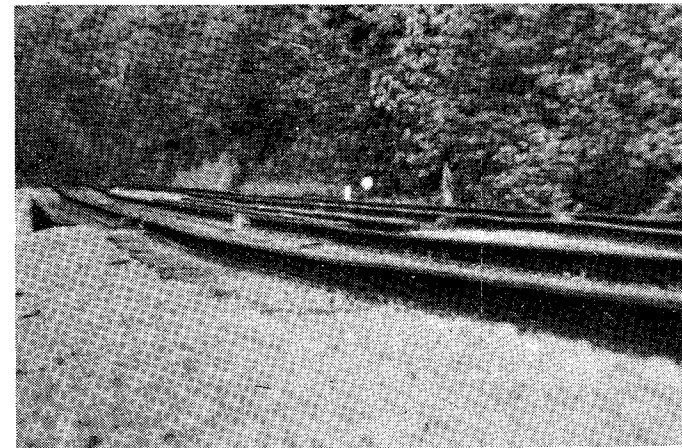
(a) Surface Geometry from Left Side,
Location A, Looking West



(b) Surface Geometry from Right Side,
Location A, Looking West



(c) Profile, Weld Cusp, Location B,
Looking West



(d) Surface, Bridge Approach,
Location C, Looking West

FIGURE 3. PHOTOGRAPHIC REPRESENTATION OF ZONE 1

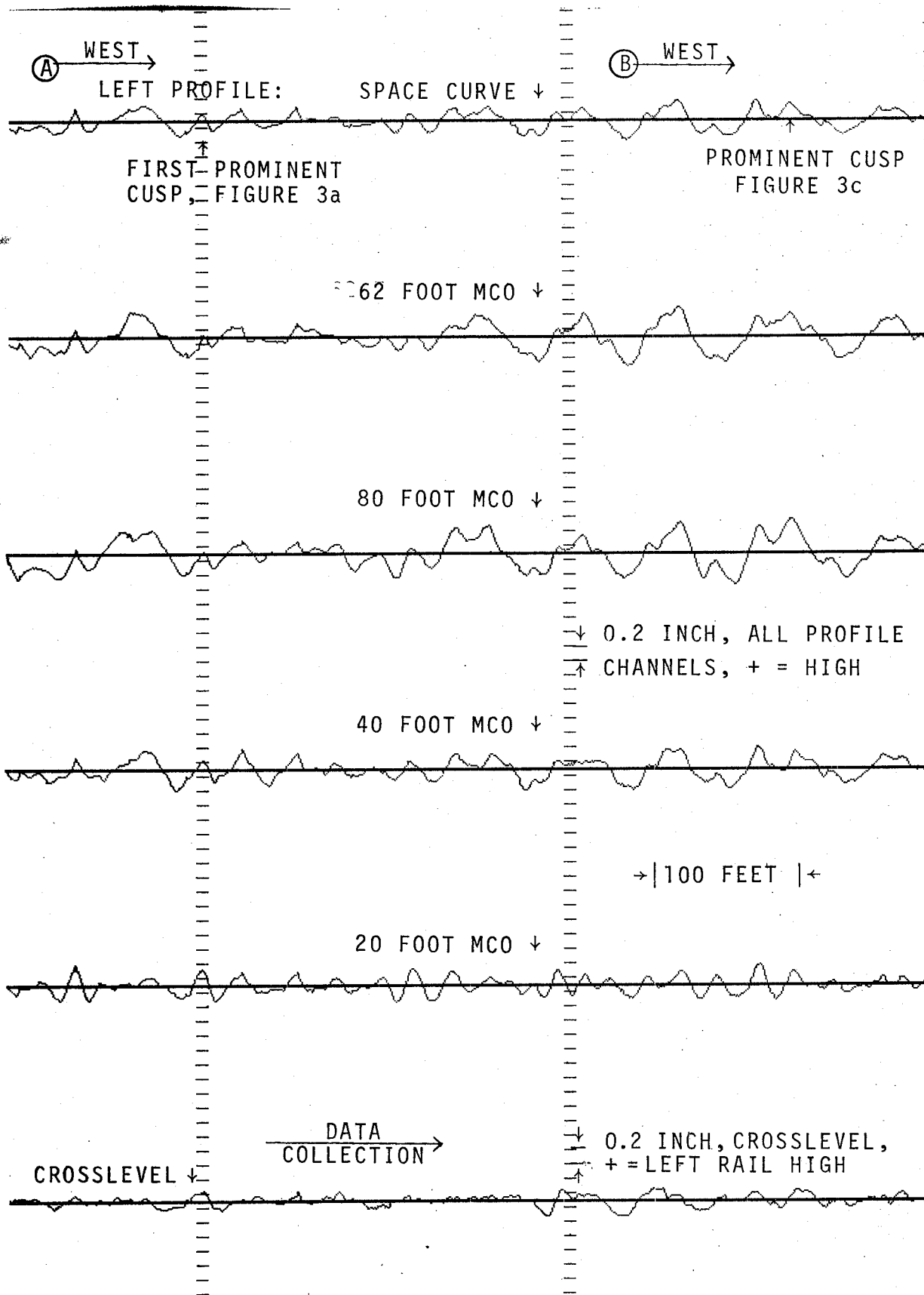


FIGURE 4. ZONE 1, LEFT SURFACE, LOCATIONS A AND B

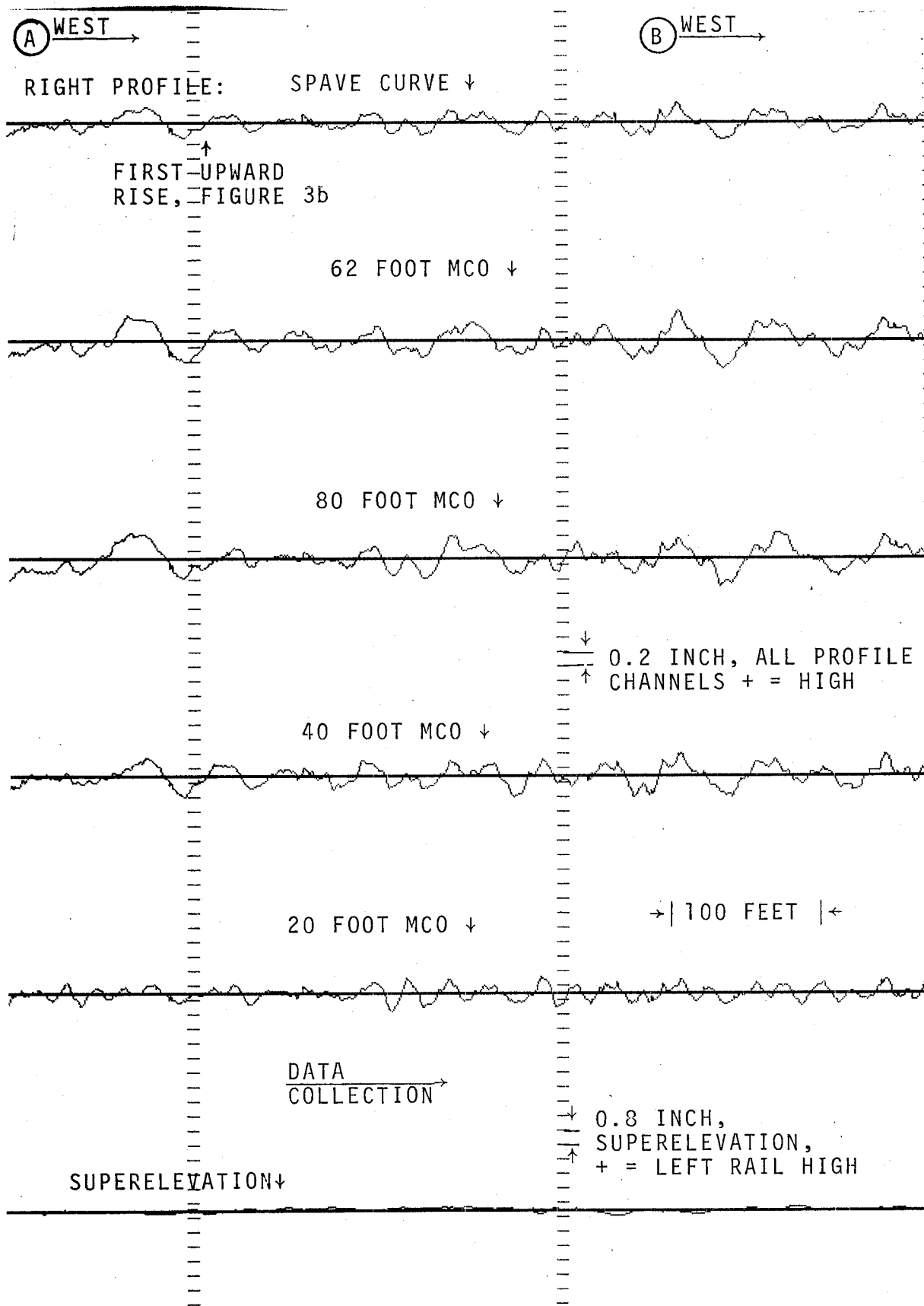


FIGURE 5. ZONE 1, RIGHT SURFACE, LOCATIONS A AND B

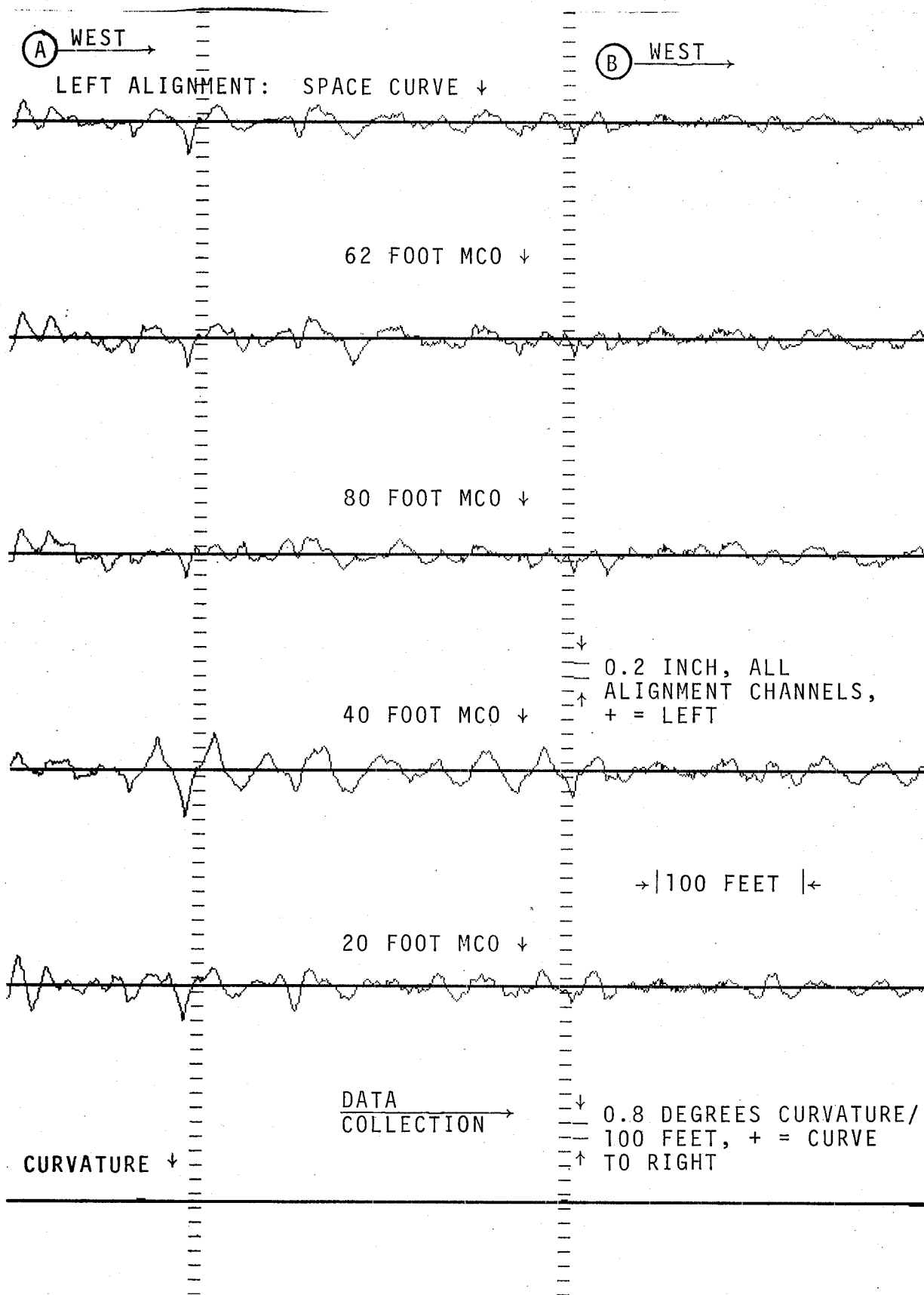


FIGURE 6. ZONE 1, LEFT LINE, LOCATIONS A AND B

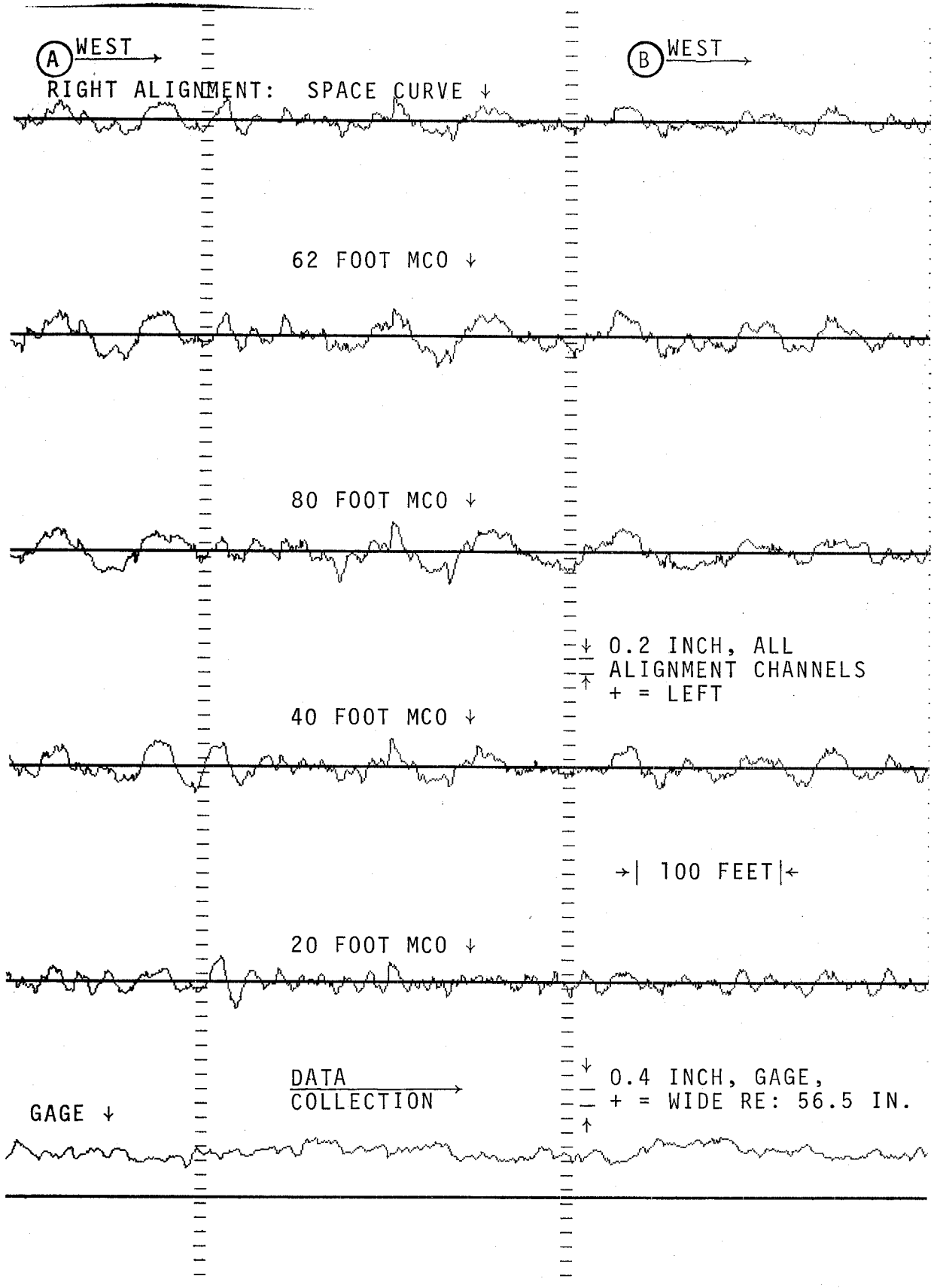


FIGURE 7. ZONE 1, RIGHT LINE, LOCATIONS A AND B

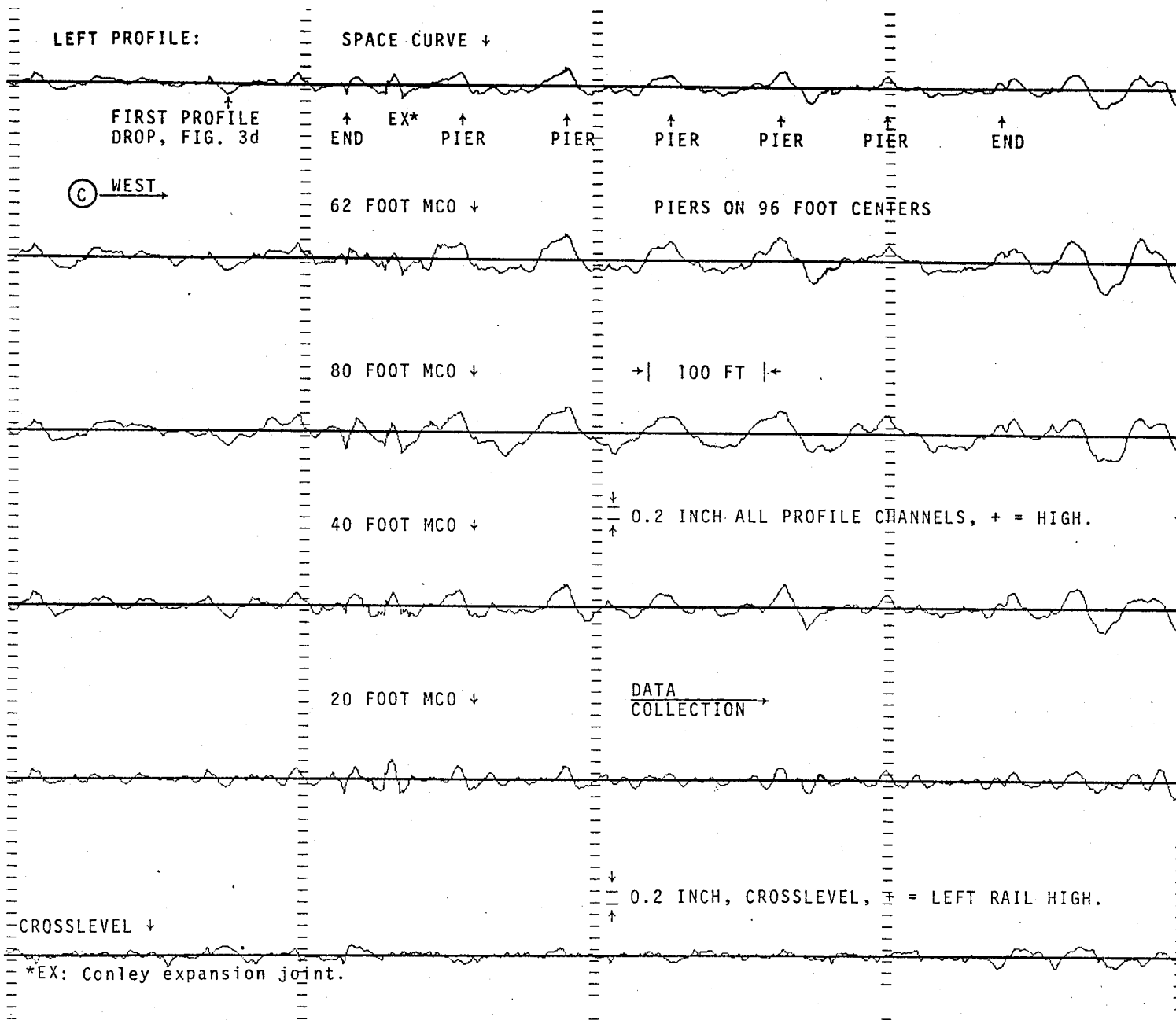
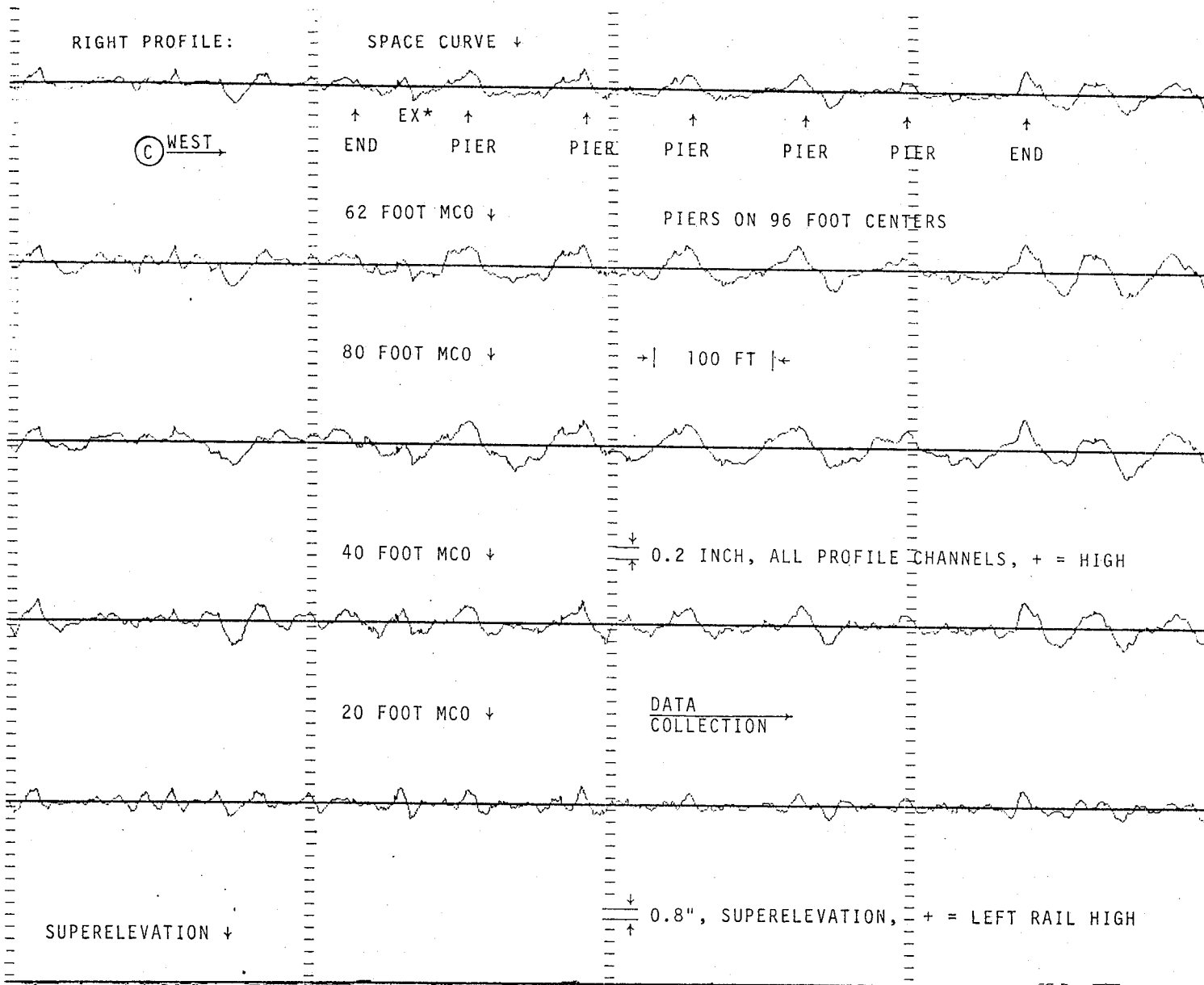
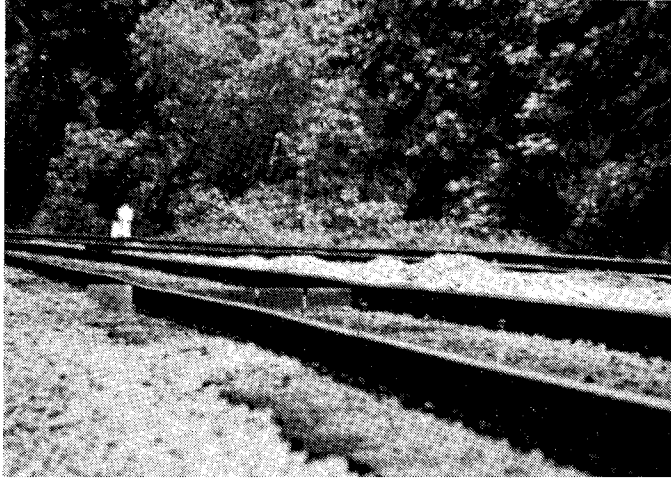


