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Track Structure Design Using Mathematical Models

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16. Abstract

The objective of the report is to demonstrate the use of mathematical track structure models in the development of design charts. The models have been developed in Task 1, Mathematical Modelling, of the Track Structures Research Program, Contract DOT-FR-30038. The charts should enable the optimal selection of track components and to evaluate the structural performance of existing track components in a given loading environment. The criterion for acceptable track design is that the strength of the track structure on a fatigue basis not be exceeded and the Miner's rule is used. The charts are based on arbitrarily chosen wheel-rail load magnitudes. For vertical wheel-rail loading, the loading configuration consists of eight wheel loads and corresponds to that of two adjacent trucks of two coupled 100 ton (90,720 kg) cars. For lateral wheel-rail loading, a single lateral load applied to the base of one rail is used.

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PREFACE

This report is the final report of Task 1, Mathematical Modelling, of the Track Structures Research Program, Contract DOT-FR-30038, sponsored by the U. S. Department of Transportation, Federal Railroad Administration.

The valuable suggestions of Dr. Gregory C. Martin, former Director of Dynamics Research, Research and Test Department, Association of American Railroads, and Dr. R. Michael McCafferty, Contracting Officer's Technical Representative of the FRA on the research program, are gratefully acknowledged. The author also wishes to thank Mr. Timothy L. Brown and Mr. Yu-Cheng Jen, students at the Illinois Institute of Technology and research assistants at the Association of American Railroads, for their assistance in performing the numerical computation and in preparing this report.

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CHAPTER 1
INTRODUCTION

1.1. General

The objective of the Track Structures Research Program, Contract DOT-FR-30038, sponsored by the U. S. Department of Transportation, Federal Railroad Administration, is to understand and solve problems in track design. A family of mathematical models, the Track Structure Models, capable of predicting states of stress in track structures due to vertical and lateral wheel loads has been developed as an earlier phase of Task 1 of the program. Discussions of the development of the models have been presented in previous reports [1,2]*. The objective here is to demonstrate the use of the models in combinations in the overall design process. For brevity, in this report, the term "design" will be used loosely. It should be understood that an inherent connection exists between the elements of design and maintenance.

The emphasis of this study is on the development of design charts using the Track Structure Models. The charts are prepared to enable the optimal selection of such track parameters as rail size, tie spacing, ballast depth, type of fastener, and fastener spacing. The components of existing track structures can be evaluated via the design charts as to their performance in a given loading environment.

A review of the literature indicates that available information on track loading environment and fatigue characteristics of track components is inadequate for proper use in the design process described above. Furthermore, available test data are incomplete for full

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*Numbers in brackets refer to corresponding entries in "References".

validation of the Track Structure Models. The development here of design charts using the Track Structure Models should be seen only as a demonstration of the utility of the models. Additional test data obtained under carefully controlled conditions are needed before the design approach presented here can be accepted as definitive. For the present limited investigation, only linear versions of the models will be used in developing design charts.

1.2 Report Organization

The mathematical models used in the study are presented in Chapter 2 with brief discussions of their characteristics and capabilities. The method used to ensure compatibility among models is described. Chapter 3 outlines the approach of the study and the criterion for design. The use of the design charts is illustrated. For the design charts, the various applied loadings are described in Chapter 4 and the track parameters are presented and justified in Chapter 5. Chapter 6 presents the design charts with brief discussions.

CHAPTER 2
MATHEMATICAL MODELS

2.1 General

For the investigation of track design parameters conducted here, the finite element models are chosen from the Track Structure Models for their versatility. The only exception is Burmister's Multi-layer Elastic System used to simulate ballast and subgrade. It requires far less computational time than the finite element model available for simulating ballast and subgrade, and the results are acceptable for the present limited investigation. The models used are shown in Table 2.1. As mentioned earlier, the nonlinear capabilities of the models are not utilized here.

2.2 Brief Description of Models

The Finite Element Vertical Track Model is used to simulate the track structure under vertical loading. It predicts vertical rail-tie reactions from vertical wheel-rail loading. The rail-tie reactions are then used as input to the Finite Element Tie Model. The model also predicts rail bending moments, shearing forces and deflections. It is formulated as a two-dimensional finite element model using one-dimensional members to simulate rails, rail joints, and tie-foundation stiffness. Because of the discrete nature of the elements, it is possible to simulate track irregularities such as ineffective ties and fasteners. It can also simulate nonlinear foundation support characteristics.

The Finite Element Lateral Track Model simulates track structure behavior under lateral loading. It is used to develop lateral rail-tie loads. It also predicts rail bending moments, deflections, and shearing forces in

Table 2.1. Mathematical Models Utilized For Design Charts

(1)	Finite Element Vertical Track Model
(2)	Finite Element Lateral Track Model
(3)	Finite Element Tie Model
(4)	Rail Fastener Model
(5)	Burmister's Multi-layer Elastic System

the lateral plane. The formulation and capabilities of the Finite Element Lateral Track Model are similar to those of the vertical track model.

The Finite Element Tie Model is also a two-dimensional model with similar formulation and capabilities. It develops vertical and lateral tie-ballast loading from the vertical and lateral rail-tie loading output of the Finite Element Vertical and Lateral Track Models. It also predicts tie deflections, bending moments, and shearing forces. The tie-ballast loading is used as input to Burmister's Multi-layer Elastic System.

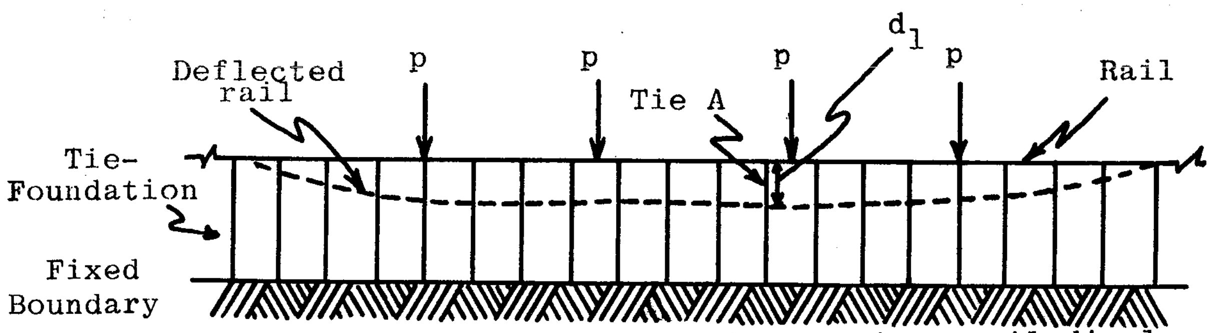
The Rail Fastener Model simulates the behavior of rail fasteners in a track structure under vertical or lateral wheel loading. It is formulated as a three-dimensional finite element model using one-dimensional elements to simulate fasteners, rails, ties, and foundation. The model can incorporate nonlinear response characteristics of foundation and fasteners as well as track irregularities.

Burmister's Multi-layer Elastic System gives stresses and displacements in multi-layer foundations due to vertical loads distributed uniformly over circular areas. Tie-ballast reactions from the Finite Element Tie Model are approximated into a series of uniform circular loads as model input. The model is formulated as a semi-infinite solid composed of multi-layers of homogeneous, isotropic, and linearly elastic materials. The stresses and displacements are assumed to be continuous at the boundaries between the layers.