FIGURE 8. ZONE 1 BRIDGE, LEFT SURFACE, LOCATION C

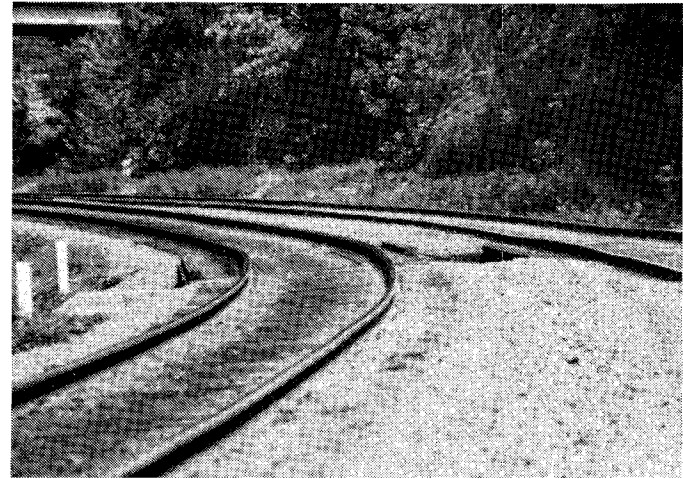


*EX: Conley expansion joint.

FIGURE 9. ZONE 1 BRIDGE, RIGHT SURFACE, LOCATION C



(a) Profile Dip at Dragging Equipment Detector, Location A, Looking West



(b) Alignment Kinks in Bridge, Location B, Looking West



(c) Alignment Kinks in Spiral, Location C, Looking West



(d) Alignment Kinks in Spiral Location D, Looking East

FIGURE 10. PHOTOGRAPHIC REPRESENTATION OF ZONE 2

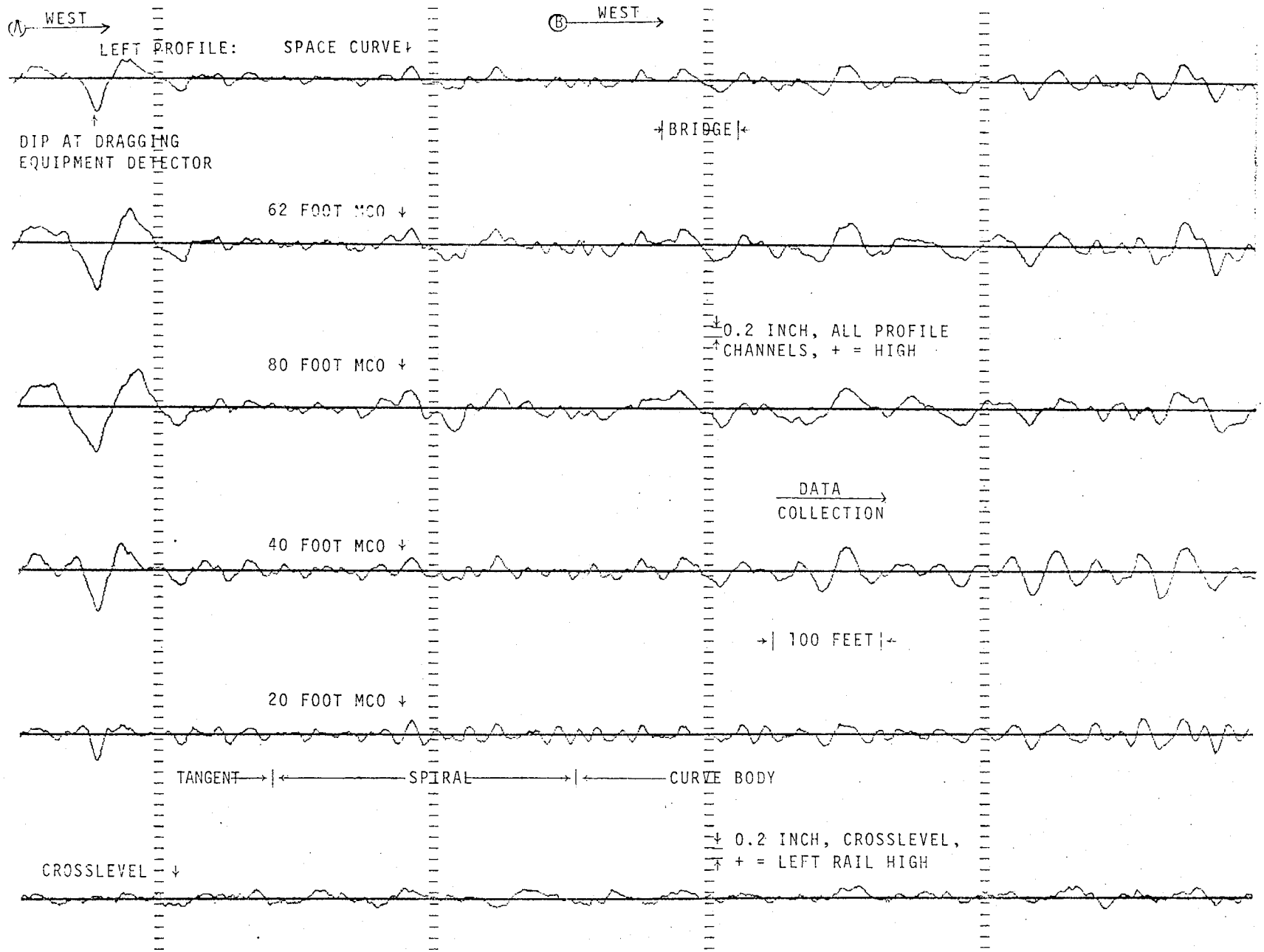


FIGURE 11. ZONE 2, LEFT SURFACE LOCATIONS A AND B

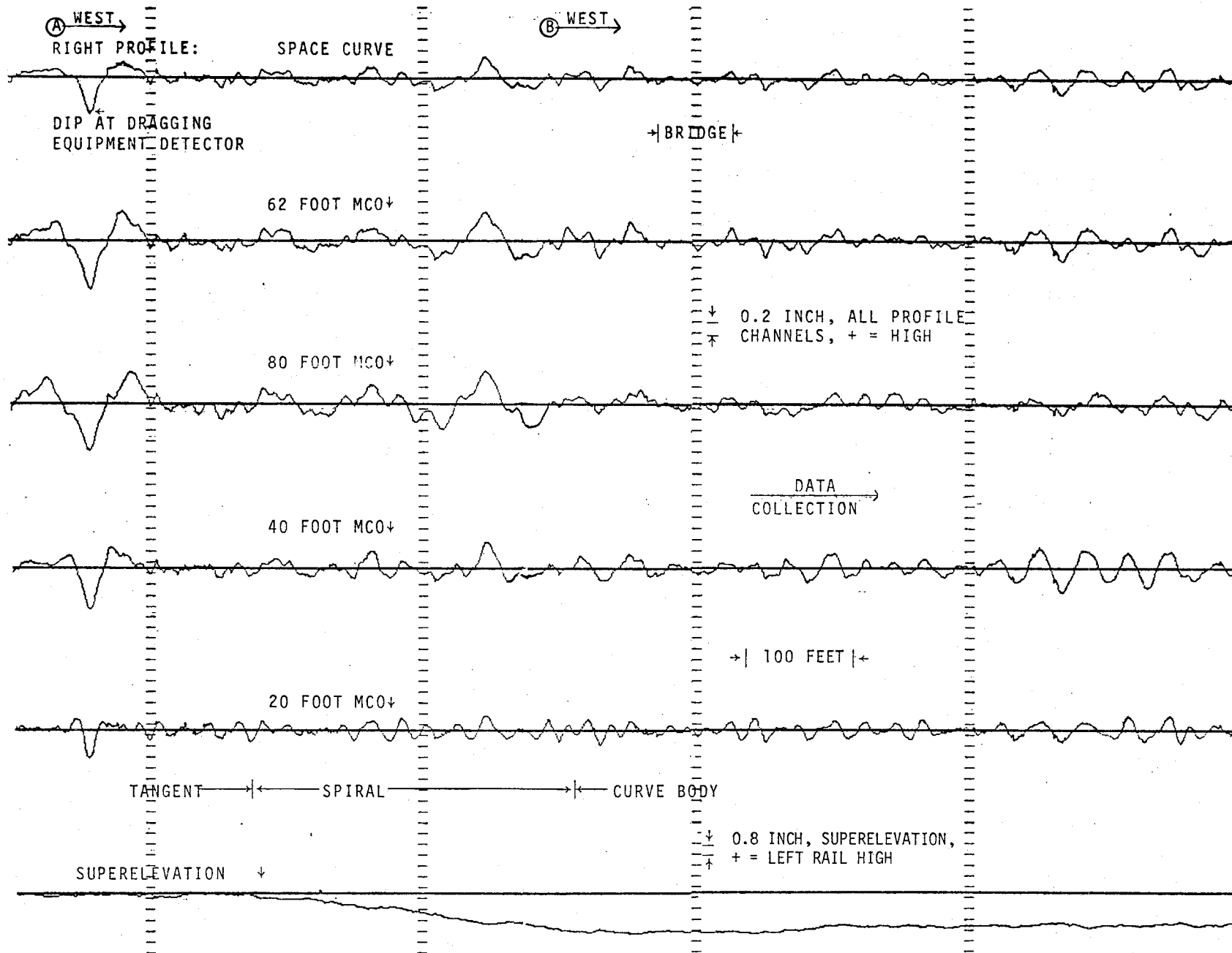


FIGURE 12. ZONE 2, RIGHT SURFACE, LOCATIONS A AND B

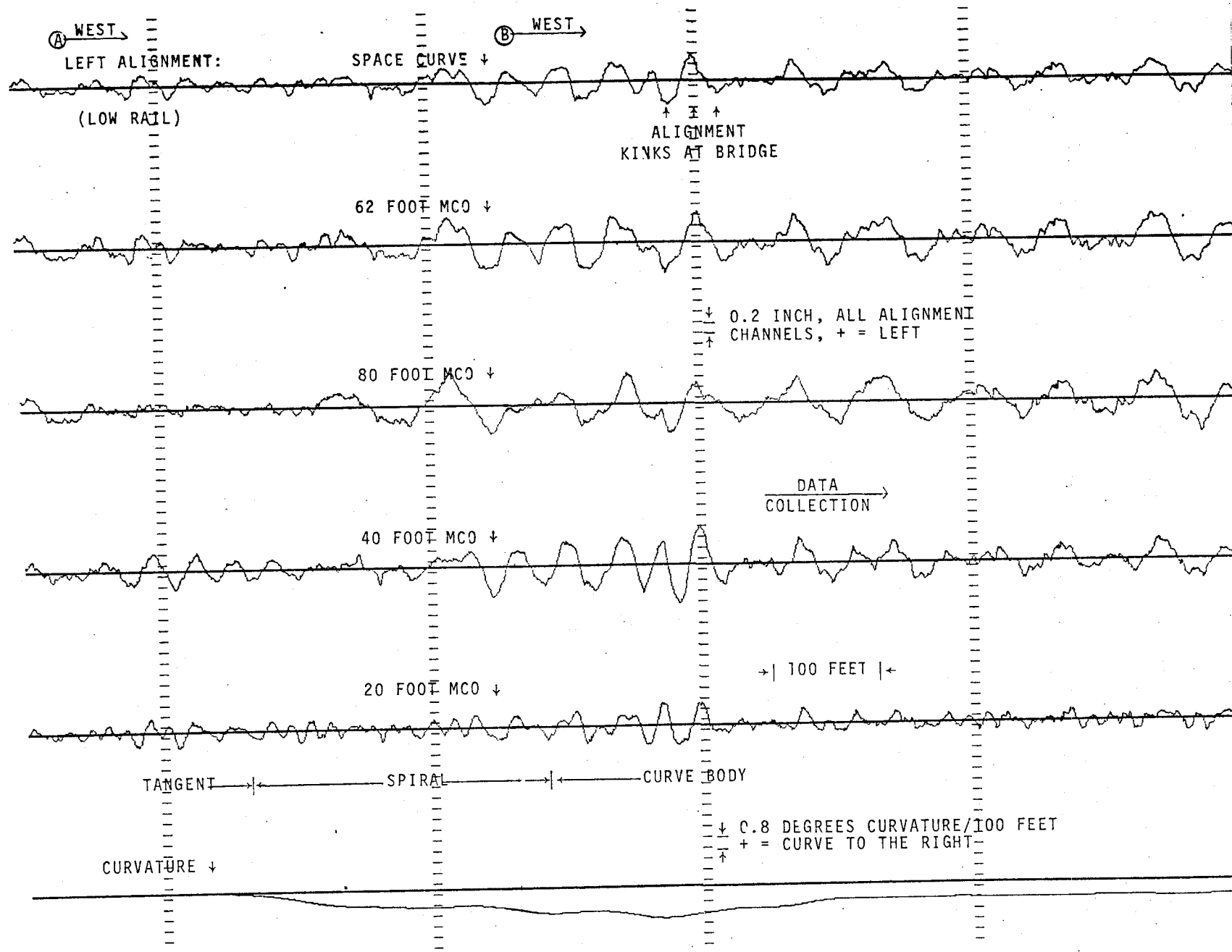