2.3 Model Compatibility

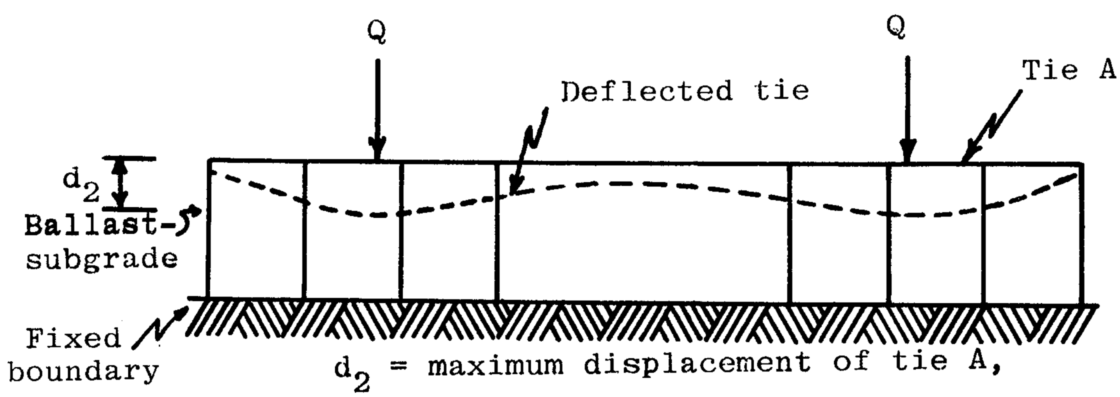
Different mathematical models, used in combination with the output of one model serving as input to another model, should be compatible at their common boundary with regard to both forces and displacements. Force compatibility is assured, since the reactions from one model are input to another model at the common boundary. Model parameters are adjusted in successive iterations to obtain displacement compatibility. In Figure 2.1, for displacement compatibility, $d_1 = d_2 = d_3$. The model parameters adjusted are tie-foundation stiffness in the Finite Element Vertical Track Model, ballast-subgrade stiffness in the Finite Element Tie Model, and Young's modulus of ballast-subgrade in Burmister's Multi-layer Elastic System.

Finite Element Vertical Track Model:



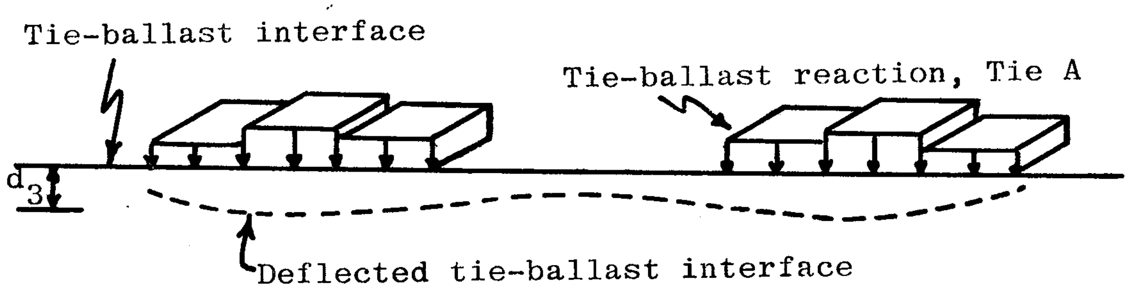
P = wheel load, d_1 = maximum rail displacement

Finite Element Tie Model, Tie A:



d_2 = maximum displacement of tie A,
Q = rail-tie reaction

Burmister's Multi-layer Elastic System:



d_3 = maximum displacement of tie-ballast

Figure 2.1. **Displacement Compatibility Between Models;** $d_1 = d_2 = d_3$

CHAPTER 3
APPROACH

3.1 General

The approach here emphasizes the development of design charts for track components and the application of the design charts to fatigue failure in track design. The following sections of this chapter describe the scope of the study, purpose and methods of design of track structures, and the theory and application of a fatigue failure criterion with illustration of the use of the design charts.

3.2 Design Approach

The scope of the study is as follows:

(a) The study is aimed at determining stresses in the track due to applied vertical and lateral rail head loads for use as a design tool.

(b) The approach assumes the existence of data on the wheel-rail loading environment.

(c) The study neglects longitudinal rail loads.

(d) The models used are quasi-static, i.e., they assume the stress in the track structure can be accurately predicted from the dynamic wheel loads without considering inertia effects of the track system.

(e) The study does not consider design in areas involving special trackwork.

(f) The study utilizes an iterative design process in which a design is chosen and then evaluated with respect to its loading environment.

(g) The design considers the ability of the track to withstand fatigue loading but not sudden instabilities such as lateral track instability.

(h) The design does not consider economics or vehicle performance.

Determination of stresses in the track structure is for the following purposes:

(a) determination of the loading environment for the ballast and subgrade for design against excessive vertical track settlement,

(b) determination of the loading environment for the ballast for design against excessive lateral displacement,

(c) determination of the loading environment for rails for use in rail design,

(d) determination of the loading environment for ties for use in tie design, and

(e) determination of the loading environment for rail fasteners for use in rail fastener design.

The essential criterion for design against excessive vertical track settlement is that the strength of the ballast and subgrade on a fatigue basis not be exceeded. The strength of the ballast and subgrade is measured by fatigue testing the ballast and subgrade materials under increasing stress levels induced by a rolling load until excessive vertical settlement occurs. Ballast and subgrade stresses in the track are determined as follows. The rail-tie load is developed in terms of the wheel-rail load via the Finite Element Vertical Track Model. The tie-ballast load is then developed from the rail-tie load with the Finite Element Tie Model. The ballast-subgrade stresses are then found from the tie-ballast loading using Burmister's Multi-layer Elastic System.

The design criterion used for design against excessive lateral displacement is that the tie-ballast load capacity on a fatigue basis not be exceeded. The tie-ballast load capacity is measured by fatigue testing the ballast material under increasing lateral tie load levels until excessive lateral tie displacement occurs. To determine the tie-ballast load in the track, the procedure used is to develop the rail-tie load from the wheel-rail load using the Finite Element Lateral Track Model. The tie-ballast load is obtained by summing the rail-tie loads.

The loading environment for rails for use in rail design is determined by use of the Finite Element Vertical and Lateral Track Models. The rail stresses from the two models are superimposed for combined vertical and lateral wheel-rail loading.

The procedure for determining the tie loading environment for tie design is similar to that used for design against vertical track settlement. To accomplish the determination of the loading environment of the tie, the Finite Element Vertical and Lateral Track Models are used to develop the rail-tie reactions from the wheel-rail loading. The rail-tie reactions are input to the Finite Element Tie Model to develop the bending moment of the tie.

The model used here for simulating the rail fastener environment is the Rail Fastener Model. This three-dimensional finite element model inputs the wheel-rail loading and predicts the fastener loading.

3.3 Design Criterion and Use of Design Charts

Based on the foregoing approach, design charts can be generated for rail bending moments, tie bending moments, fastener moments, and ballast-subgrade stresses due to vertical and lateral wheel loads by utilizing the mathematical models in combinations. The design charts enable the determination of the maximum stress level reached in a track component given the wheel loading on the track and the track structure design. A typical design chart is shown in Figure 3.1 where the maximum component stress S is plotted against the component design D for various wheel or car loads P.

Thus, track designs can be evaluated using the charts and the design procedure can be iterated until acceptable designs are reached, using the following criterion for component stress levels.

Since the track structure is subjected to cyclic loading, the criterion for acceptable stress level is fatigue failure. The cyclic nature of the loading is illustrated in a typical load spectrum in Figure 3.2. The magnitude of the wheel or car load P is plotted versus the number of cycles n for a certain period. From the load spectrum and the design charts, the number of cycles of occurrence of any stress level reached in a track component can be obtained. For example, the stress levels reached for a track component design D_1 under wheel loads P_1, P_2 and P_3 are respectively S_1, S_2 and S_3, from Figure 3.1. From Figure 3.2, it can be seen that these stress levels occur n_1, n_2 and n_3 cycles, respectively.

A commonly used design criterion for fatigue is Miner's rule [3];

$$\frac{n_1}{N_1} + \frac{n_2}{N_2} + \frac{n_3}{N_3} + \ldots + \frac{n_m}{N_m} = 1$$

or; $\Sigma \frac{n}{N} = 1$

where n_1, n_2, n_3, ... n_m are the numbers of cycles applied at the different stress levels S_1, S_2, S_3, ..., S_m respectively and N_1, N_2, N_3, ..., N_m are the numbers of cycles to failure at the corresponding stress levels S_1, S_2, S_3, ..., S_m.

According to this equation, the damage at a stress level is directly proportional to the number of cycles at that level and the total damage at mixed stress levels is equal to the sum of damages at all stress levels.