FIGURE 13. ZONE 2, LEFT LINE, LOCATIONS A AND B

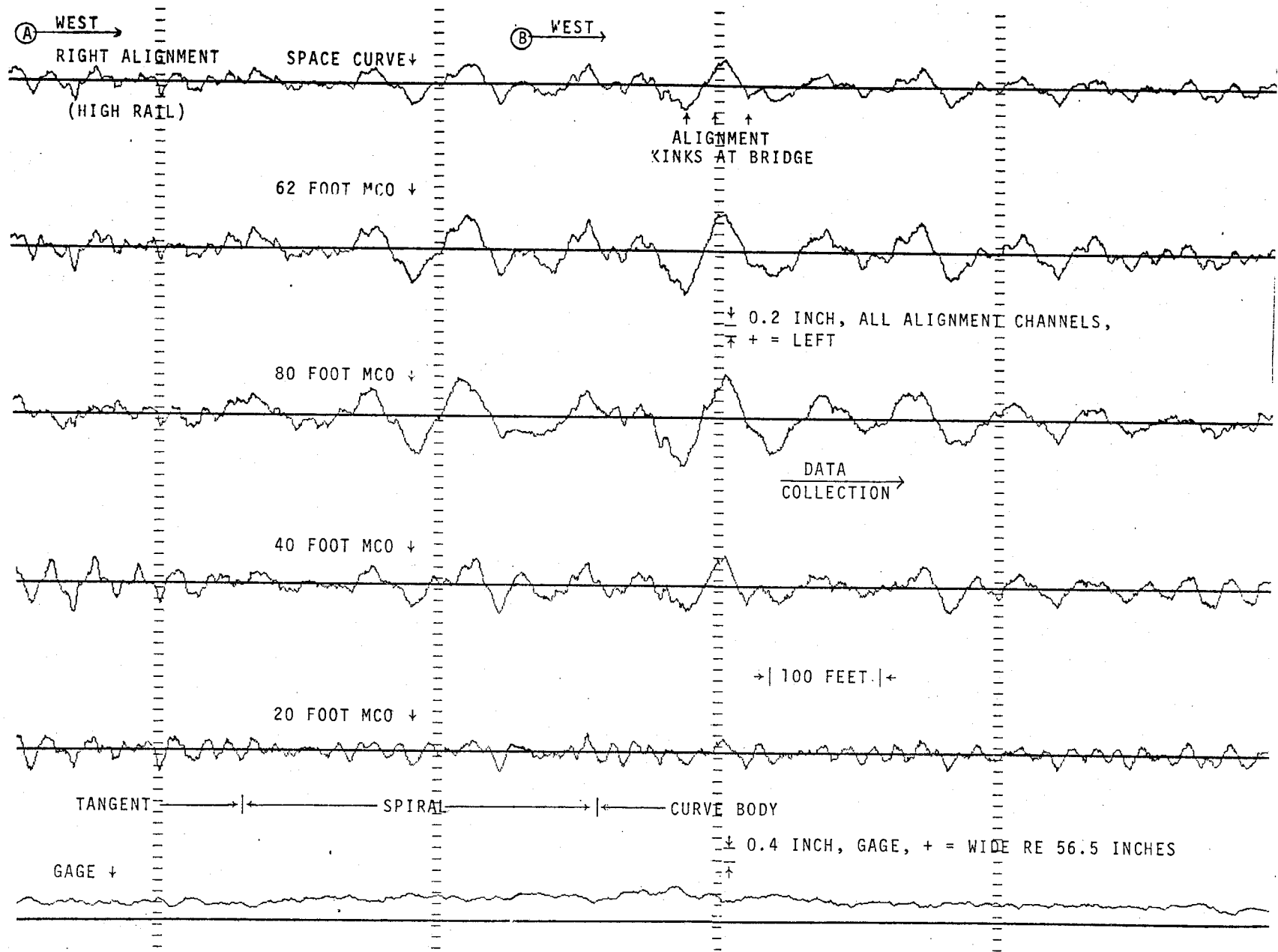


FIGURE 14. ZONE 2, RIGHT LINE, LOCATIONS A AND B

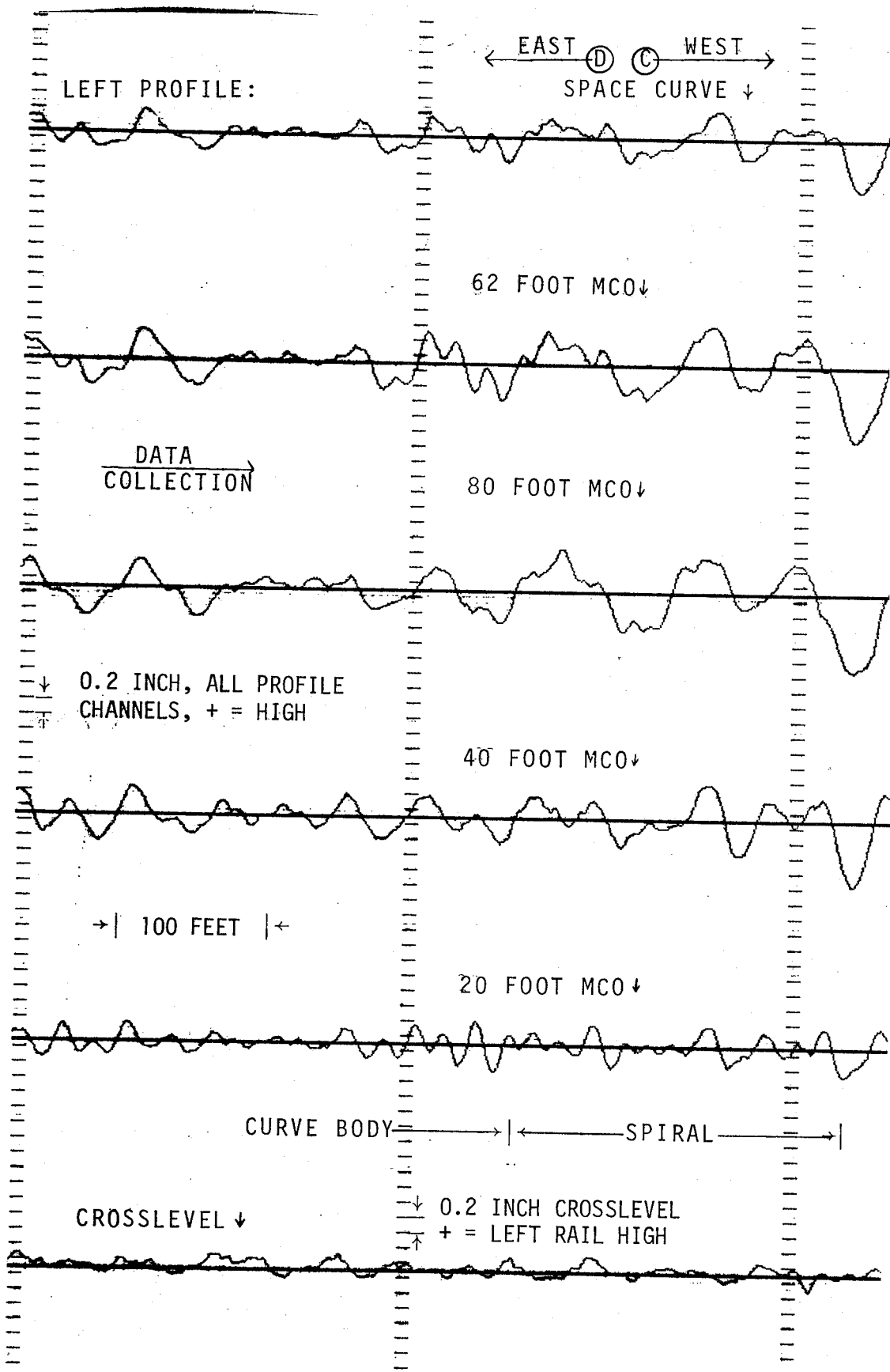


FIGURE 15. ZONE 2, LEFT SURFACE, LOCATIONS C AND D

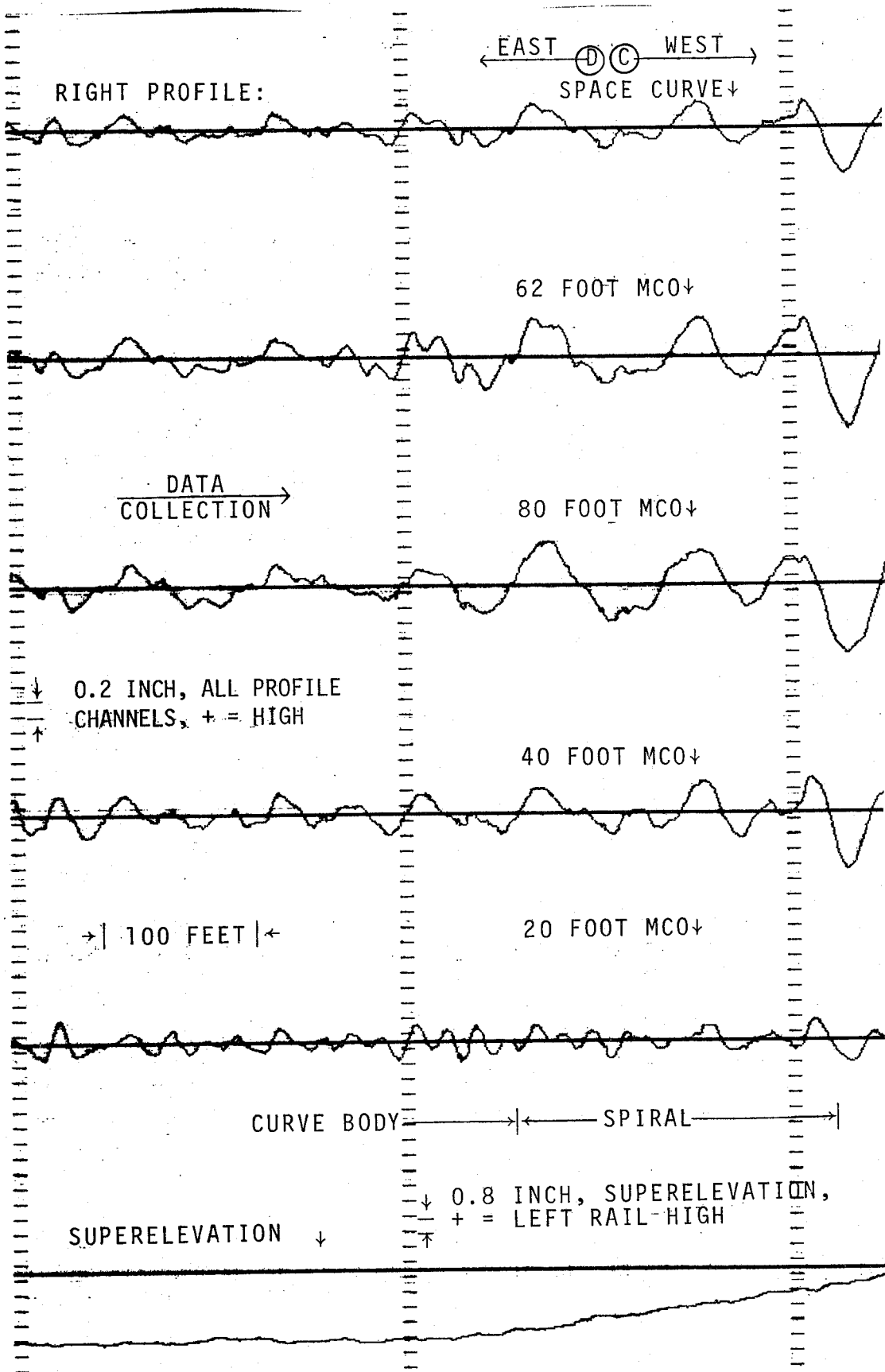


FIGURE 16. ZONE 2, RIGHT SURFACE, LOCATIONS C AND D

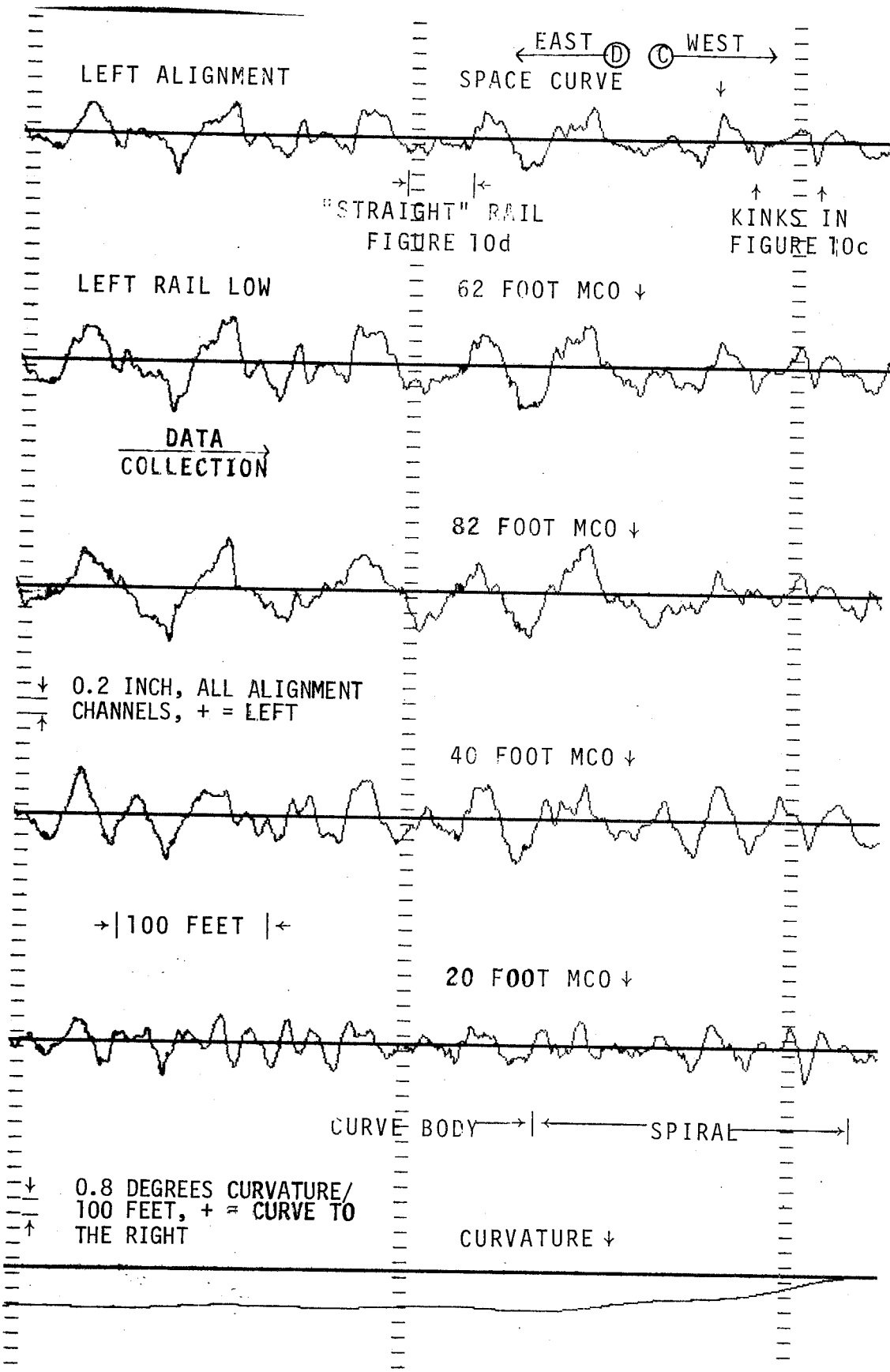


FIGURE 17. ZONE 2, LEFT LINE, LOCATIONS C AND D

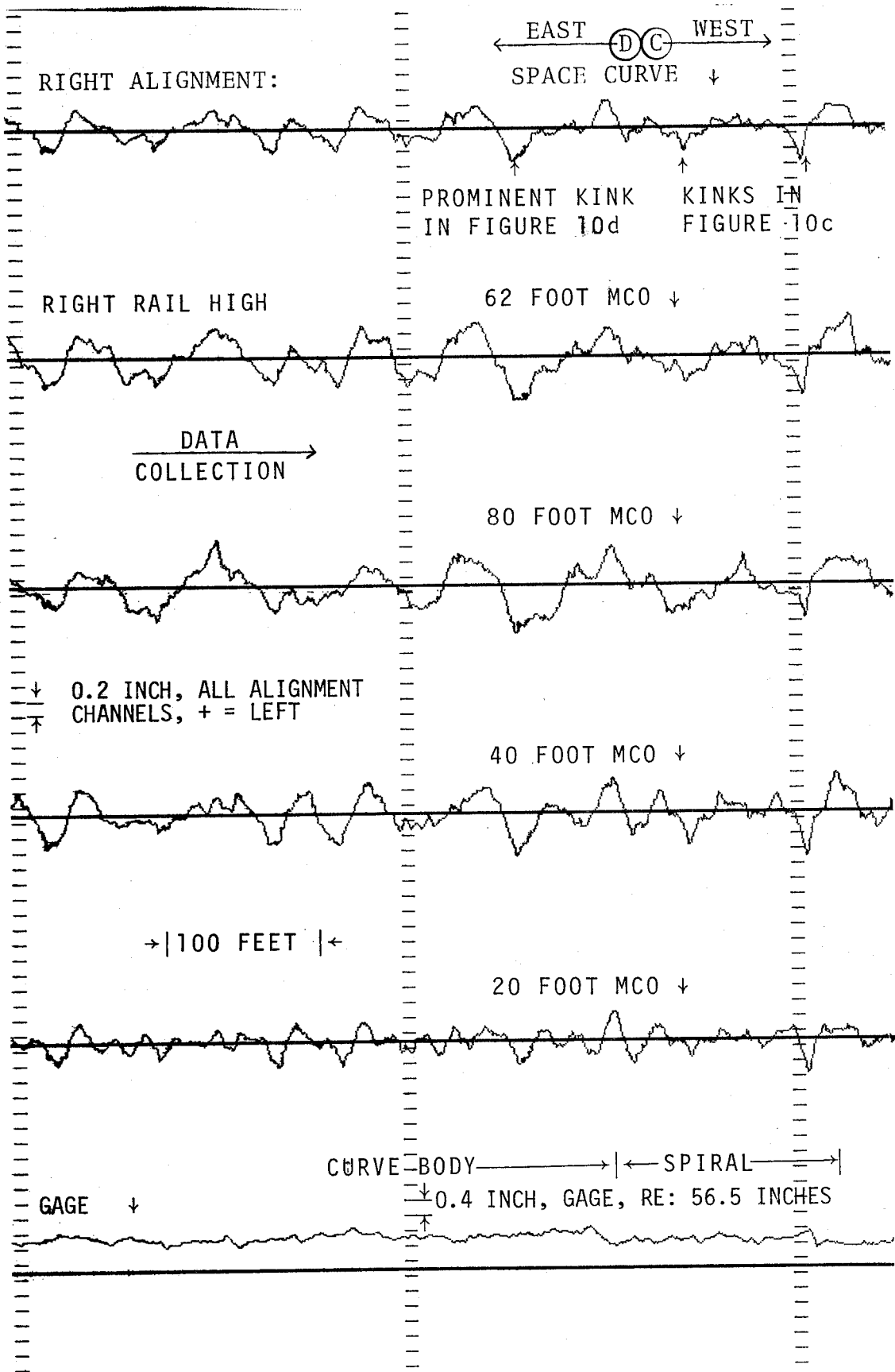
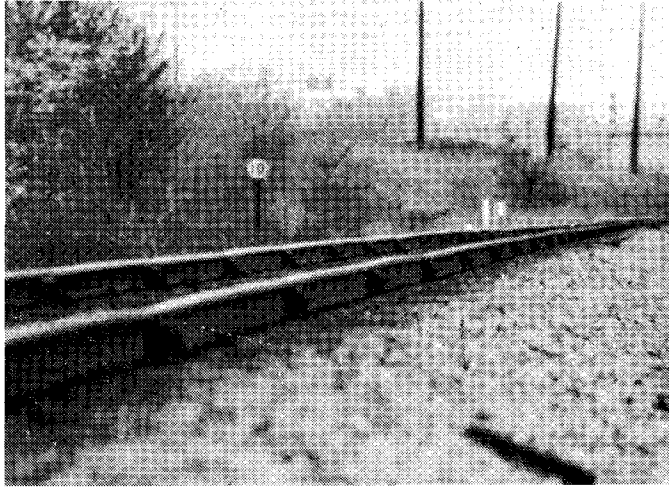
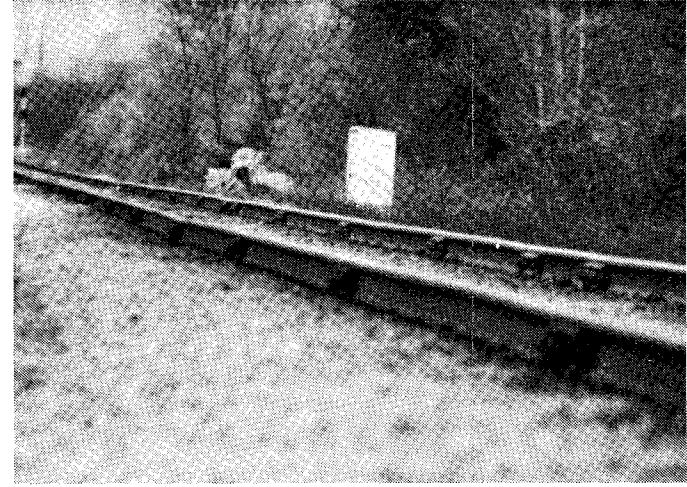


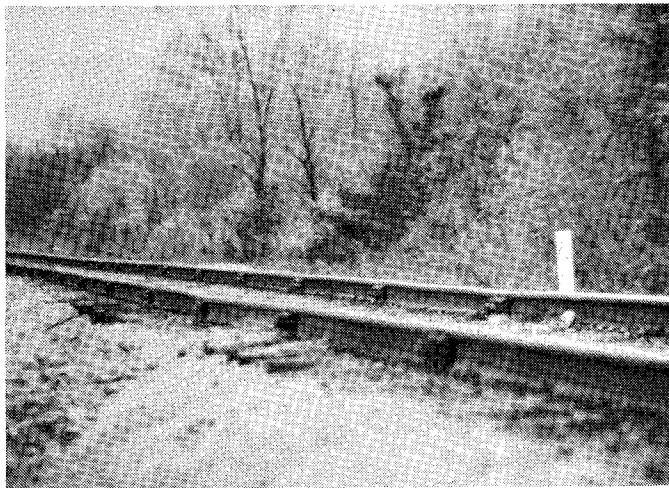
FIGURE 18. ZONE 2, RIGHT LINE, LOCATIONS C AND D



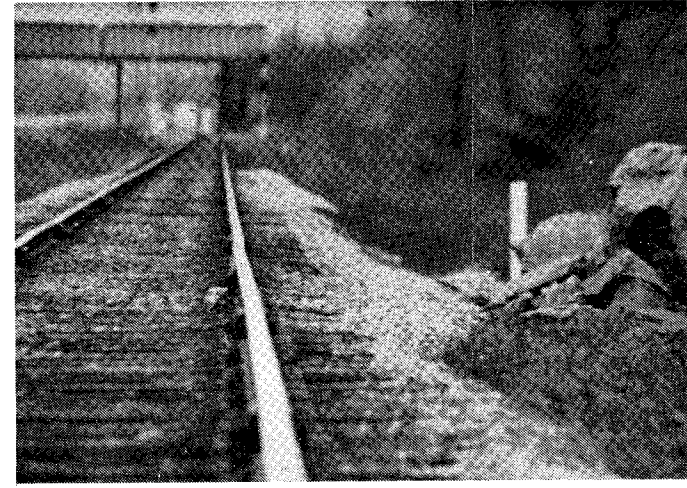
(a) Surface Geometry, Location A, Looking East



(b) Surface Geometry, Location A Looking West



(c) Surface Geometry Over Bridge, Location B, Looking West



(d) Alignment Kink at Bridge Approach, Location B, Looking West

FIGURE 19. PHOTOGRAPHIC REPRESENTATION OF ZONE 3

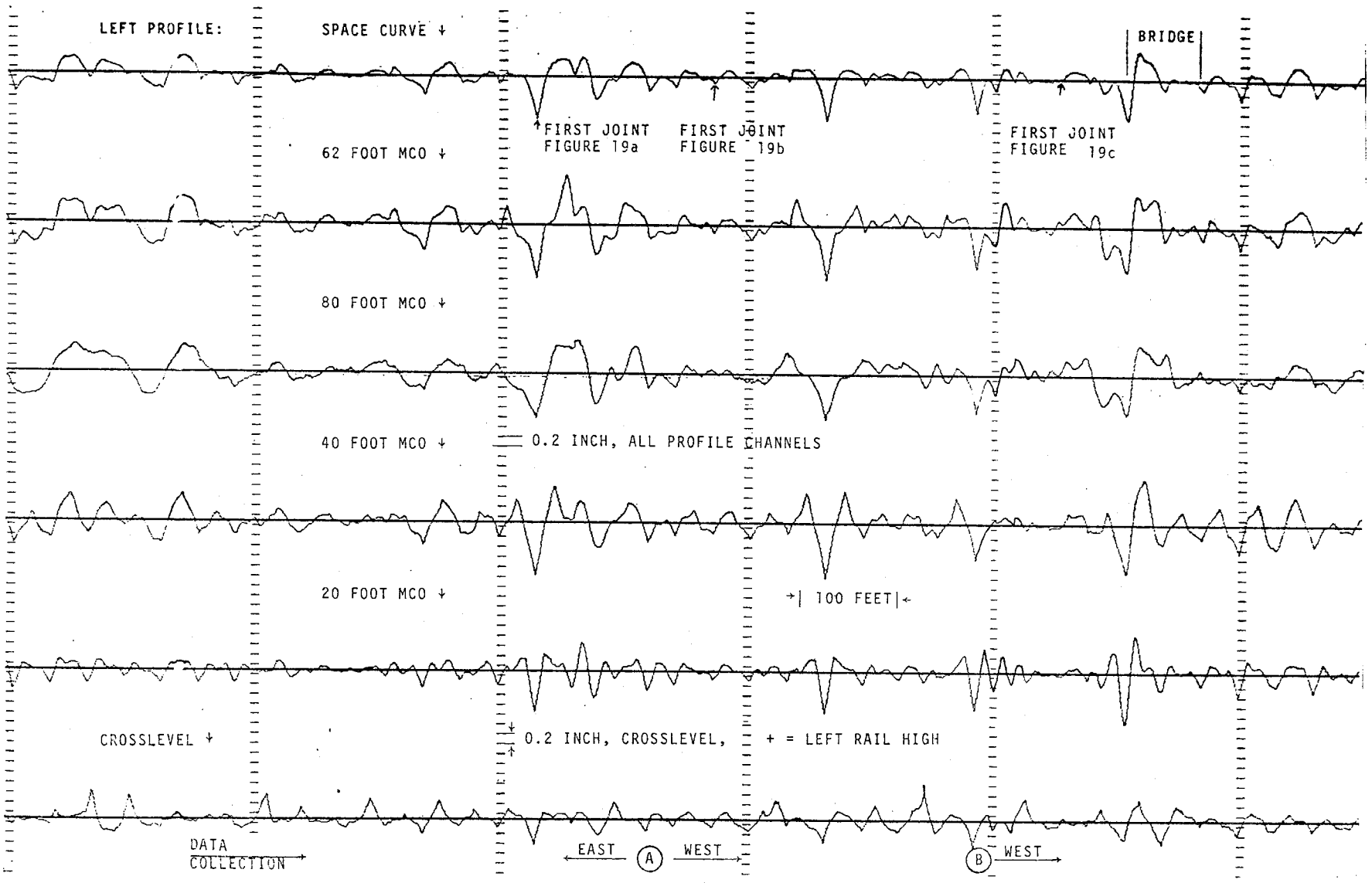
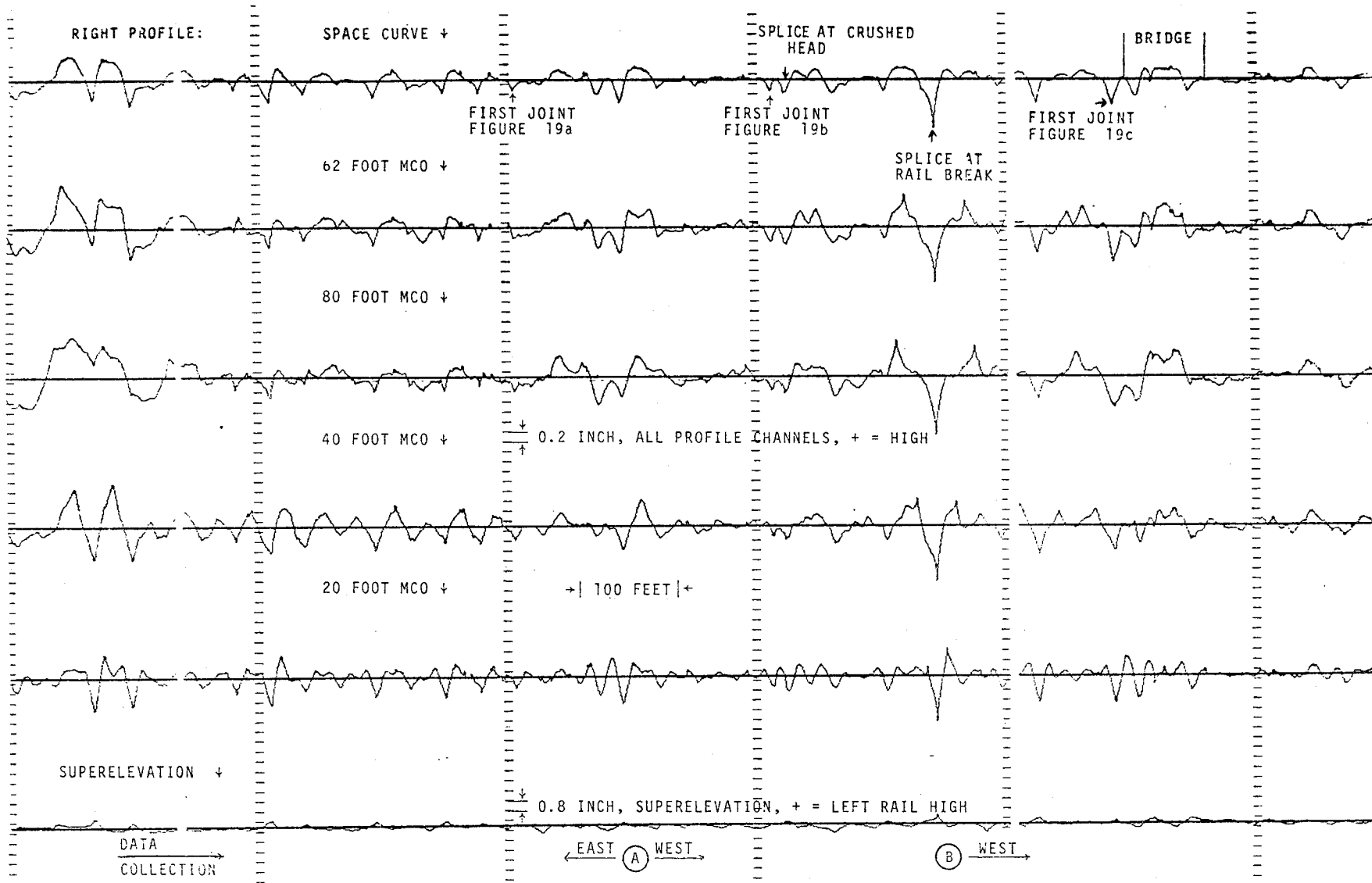


FIGURE 20. ZONE 3, LEFT SURFACE



37

FIGURE 21. ZONE 3, RIGHT SURFACE

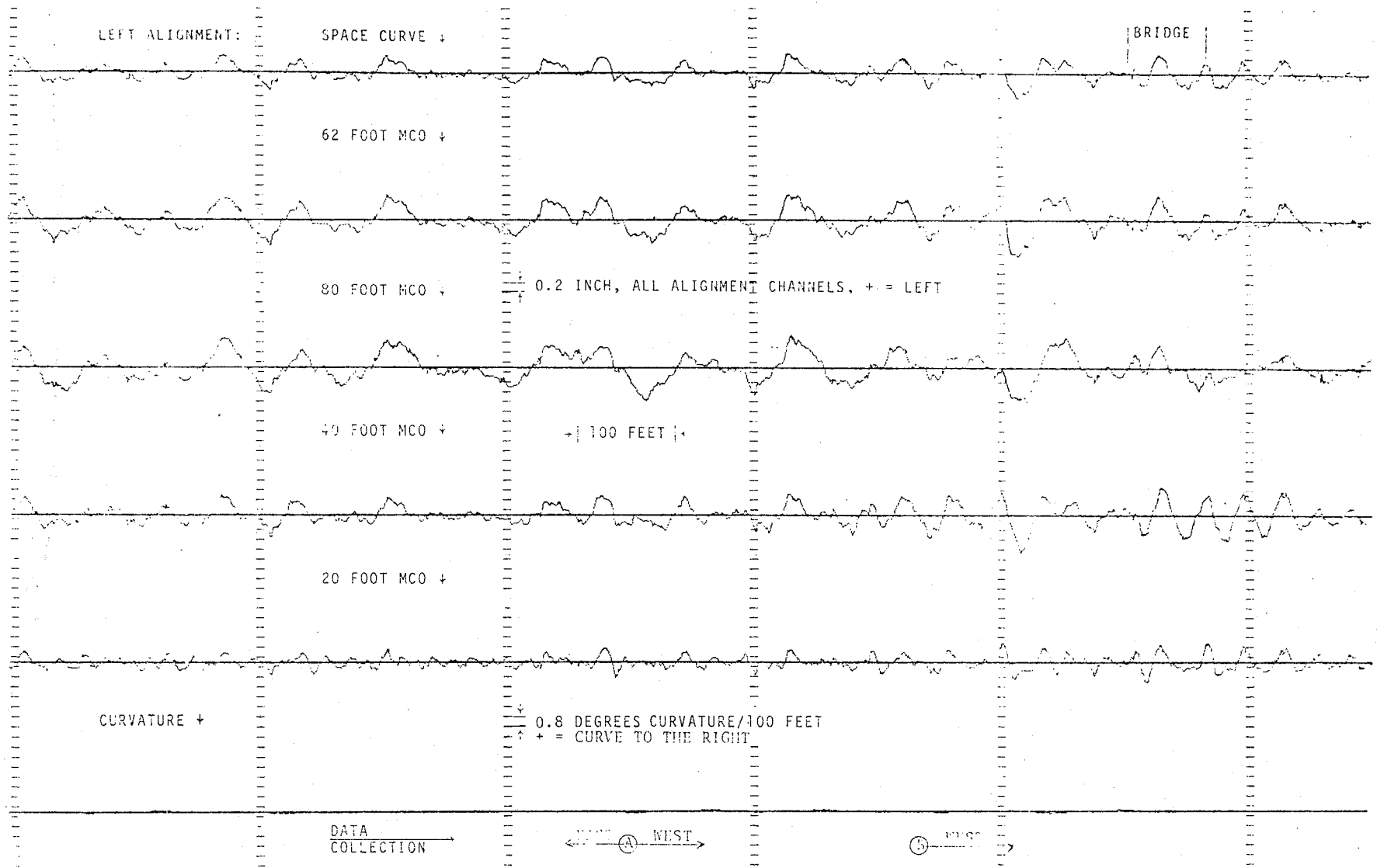


FIGURE 22. ZONE 3, LEFT LINE

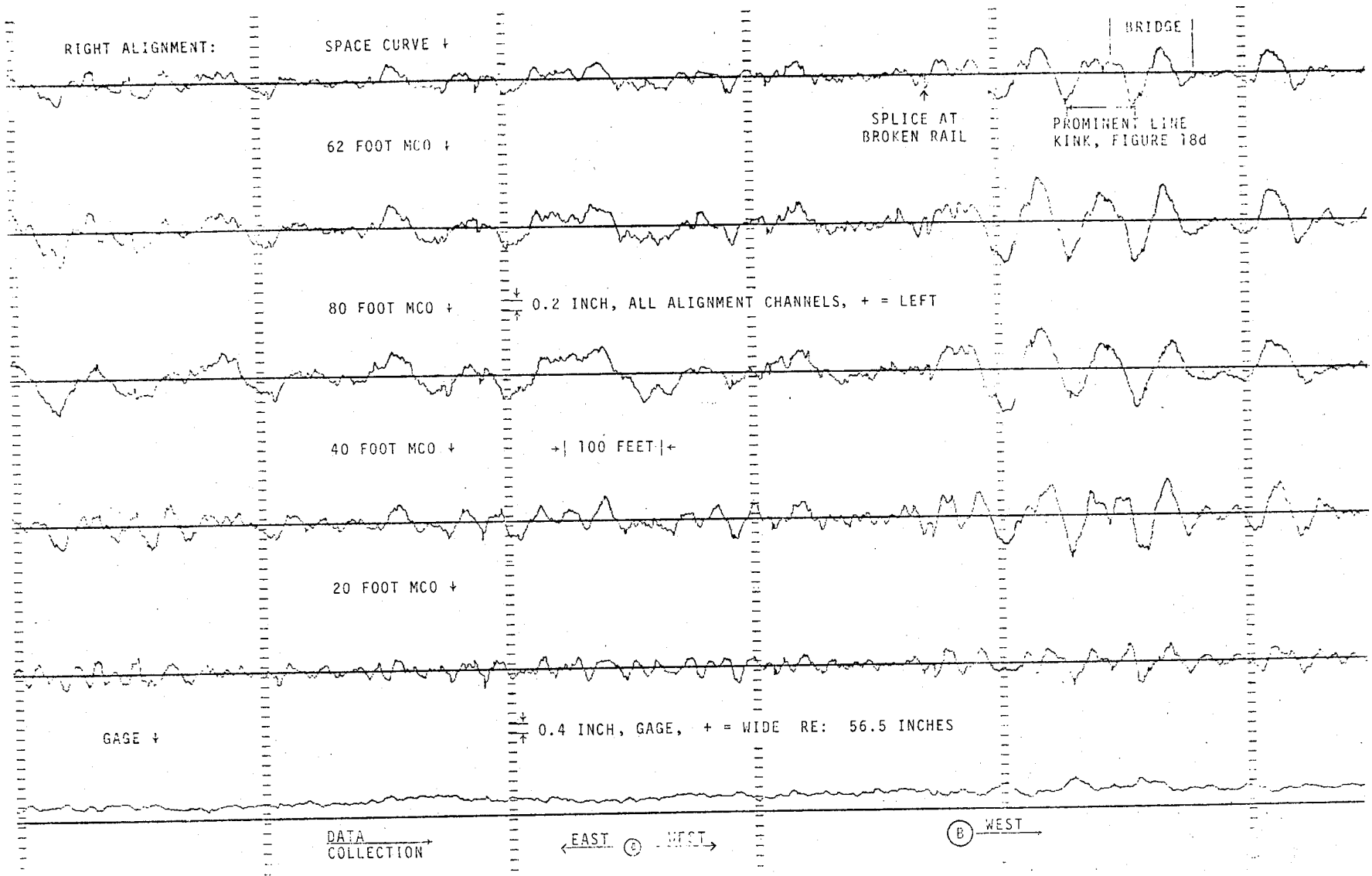
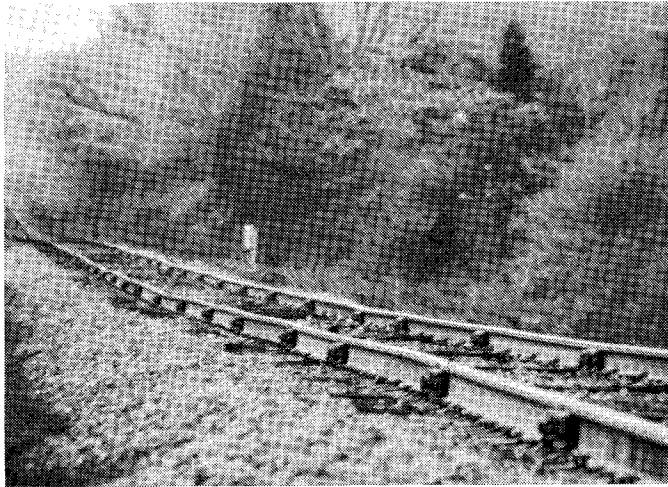
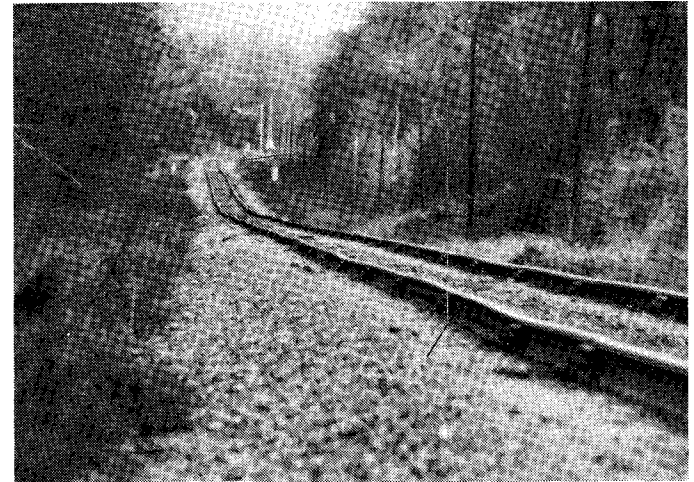


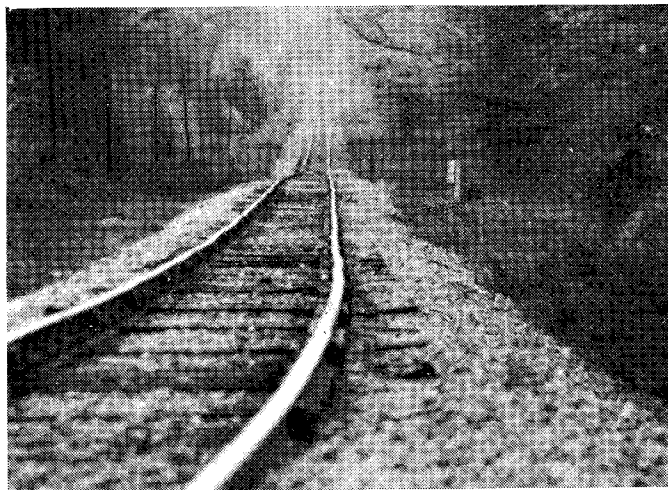
FIGURE 23. ZONE 3, RIGHT LINE



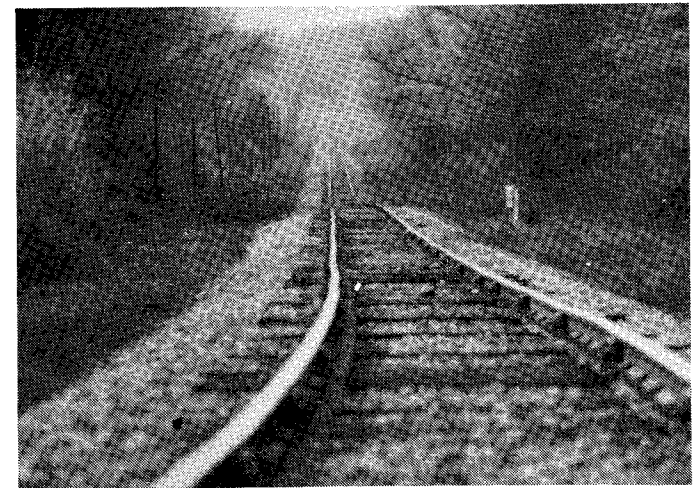
(a) Surface Geometry from Left Side,
Location A, Looking West