The number of cycles to failure, N, is obtained by fatigue testing the structural component at constant load

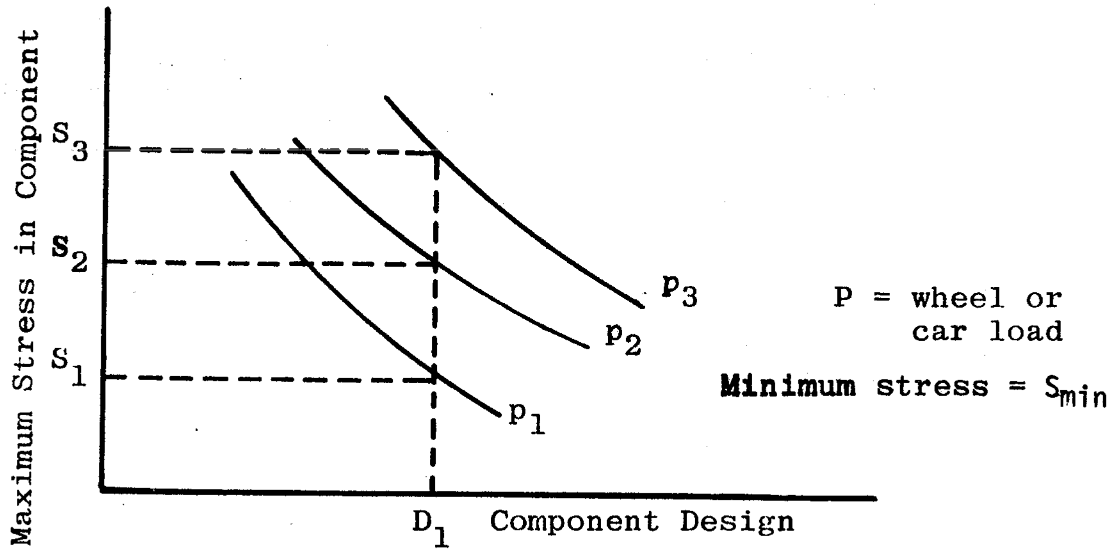


Figure 3.1. Typical Design Chart

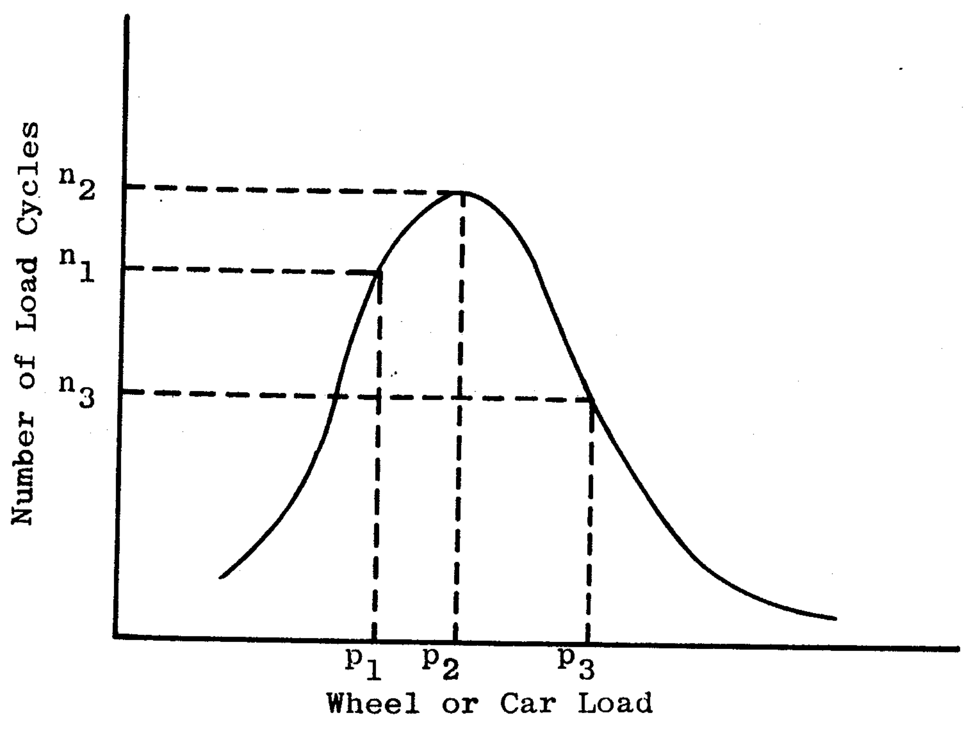


Figure 3.2. Typical Load Histogram

amplitude. A typical S-N curve is plotted in Figure 3.3. It should be noted that S-N data are defined by S_{max} and S_{min}, the maximum and minimum stress levels for the load amplitude used in testing. The same data are also commonly plotted as S_{max} versus S_{min} curves for various failure cycles N (called Goodman diagrams). With these diagrams and the track design charts showing S_{max} and S_{min}, failure cycles N can be found. For example, the failure cycles for the maximum stress levels S_1, S_2 and S_3 in a track component design D_1 are respectively N_1, N_2 and N_3, from Figure 3.3. Using Miner's rule, the track component design D_1 can then be accepted or rejected.

To evaluate an existing track structure, Miner's rule can be rewritten as:

$$T\left(\frac{f_1}{N_1} + \frac{f_2}{N_2} + \frac{f_3}{N_3} + \ . \ . \ . + \frac{f_m}{N_m}\right) = 1$$

$$T = 1 \ / \left(\frac{f_1}{N_1} + \frac{f_2}{N_2} + \frac{f_3}{N_3} + \ . \ . \ . + \frac{f_m}{N_m}\right) = 1 \ / \Sigma\frac{f}{N}$$

where T is the expected life of a track structure component in years,

and f_1, f_2, f_3, . . ., f_m are the numbers of cycles per year applied at the different stress levels S_1, S_2, S_3, . . ., S_m respectively.

The expected life of track structure components can thus be computed and maintenance can be determined on this basis.