(b) Surface Geometry from Right Side,
Location B, Looking East



(c) Alignment Geometry of Right Rail,
Location A, Looking West



(d) Alignment Geometry of Left Rail,
Location A, Looking West

FIGURE 24. PHOTOGRAPHIC REPRESENTATION OF ZONE 4

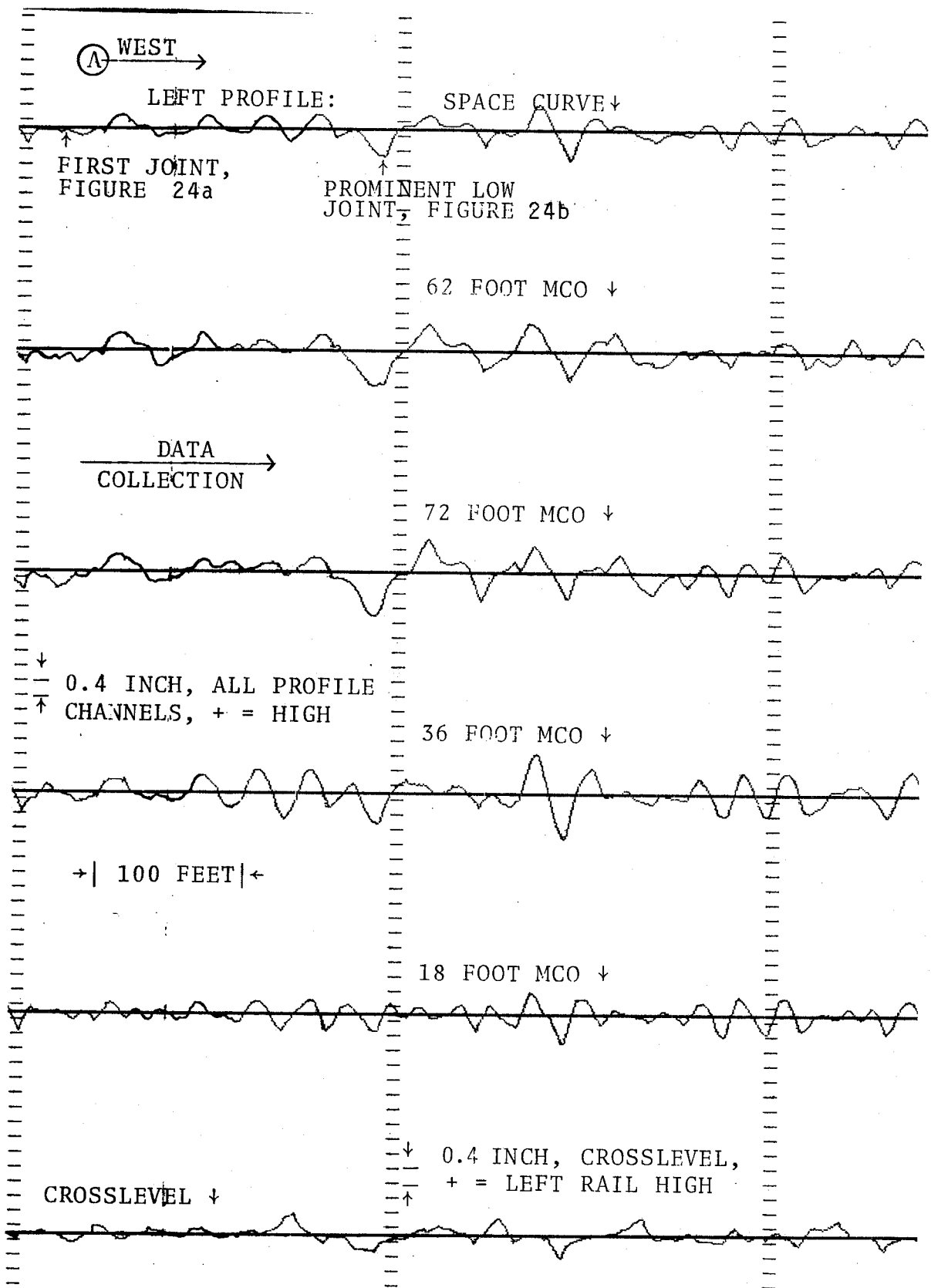


FIGURE 25. ZONE 4, LEFT SURFACE, LOCATION A

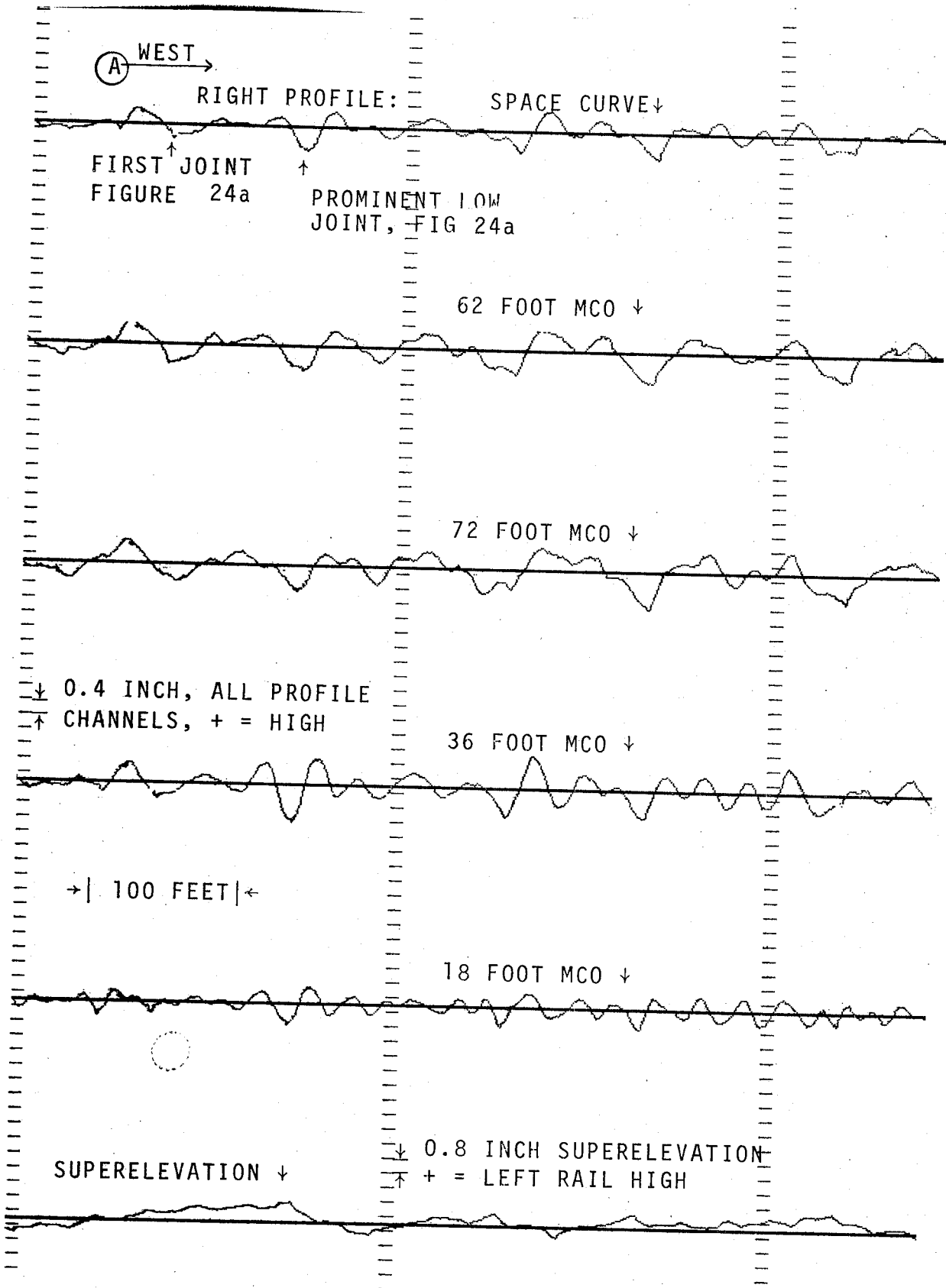


FIGURE 26. ZONE 4, RIGHT SURFACE, LOCATION A

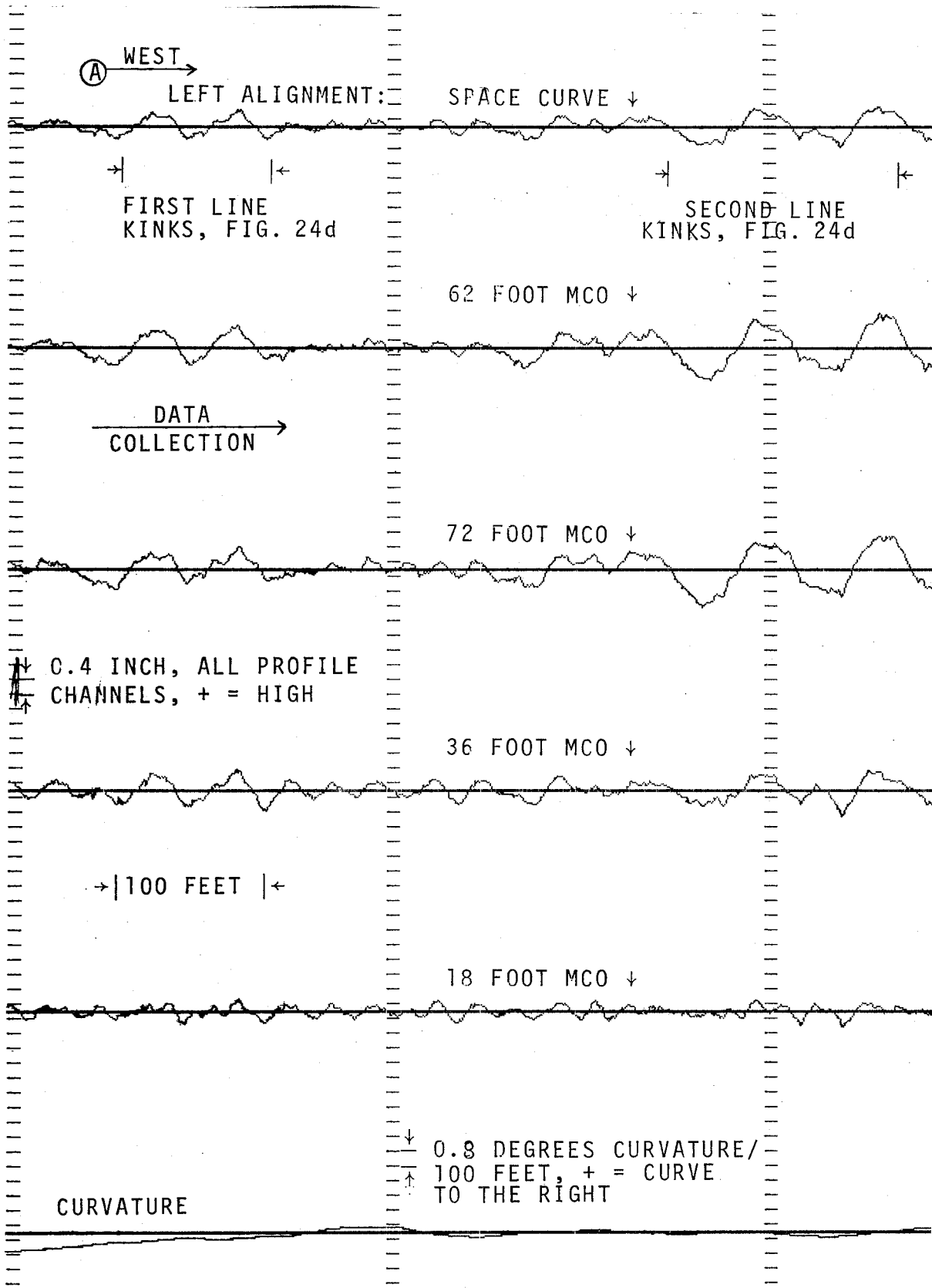


FIGURE 27. ZONE 4, LEFT LINE, LOCATION A.

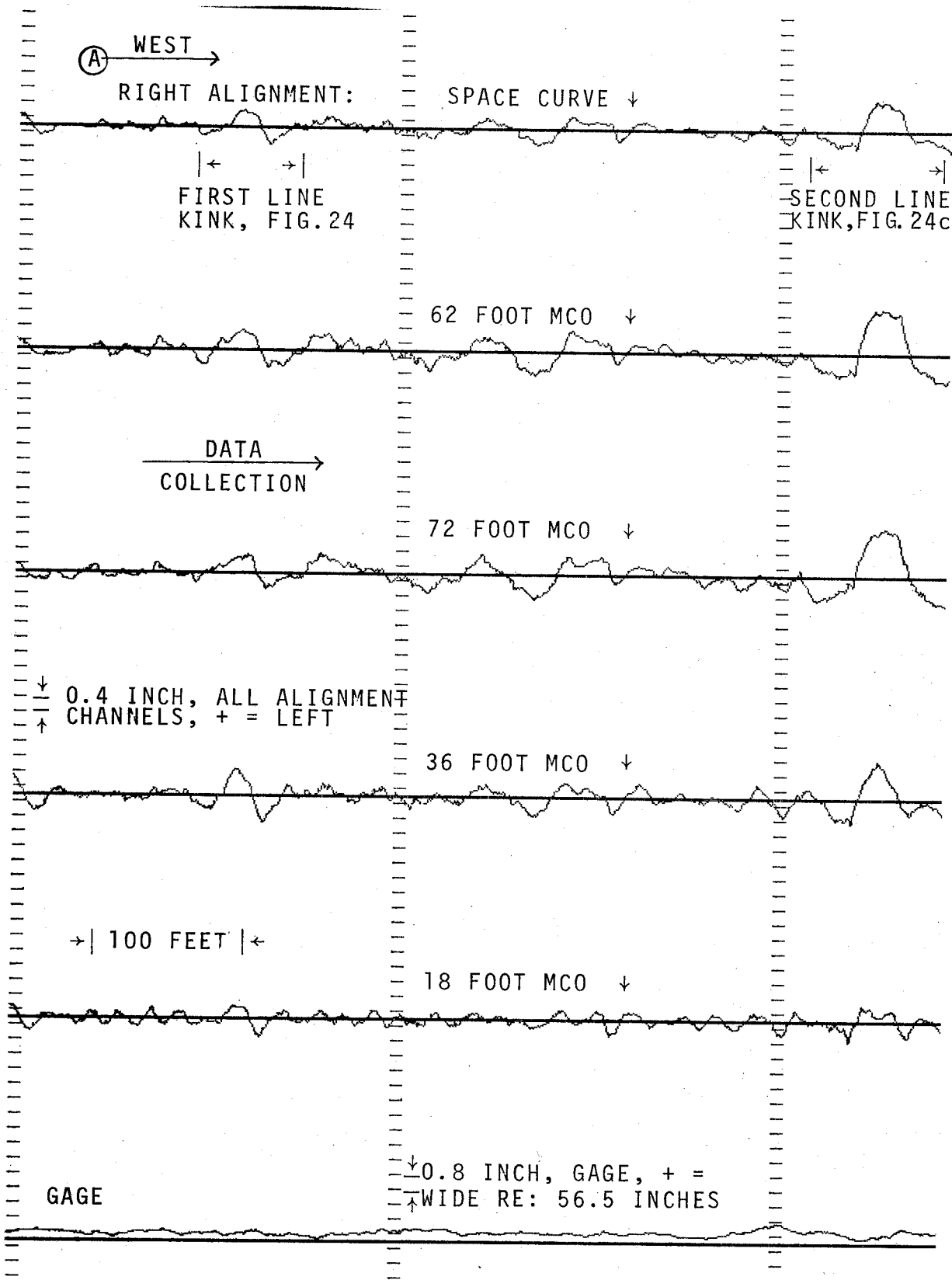


FIGURE 28. ZONE 4, RIGHT LINE, LOCATION A

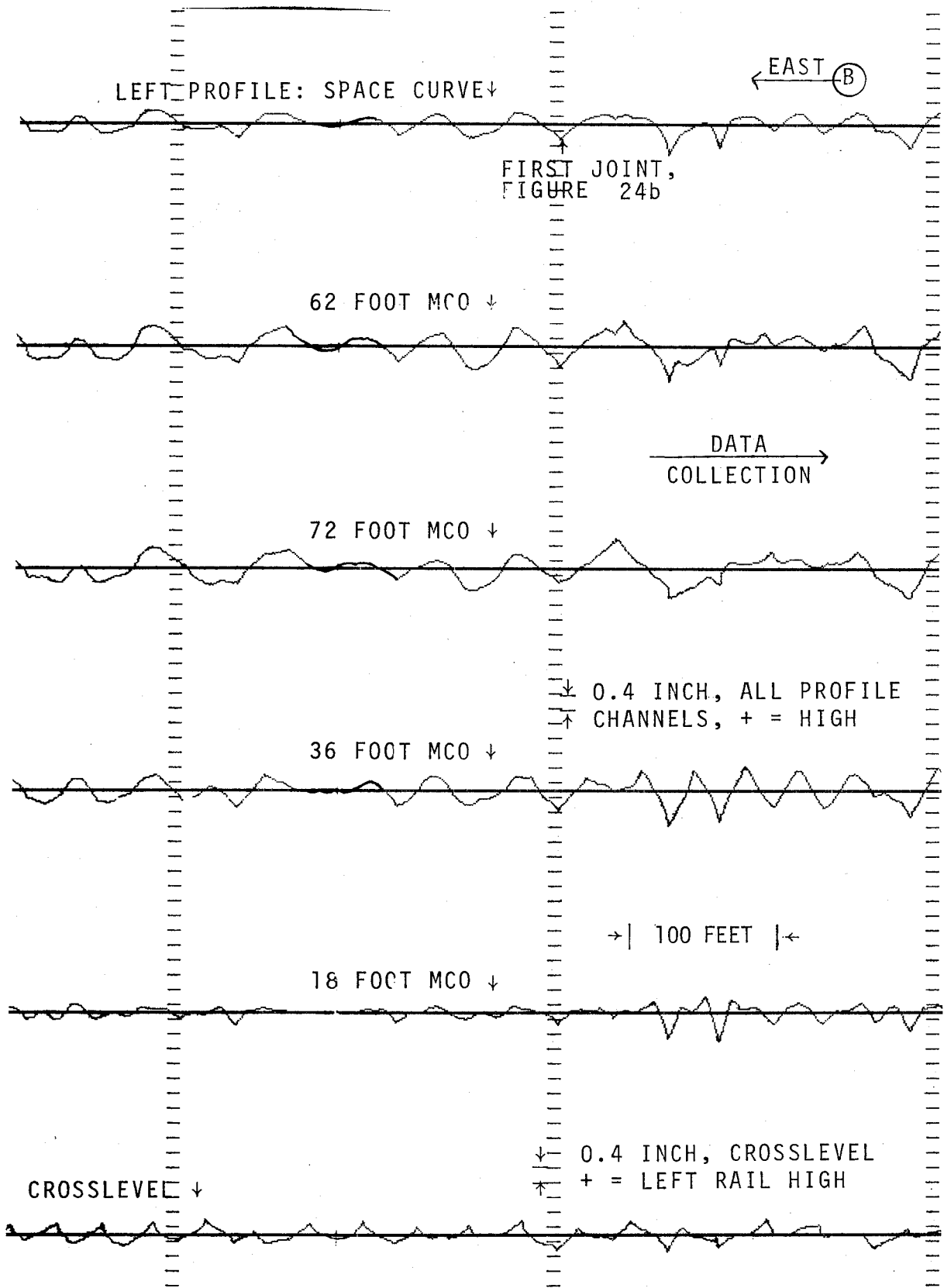


FIGURE 29. ZONE 4, LEFT SURFACE, LOCATION B

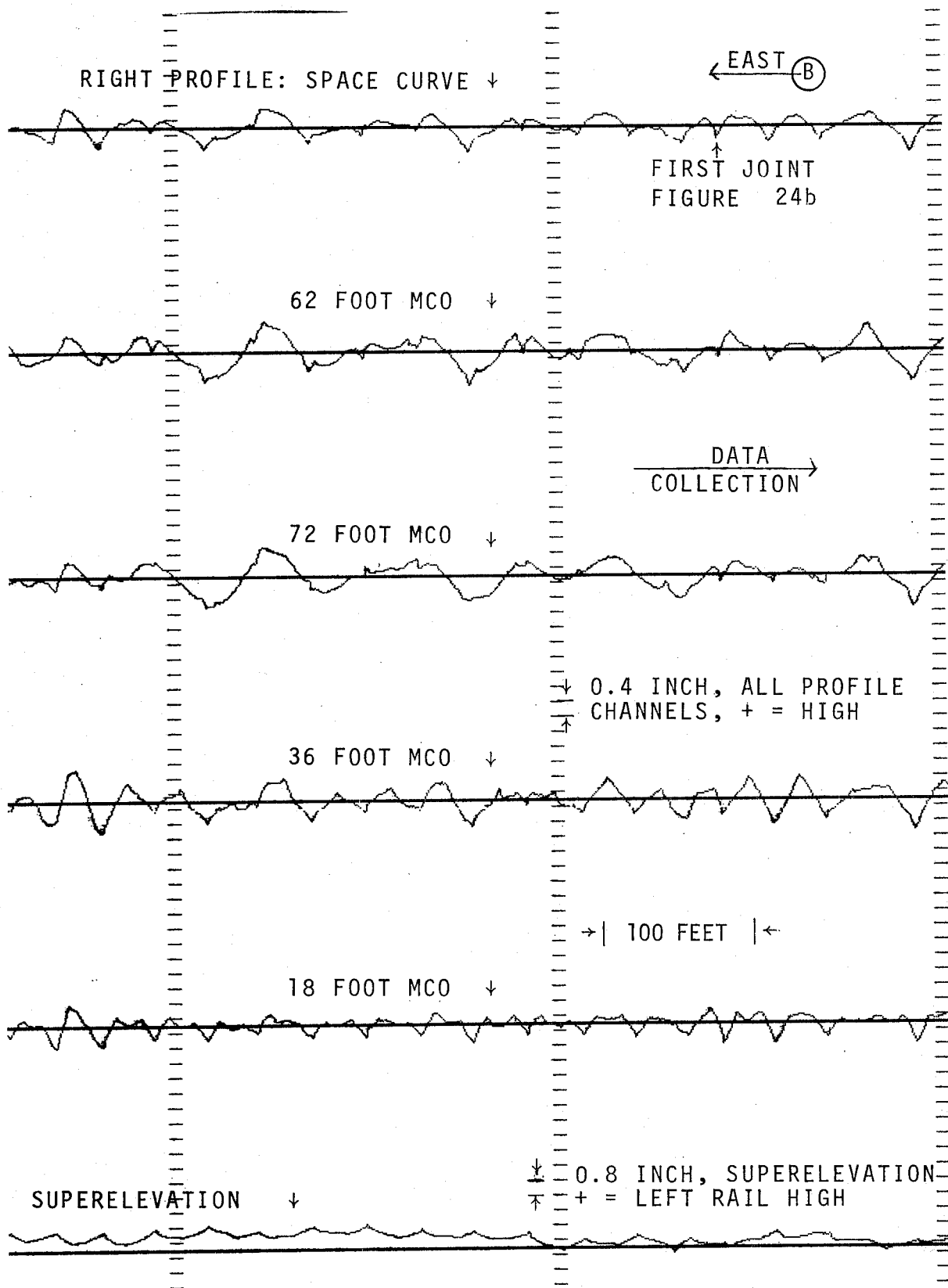
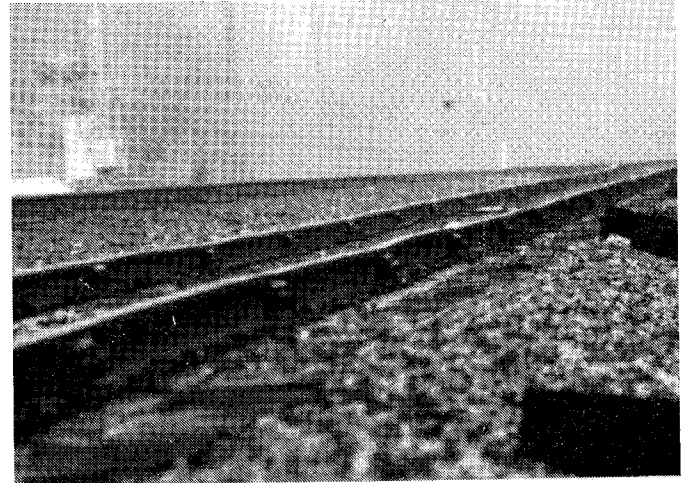


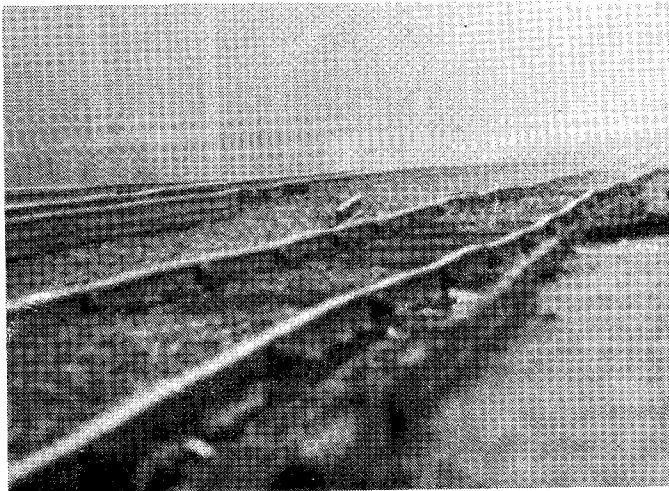
FIGURE 30. ZONE 4, RIGHT SURFACE, LOCATION B



(a) Surface Geometry from Right Side,
Location A, Looking West



(b) Surface Geometry from Right Side,
Location B, Looking East



(c) Surface Geometry from Right Side,
Location C, Looking East



(d) Surface Geometry from Right Side,
Location C, Looking West

FIGURE 31. PHOTOGRAPHIC REPRESENTATIONS OF ZONE 5

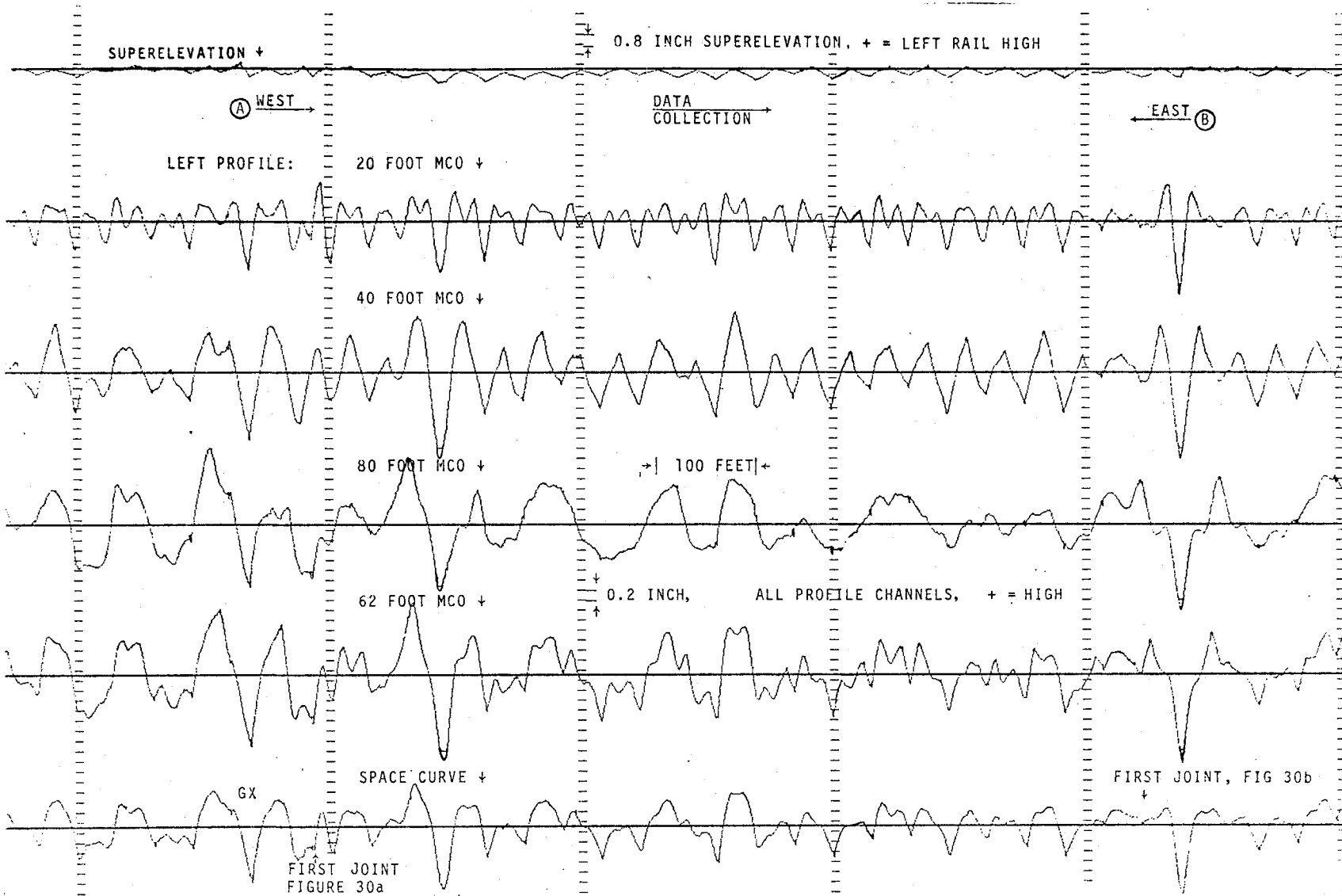


FIGURE 32. ZONE 5, LEFT SURFACE, LOCATIONS A AND B

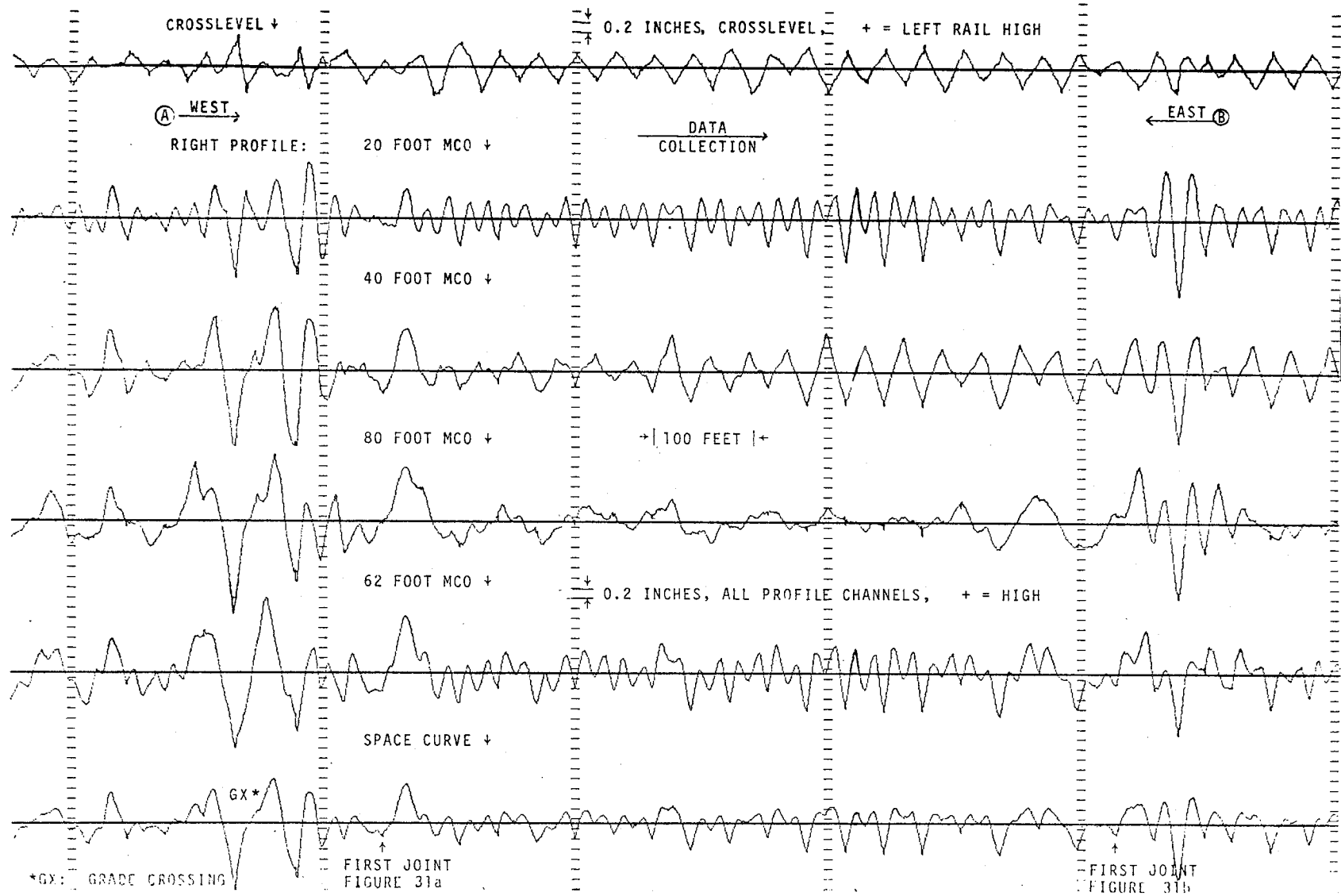


FIGURE 33. ZONE 5, RIGHT SURFACE, LOCATIONS A AND B

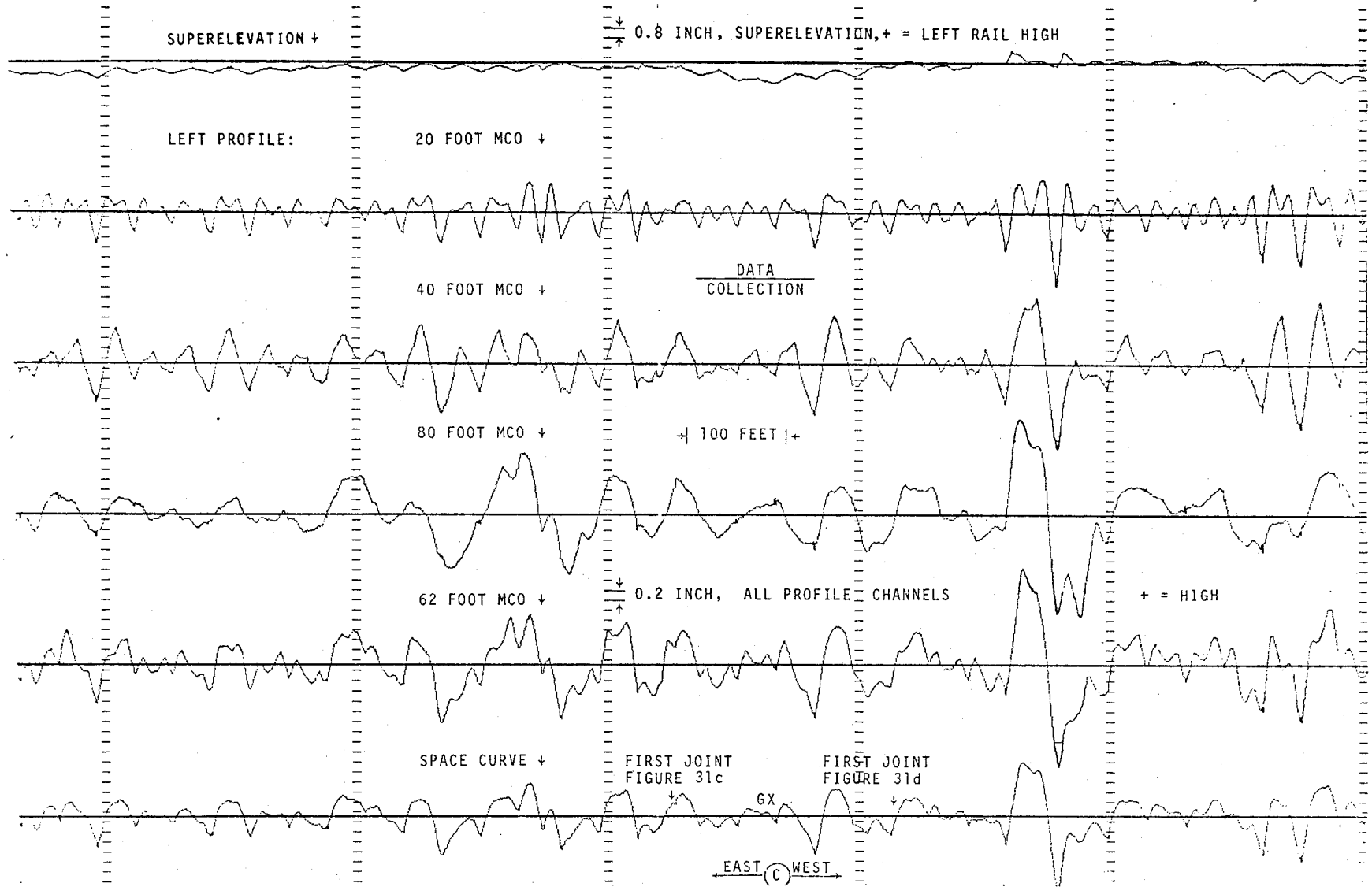


FIGURE 34. ZONE 5, LEFT SURFACE, LOCATION C

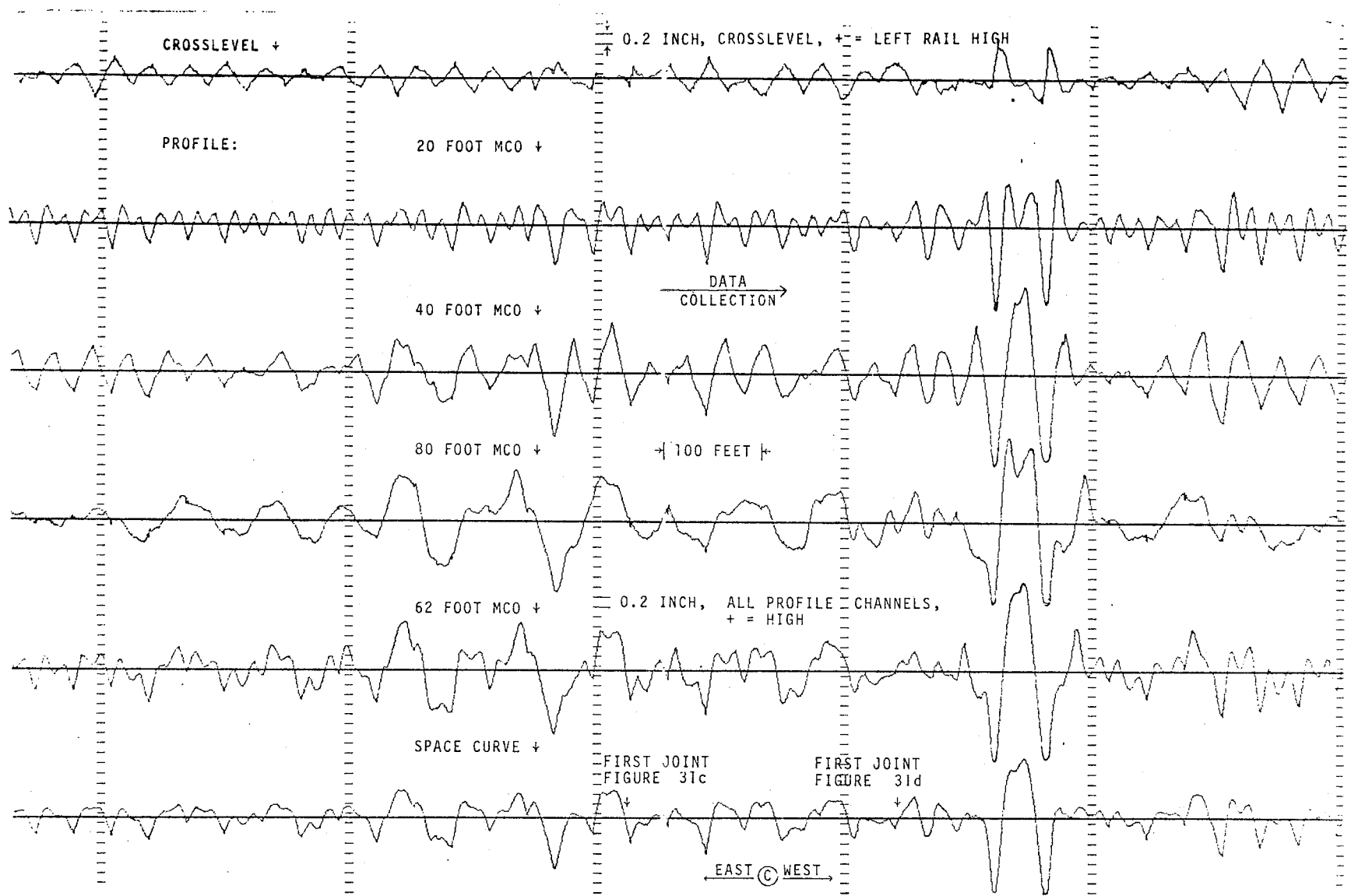


FIGURE 35. ZONE 5, RIGHT SURFACE, LOCATION C

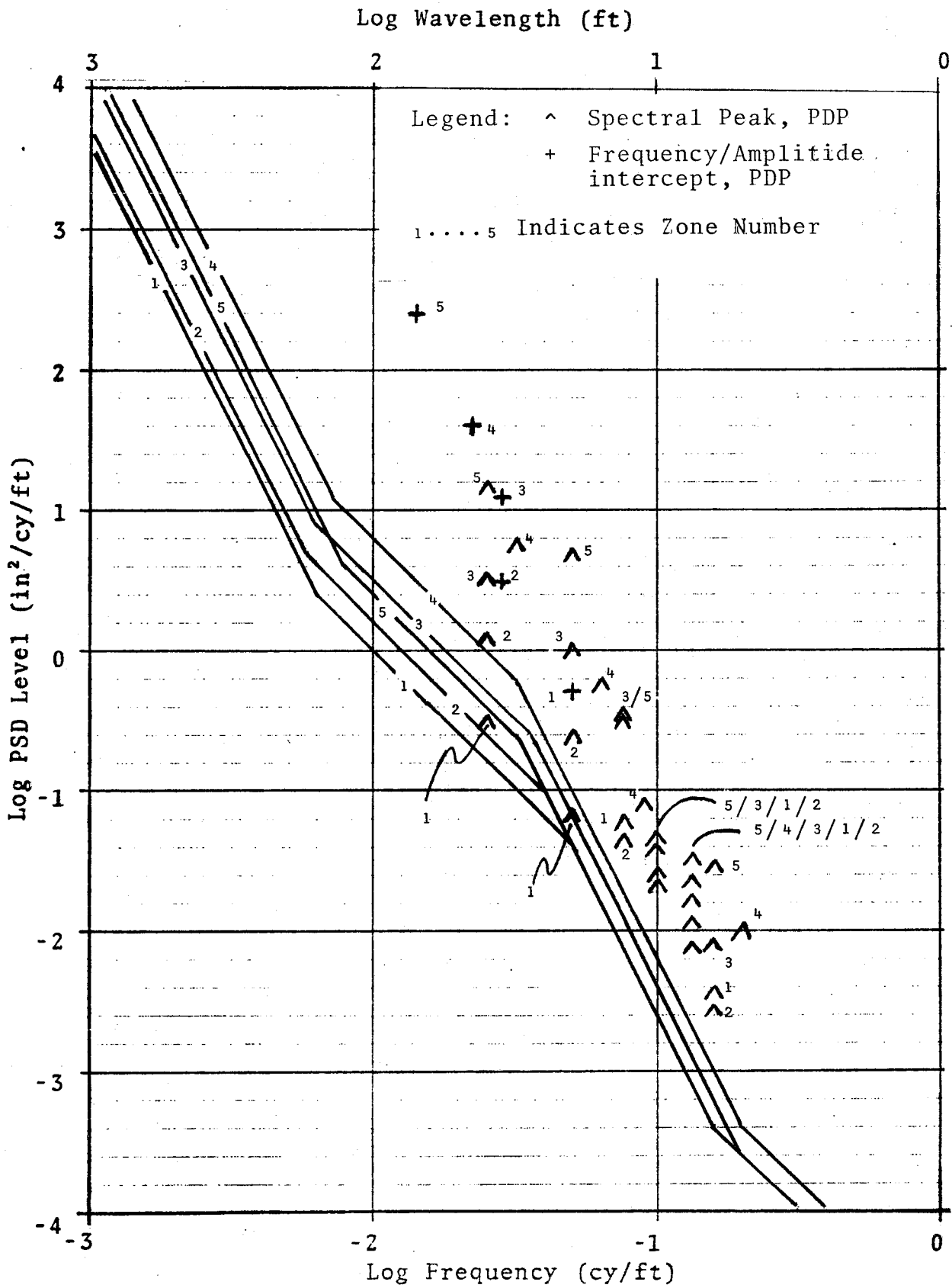


FIGURE 36. PSD'S OF INDIVIDUAL RAIL PROFILE

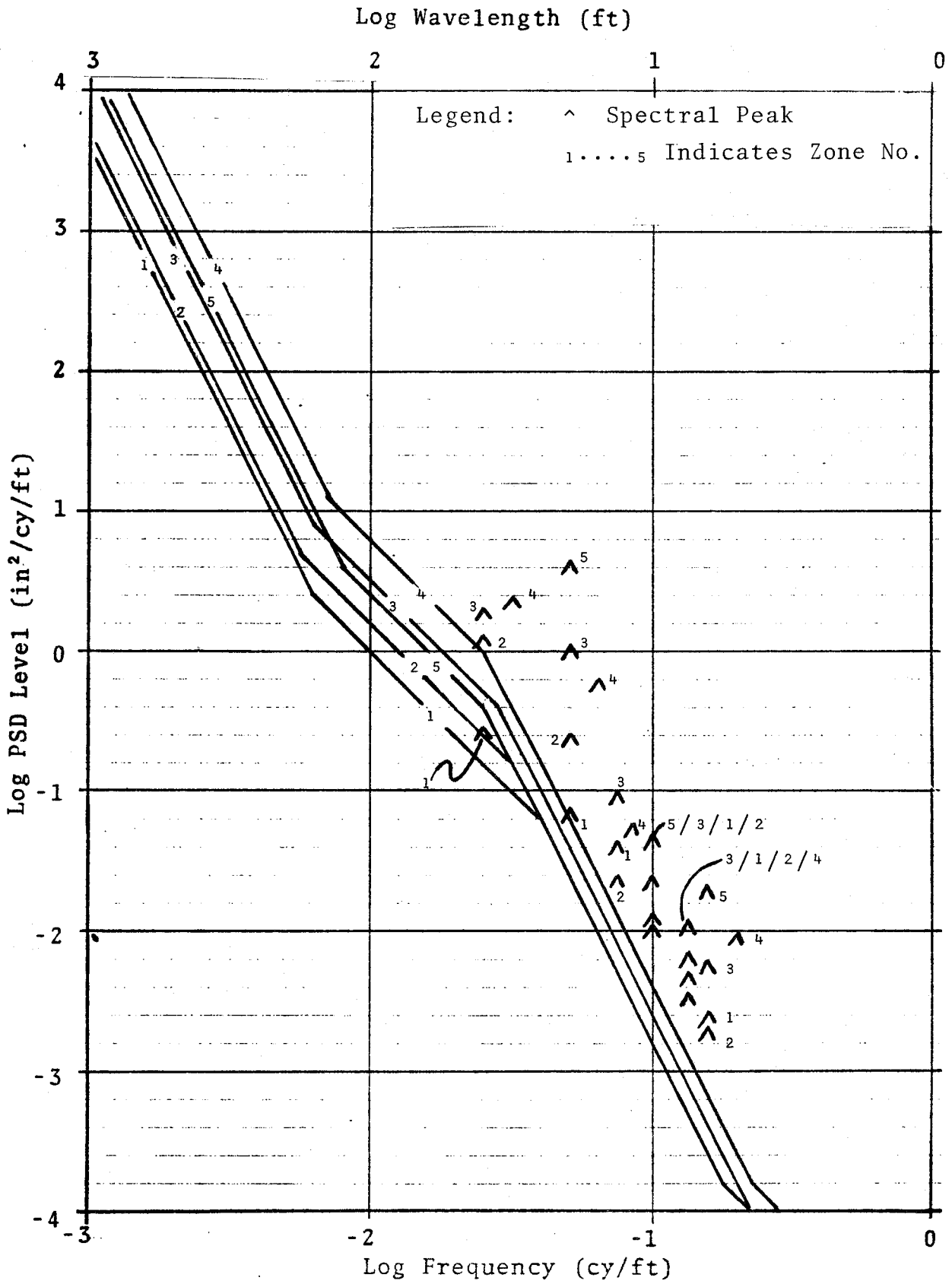


FIGURE 37. PSD'S OF MEAN PROFILE

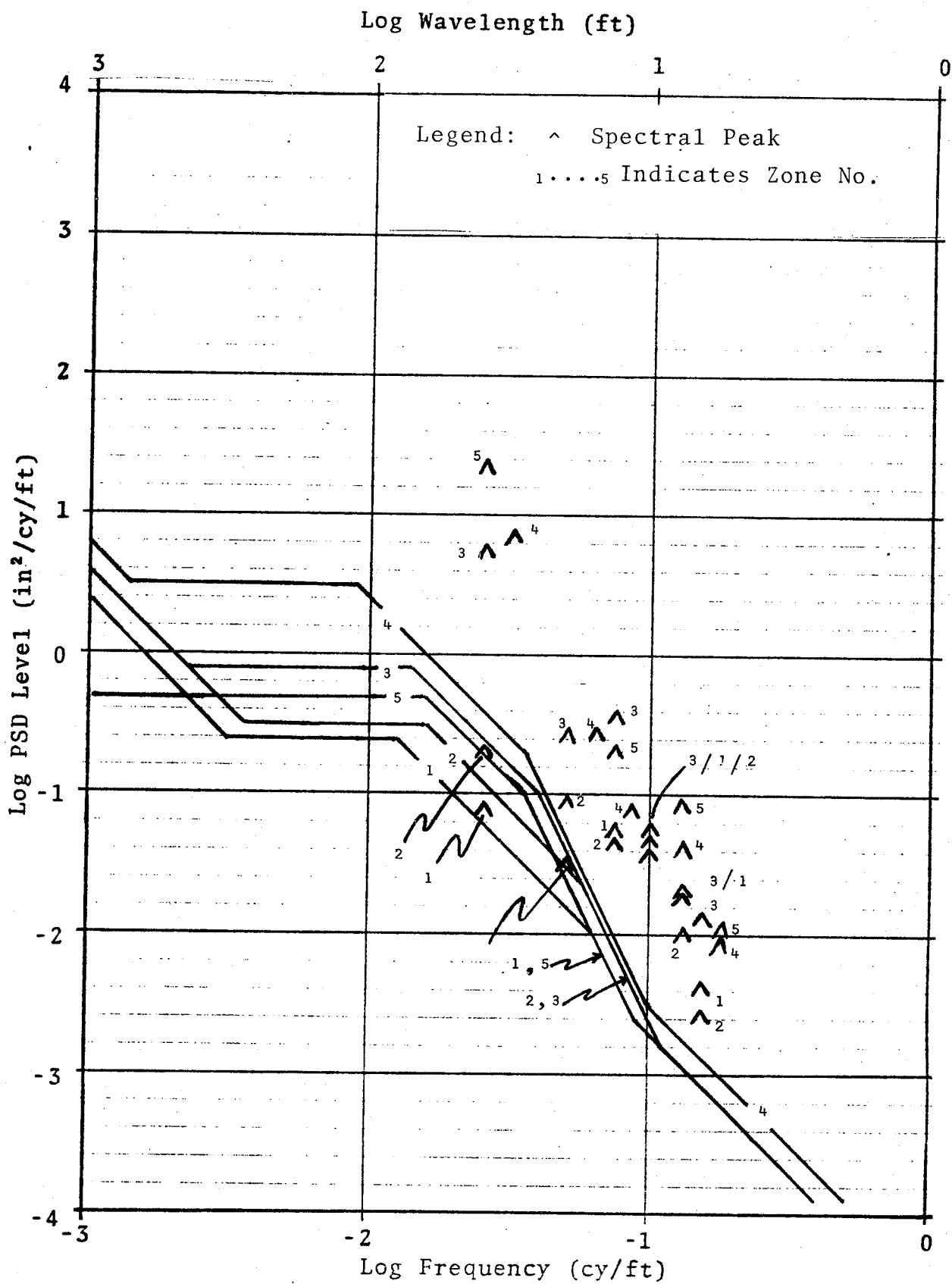


FIGURE 38. PSD'S OF CROSSLLEVEL

borne out by the high spectral peaks accompanied by a low-level continuum background. Also, the long wavelength continuum of Zone 4 is relatively high corresponding to the fact that this zone is in rolling terrain.

PSD models of line for Zones 1-4 are shown in Figures 39, 40 and 41. Note that modeling Zone 2 required a ϕ^{-6} -segment in the long wavelength segment. This is a result of several transition spirals and curvature kinks located in the zone.

3.4 PARAMETRIC REPRESENTATION

Most of the parameters are taken directly from the PSD models shown in Figures 36 through 41. These are tabulated in Table 3 for surface and Table 4 for line. Additional data, such as variances of geometry measures, are generated in accordance with the formulae in Appendix B.

3.5 JOINT AMPLITUDE HISTOGRAMS

For bolted rail, joints are usually distinct in the space curve profile plots (see Figures 20-21 for Zone 3, for example). This allows the joint amplitudes to be measured directly from the plots. Figures 42-44 are histograms that illustrate how joint amplitudes are distributed for Zones 3-5, respectively. The three histograms are similar, even though the quality of the track in the three zones varies significantly.