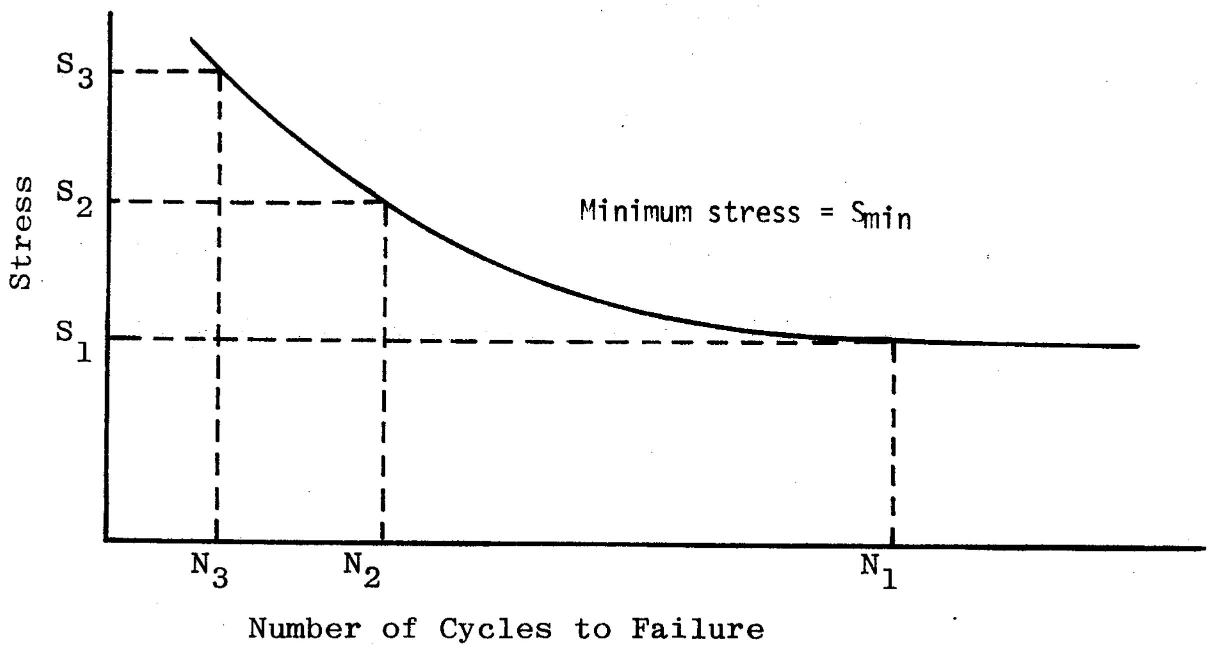


Figure 3.3. Typical S-N Curve

CHAPTER 4
LOADING

4.1 General

The relation between the applied wheel-rail loading and the resulting stresses in track components is linear for the models used in this study. Hence the stresses in track components due to any given wheel-rail load can be obtained by proportion from the stresses due to a single arbitrary load magnitude.

Design charts are presented in Chapter 6 giving stresses or moments in track components due to the following quasi-static loads: 10,000 lb (4,536 kg) vertical wheel loads on both rails, a 10,000 lb (4,536 kg) lateral load applied to one rail, and a 10,000 lb.in (1,130 N.m) moment about the longitudinal axis of one rail. The track stresses due to any given wheel-rail load are obtained by first resolving the load into vertical and lateral components. The ratio of each load component to 10,000 lb (4,536 kg) is multiplied by the track stresses due to the 10,000 lb (4,536 kg) load. The resulting track stresses due to vertical and lateral load components are superimposed to obtain the track stresses due to the given wheel-rail load. The moment about the longitudinal rail axis is used in Chapter 6 to develop the fastener loading environment for use in fastener design. Direct proportioning also applies to moments.

4.2 Vertical Wheel-Rail Loading

A typical train consist currently operating on a main line would have roughly the distribution of cars given in Table 4.1. There is a trend for 100 ton (90,720 kg) cars to replace 70 ton (63,500 kg) cars. Consequently, a loading configuration formed by the two adjacent trucks of two coupled 100 ton (90,720 kg) cars is chosen here as the typical loading configuration for the development of

Table 4.1. Typical Distribution of Cars in Train Consist

100 ton (90,720 kg)	20%
70 ton (63,500 kg)	50%
50 ton (45,360 kg)	25%
Under 50 ton (45,360 kg)	5%

design charts. This loading configuration is shown in Figure 4.1. The magnitude of each wheel load on both rails, as explained earlier, is arbitrarily chosen to be 10,000 lb (4,536 kg). To limit the scope of the present investigation, unequal wheel loads from a wheel-axle set are not considered.

4.3 Lateral Wheel-Rail Loading

A single isolated lateral load of 10,000 lb (4,536 kg) applied to one rail is used as model input for determining stresses due to lateral wheel-rail loading. The total stress field in the track structure is obtained by superimposing stresses due to lateral and vertical loads. However, statistical information on the occurrence of lateral loads in conjunction with vertical loads is required for meaningful superposition.

4.4 Rail Fastener Loading

Two components of fastener loading are input separately to the Rail Fastener Model, a lateral load of 10,000 lb (4,536 kg) applied at the base of one rail and a 10,000 lb.in (1,130 N.m) moment about the longitudinal axis of the same rail. The effects of each loading component are presented separately. For fastener design, the stresses due to the two effects are superimposed in the appropriate ratio to produce the stresses due to a lateral load applied at a distance above the base of one rail. Other loading conditions, such as vertical loads causing rotation of fasteners about the transverse axis of the rail, are relatively insignificant and are omitted from the study.