The mean value of low joints is given by:

$$\bar{c} = \frac{1}{N} \sum_{n=1}^N c_n = \frac{1}{N} \sum_{m=1}^M n_m c_m,$$

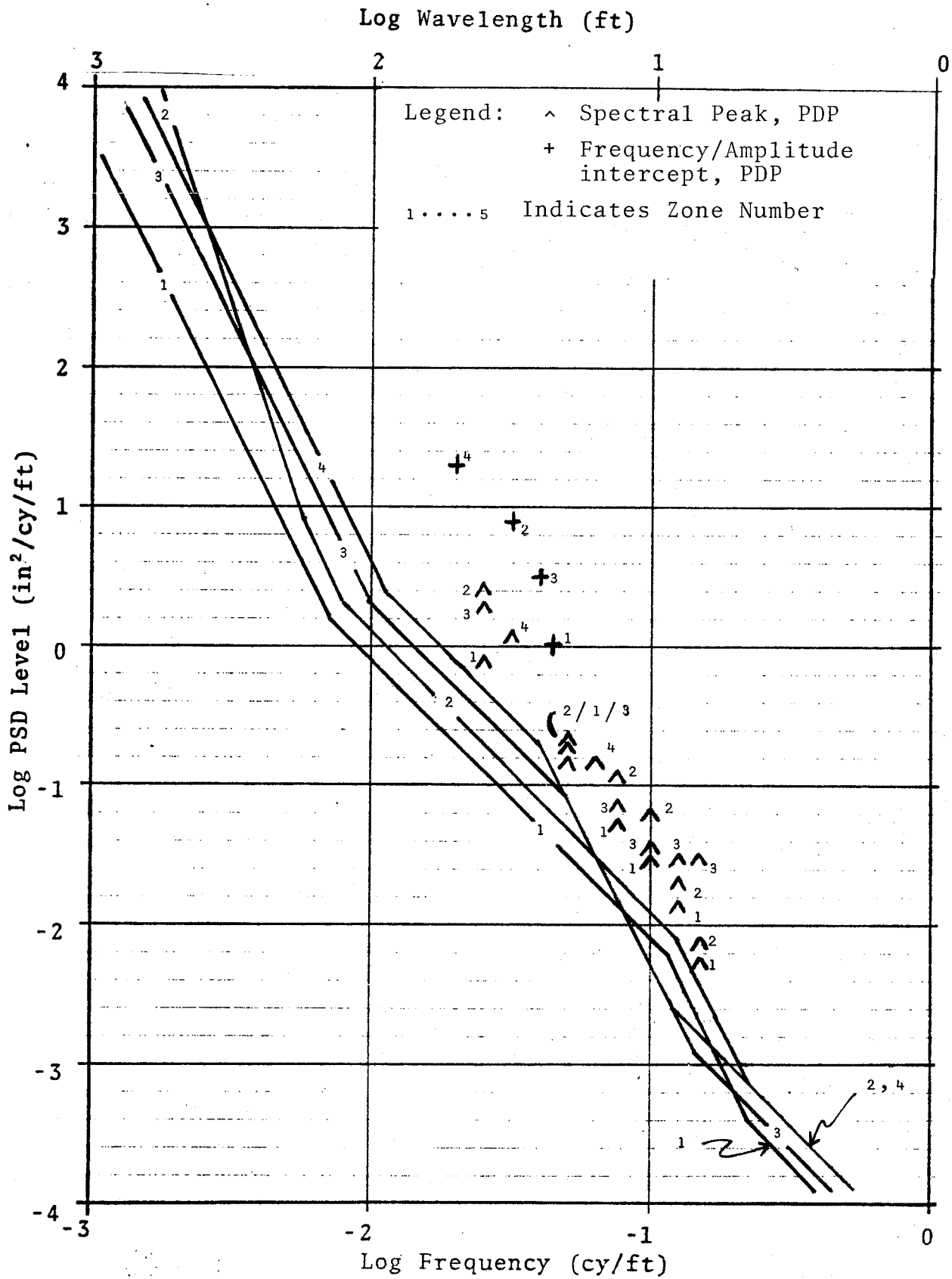


FIGURE 39. PSD'S OF INDIVIDUAL RAIL ALIGNMENT

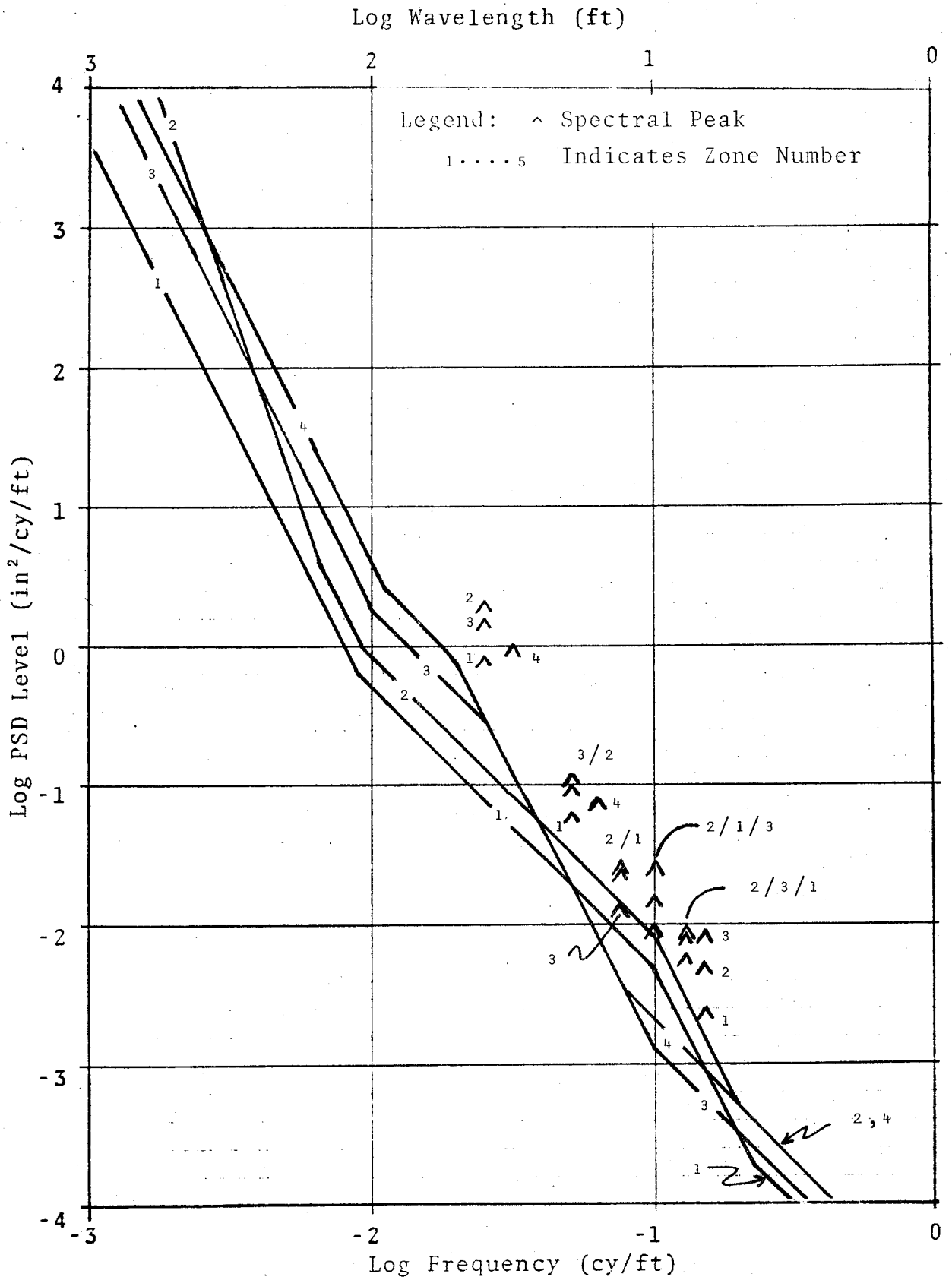


FIGURE 40. PSD'S OF MEAN ALIGNMENT

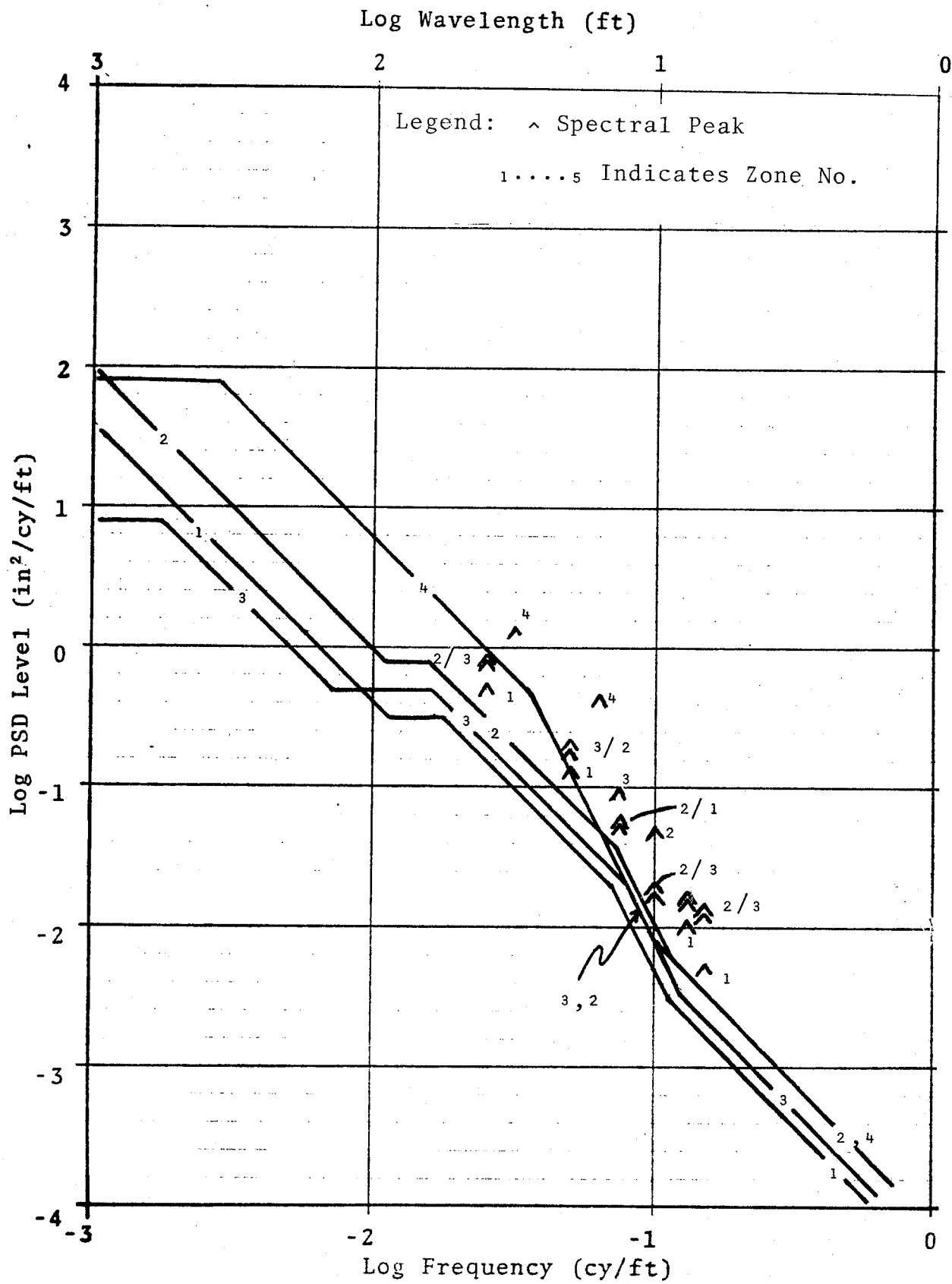


FIGURE 41. PSD'S OF GAGE

TABLE 3. SURFACE PARAMETERS

| Zone Number | | 1 | 2 | 3 | 4 | 5 | |
|--|-------------------------|--|------|------|------|-------|------|
| Track Class by Geometry | | 4.5 | 4.5 | 3.5 | 2.5 | 3.0 | |
| Continuum: Units: A_n : in ² -cy/ft; ϕ_{nm} ; cy/ft; all σ 's (standard error) in inches. | Individual Rail Profile | $A_1' \times 10^{-4}$ | 1.00 | 1.58 | 3.16 | 6.31 | 2.51 |
| | | $\phi_{13} \times 10^{-3}$ | 6.31 | 5.62 | 6.31 | 7.08 | 7.94 |
| | | $\phi_{14} \times 10^{-2}$ | 5.01 | 3.98 | 3.55 | 3.16 | 3.16 |
| | | $\phi_{15} \times 10^{-1}$ | 1.58 | 1.58 | 2.00 | 2.00 | 1.58 |
| | | $\sigma_1, 62$ ft MCO | 0.20 | 0.24 | 0.35 | 0.51 | 0.34 |
| | Mean Profile | $A_3 \times 10^{-4}$ | 1.00 | 1.58 | 3.16 | 6.31 | 2.51 |
| | | $\phi_{33} \times 10^{-3}$ | 6.31 | 5.62 | 6.31 | 7.08 | 7.94 |
| | | $\phi_{34} \times 10^{-2}$ | 3.98 | 3.16 | 2.82 | 2.51 | 2.51 |
| | | $\phi_{35} \times 10^{-1}$ | 1.78 | 1.78 | 2.24 | 2.24 | 1.78 |
| | Crosslevel | $A_4 \times 10^{-4}$ | 0.40 | 0.79 | 1.58 | 2.51 | 1.26 |
| | | $\phi_{43} \times 10^{-2}$ | 1.26 | 1.58 | 1.41 | 0.89 | 1.58 |
| | | $\phi_{44} \times 10^{-2}$ | 6.31 | 5.62 | 3.98 | 3.55 | 3.55 |
| | | $\phi_{45} \times 10^{-1}$ | 0.89 | 1.12 | 1.12 | 1.00 | 0.89 |
| | | σ_4, X -level | 0.07 | 0.09 | 0.11 | 0.21 | 0.11 |
| | | $\sigma_4, Warp$ | 0.10 | 0.13 | 0.15 | 0.30 | 0.16 |
| Periodic Deterministic Process | Individual Rail Profile | Rail Length, ft | 39 | 39 | 39 | 28-36 | 39 |
| | | Ordinate Level, $S_e(0)$ in ² /cy/ft | 0.50 | 3.16 | 12.6 | 31.6 | 251 |
| | | Corner Frequency, ϕ_0 (cy/ft $\times 10^{-2}$) | 5.01 | 2.82 | 2.82 | 2.24 | 1.41 |
| | | Joint Amplitude, C (in) | 0.10 | 0.14 | 0.27 | 0.34 | 0.63 |
| | | Decay Rate, k (ft ⁻¹) | 0.31 | 0.18 | 0.18 | 0.14 | 0.09 |
| | | Peak 62' MCO, in | 0.09 | 0.10 | 0.21 | 0.08 | 0.31 |

TABLE 4. LINE PARAMETERS

| Zone Number | | 1 | 2 | 3 | 4 | |
|---|---------------------------|---|------|------|------|-------|
| Track Class by Geometry | | 4.5 | 4.5 | 3.5 | 2.5 | |
| Continuum: Units: A_n : in ² -cy/ft; ϕ_{mn} ; cy/ft; all σ 's (standard errors) in inches. | Individual Rail Alignment | $A_5 \times 10^{-4}$ | 0.79 | 1.26 | 2.00 | 3.16 |
| | | $\phi_{53} \times 10^{-2}$ | 0.71 | 0.71 | 1.00 | 1.12 |
| | | $\phi_{54} \times 10^{-2}$ | 11.2 | 12.6 | 5.01 | 3.98 |
| | | $\phi_{55} \times 10^{-1}$ | 2.24 | 2.24 | 1.41 | 1.12 |
| | | σ_5 , 62 ft MCO | 0.19 | 0.25 | 0.36 | 0.48 |
| | Mean Alignment | $A_7 \times 10^{-4}$ | 0.50 | 0.79 | 2.00 | 3.16 |
| | | $\phi_{73} \times 10^{-2}$ | 0.89 | 0.89 | 1.00 | 1.12 |
| | | $\phi_{74} \times 10^{-2}$ | 10.0 | 10.0 | 2.51 | 2.00 |
| | | $\phi_{75} \times 10^{-1}$ | 2.24 | 2.00 | 1.00 | 0.79 |
| | Gage | $A_8(3) \times 10^{-4}$ | 1.00 | 2.00 | 1.26 | 6.31 |
| | | $\phi_{83} \times 10^{-2}$ | 1.78 | 1.58 | 1.58 | 0.28 |
| | | $\phi_{84} \times 10^{-2}$ | 7.08 | 7.08 | 7.94 | 3.55 |
| | | $\phi_{85} \times 10^{-1}$ | 1.12 | 1.12 | 1.26 | 1.00 |
| | | σ_8 Gage | 0.09 | 0.14 | 0.11 | 0.59 |
| | | σ_8' , GR/C* | 0.13 | 0.20 | 0.16 | 0.84 |
| Periodic Deterministic Process | Individual Rail Alignment | Rail Length, ft | 39 | 39 | 39 | 28-36 |
| | | Ordinate Level, $s_e(0)$ in ² /cy/ft | 1.00 | 7.94 | 3.16 | 20.0 |
| | | Corner Frequency, ϕ_0 , cy/ft $\times 10^{-2}$ | 4.47 | 3.16 | 3.98 | 2.00 |
| | | Joint Amplitude \bar{c} (in) | 0.12 | 0.24 | 0.19 | 0.20 |
| | | Decay Rate, k (ft ⁻¹) | 0.28 | 0.20 | 0.25 | 0.13 |
| | | Peak 62' MCO, (in) | 0.11 | 0.19 | 0.17 | 0.05 |

*Gage Rate of Change

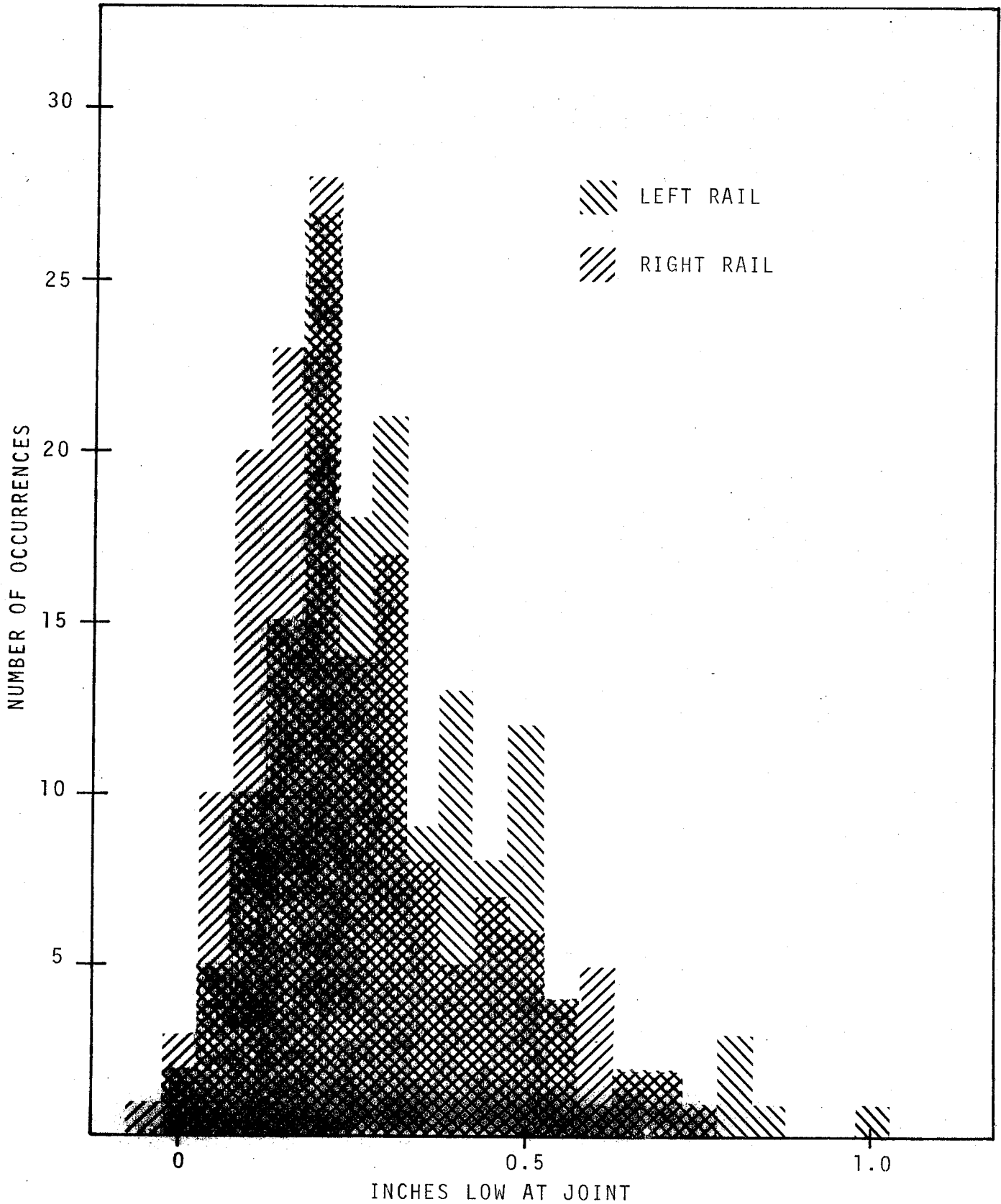


FIGURE 42. ZONE 3 HISTOGRAM FOR 156 LEFT RAIL AND 156 RIGHT RAIL LOW JOINTS

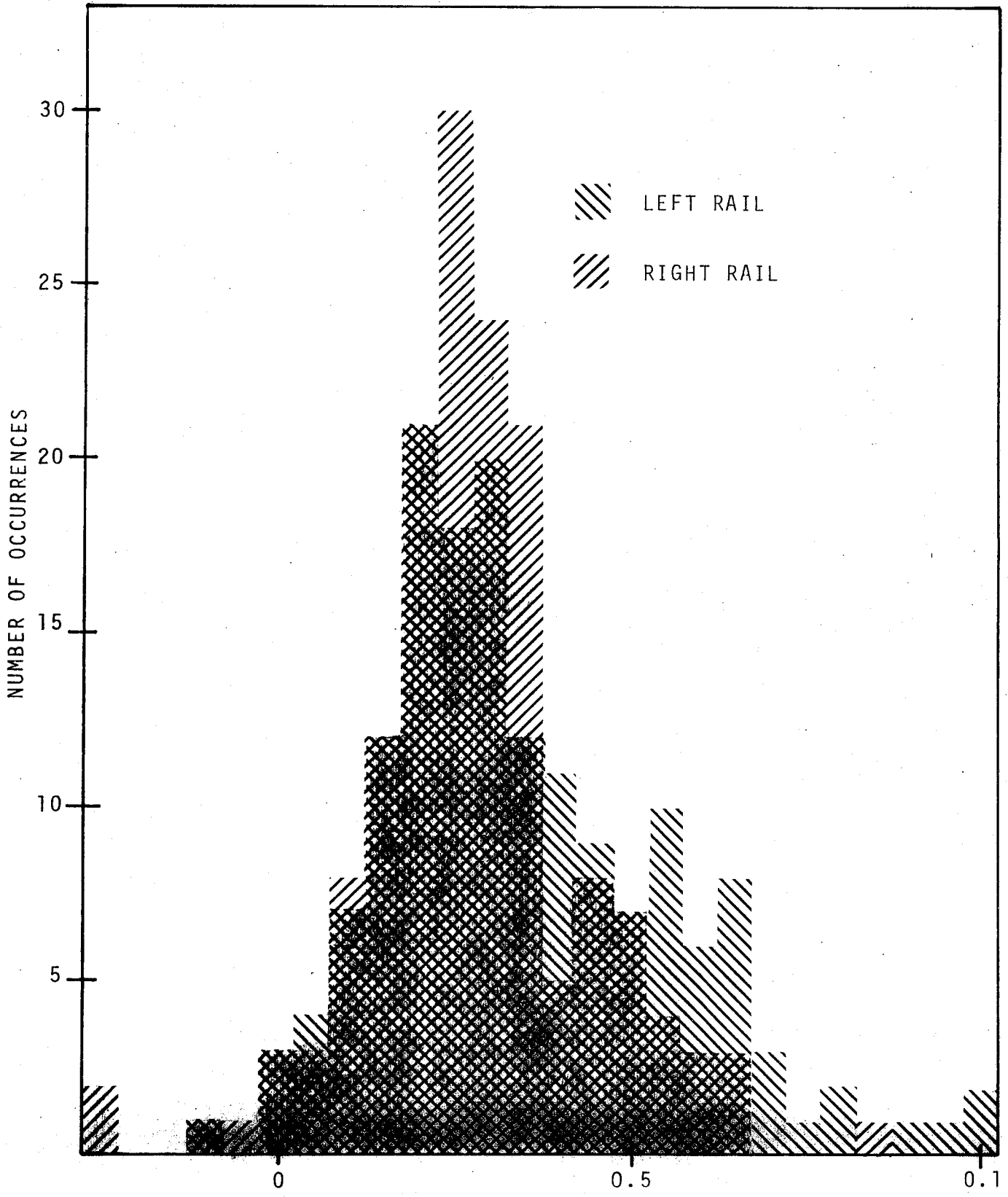


FIGURE 43. ZONE 4 HISTOGRAM FOR 159 LEFT RAIL AND 157 RIGHT RAIL LOW JOINTS

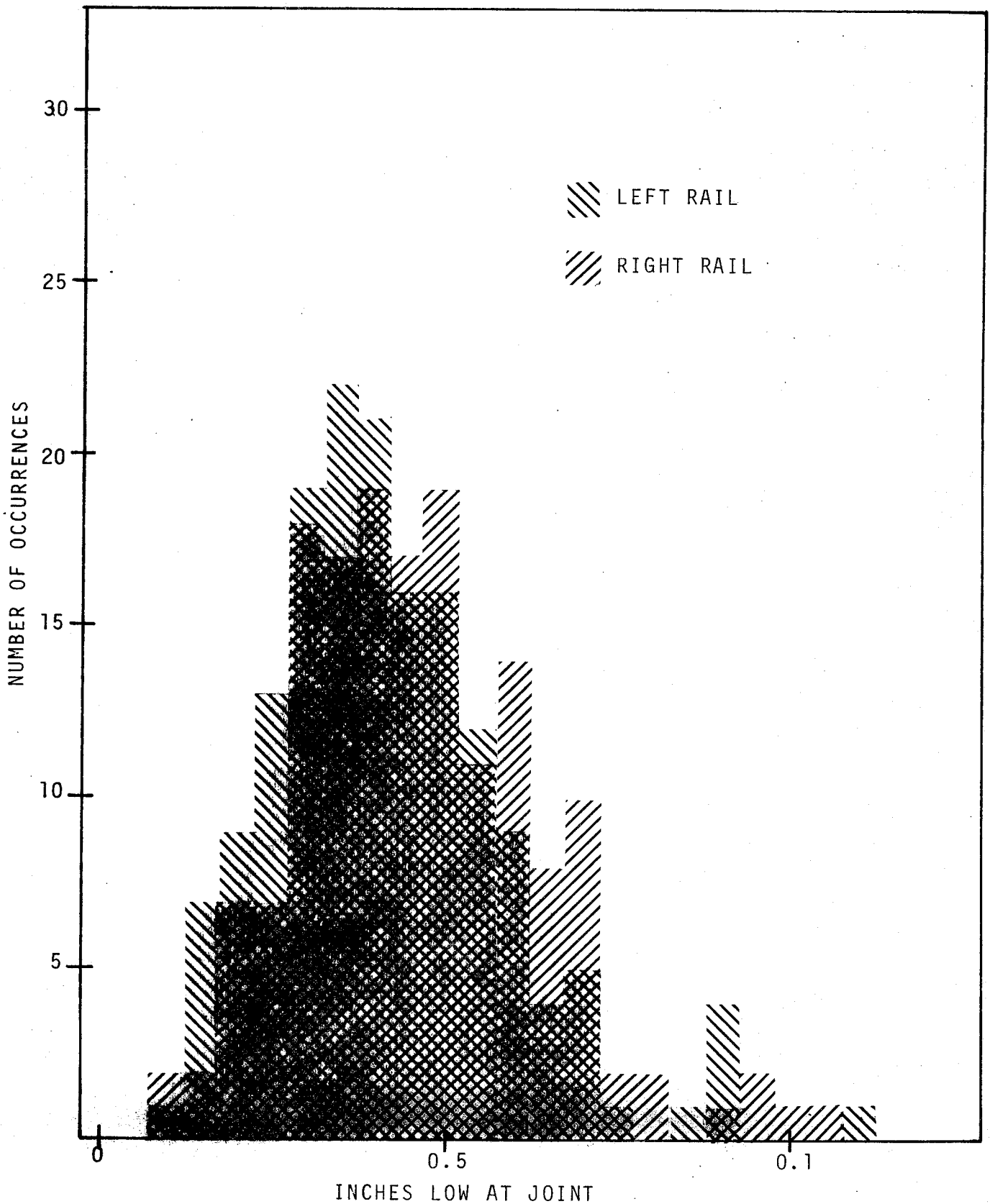


FIGURE 44. ZONE 5 HISTOGRAM FOR 161 LEFT RAIL AND 160 RIGHT RAIL LOW JOINTS

where

c_n = depth of n^{th} joint,

c_m = center amplitude of m^{th} bin

\bar{c} = mean amplitude of low joints,

N = total number of joints surveyed,

n_m = number of joints in amplitude bin c_m

$$N = \sum_{m=1}^M n_m$$

M = total number of amplitude bins.

If data from Figure 42 is used in this formulation, then

$$\bar{c} = 0.28 \text{ inch}$$

which compares favorably with the result of 0.27 inch given in Table 3.

APPENDIX A
 MATHEMATICAL DESCRIPTION OF SPACE CURVE

The space curve data presented in this report is obtained by passing MCO data through a filter with (θ) , that is given as follows:

$$F(\theta) = 1 - \left(\frac{\sin M \theta}{M \sin \theta} \right)^3 - B \sin^2 \theta \prod_{i=1}^5 \frac{\sin N_i \theta}{N_i \sin \theta}$$

where:

- $M = 67, N_1 \dots N_5 = 67, 59, 47, 41, 37$
- $\theta = \text{dimensionless frequency} = \pi x / \lambda$
- $\lambda = \text{wavelength (feet)}$
- $x = \text{sample interval} = 1 \text{ foot, and}$
- $B = \frac{1}{2}(M^2 - 1) = 2244$

A graph of this function is given in Figure A-1.

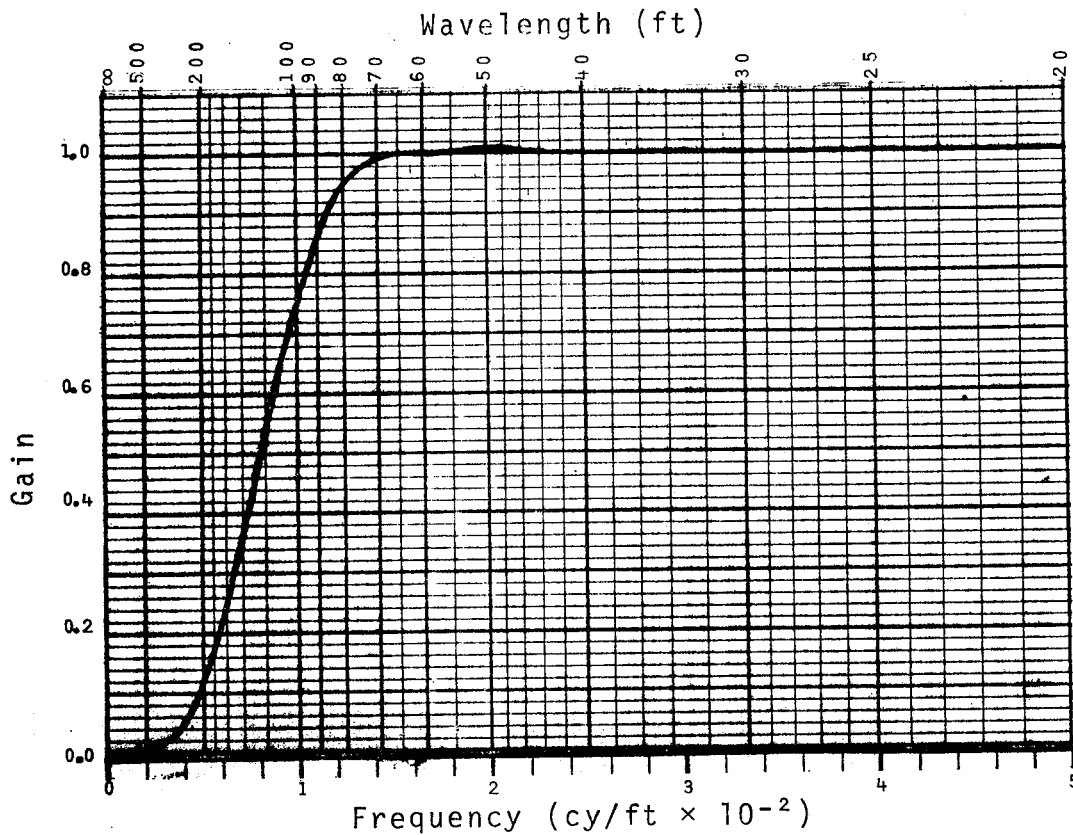


FIGURE A-1. WAVELENGTH FREQUENCY RESPONSE CHARACTERISTICS OF SPACE CURVE

APPENDIX B

FORMULAE FOR VARIANCES AND PEAK VALUES OF TRACK SAFETY MEASURES

The following formulae, extracted from Corbin², relate parameters derived from PSD models to parameters defined in the FRA Track Safety Standards¹.

The standard deviation of a MCO constructed from typical P/A spectra is given by:

$$\sigma_n^2 = \pi^2 A_n s Q(\rho_n, u_n),$$

where

σ_n = standard error (in),

n = parameter index: = 1 and 2 for individual rail profile; = 5 and 6 for individual rail alignment,

A_n = roughness constant for individual rail P/A (in²-cy/ft), and

s = half chord length (ft).

In the above,

$$Q(\rho_n, u_n) = \frac{1}{3} \rho_n u_n^2 + (1 - \rho_n) \left[1 - \frac{1}{u_n} \left(\frac{3}{2} - 2e^{-u_n} + \frac{1}{2}e^{-2u_n} \right) \right]$$

²Ibid.

¹Ibid.

where

$$u_n = 2\pi\phi_{n_4} S$$

$$\rho_n = \left(\phi_{n_3}/\phi_{n_4}\right)^2, \text{ and}$$

ϕ_{n_3} , ϕ_{n_4} are appropriate break frequencies (cy/ft).

Standard errors for crosslevel and gage are given by:

$$\sigma_n^2 = \frac{\pi}{2} \frac{A_n}{\phi_{n_3}},$$

where

$n = 4$ for crosslevel, $= 8$ for gage,

$A_n =$ roughness constant for crosslevel or gage,
and

$\phi_{n_3} =$ appropriate corner frequencies.

Standard errors for warp (crosslevel rate) and for gage rate are given by:

$$\sigma_n'^2 = \frac{\pi A_n}{\phi_{n_3}} = 2\sigma_n^2.$$

The PDP gives rise to harmonically related spectral peaks in the PSD. These are first corrected for the level of the background continuum. The exponential joint model described by Corbin² gives the following envelope function which is applied in a least squares fit to these spectral peaks:

² Ibid.

$$s(\phi) = \frac{s_e(0)}{[1 + (2\pi\phi/k)^2]^2}$$

where k = the joint decay rate (ft^{-1}). This function has the following asymptotic properties:

$$\lim_{\phi \rightarrow 0} s_e(\phi) = s_e(0) \text{ (in}^2/\text{cy/ft)},$$

$$\lim_{\phi \rightarrow \infty} s_e(\phi) = \frac{s_e(0) (k/2\pi)^4}{\phi^4}$$

In the log PSD-level vs. log frequency graphs, these are straight line asymptotes having a slope of 0 and -4, respectively. Their point of intersection defines $s_e(0)$ and a spatial frequency, ϕ_0 , which are related to decay rate and mean joint amplitude, \bar{c} (in), as follows:

$$k = 2\pi\phi_0, \text{ and}$$

$$\bar{c} = \pm\pi L\phi_0 \left(\frac{BS_e(0)}{2} \right)^{1/2}$$

where

L = rail length (ft), and

B = resolution bandwidth of the spectral plot = 10^{-3} cy/ft.

The peak MCO measurement constructed on the exponential joint model PDP will be given by:

$$\hat{\delta} = \bar{c} \left(1 - e^{-k|S-L|} \right).$$

This formula is valid for the 62 foot MCO ($s = 31$) and for typical rail lengths encountered in U.S. practice, i.e. 28-39 feet.