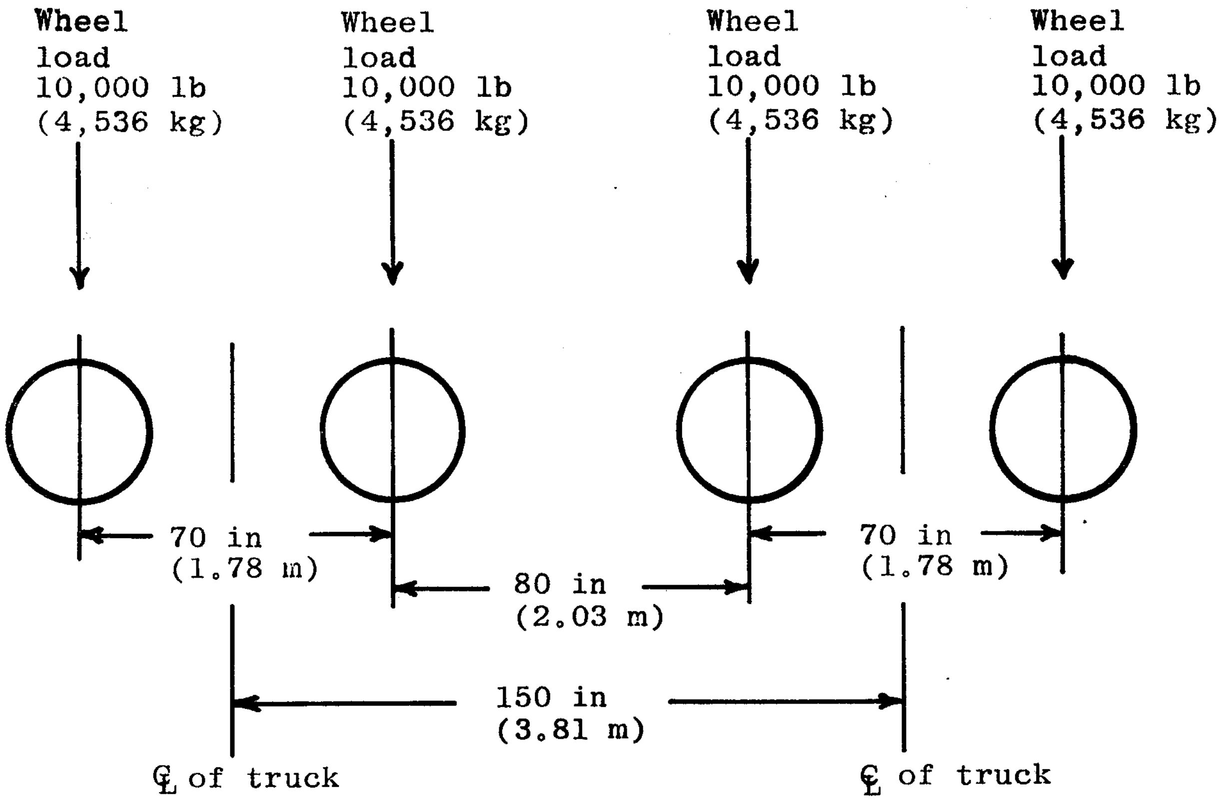


Figure 4.1. Quasi-Static Loading Configuration Formed by Two 100 ton (90,720 kg) Cars Coupled Together

CHAPTER 5
TRACK PARAMETERS

5.1 General

Variations in the following track parameters are incorporated in the design charts; rail size, tie spacing, rail fastener rotational stiffness, tie support conditions, ballast depth, lateral tie-foundation stiffness, and Young's modulus of subgrade. In order to limit the scope of the study, no track irregularities such as ineffective ties and rail joints are considered although the models have such capabilities. In the following sections, the ranges of variation for the different parameters are established and a reference track is established as a basis for the parameter variation.

5.2 Parameter Variations

It is important that the ranges of parameter variation encompass the full range of track design practice so that the need to extrapolate results can be minimized. The ranges of parameter variation are discussed in the following paragraphs.

115 RE (57.0 kg/m) rail and all heavier recommended AREA rail sizes [4] are represented in the design charts.

Tie spacing is varied in the design charts from 20 in (50.8 cm) to 28 in (71.1 cm) with 2 in (5.1 cm) increments. This covers the range of present practice of U. S. railroads for both wood and concrete ties. The 7 in x 9 in x 8.5 ft (17.8 cm x 22.9 cm x 2.59 m) tie size is the standard wood tie being used in track design. As such, no variation in tie size is considered. The tie support conditions represented in the design charts are shown in Figure 5.1.

Spikes are the most commonly used rail fasteners. Various types of elastic fasteners are used with concrete ties. Complete test data on the ranges of fastener

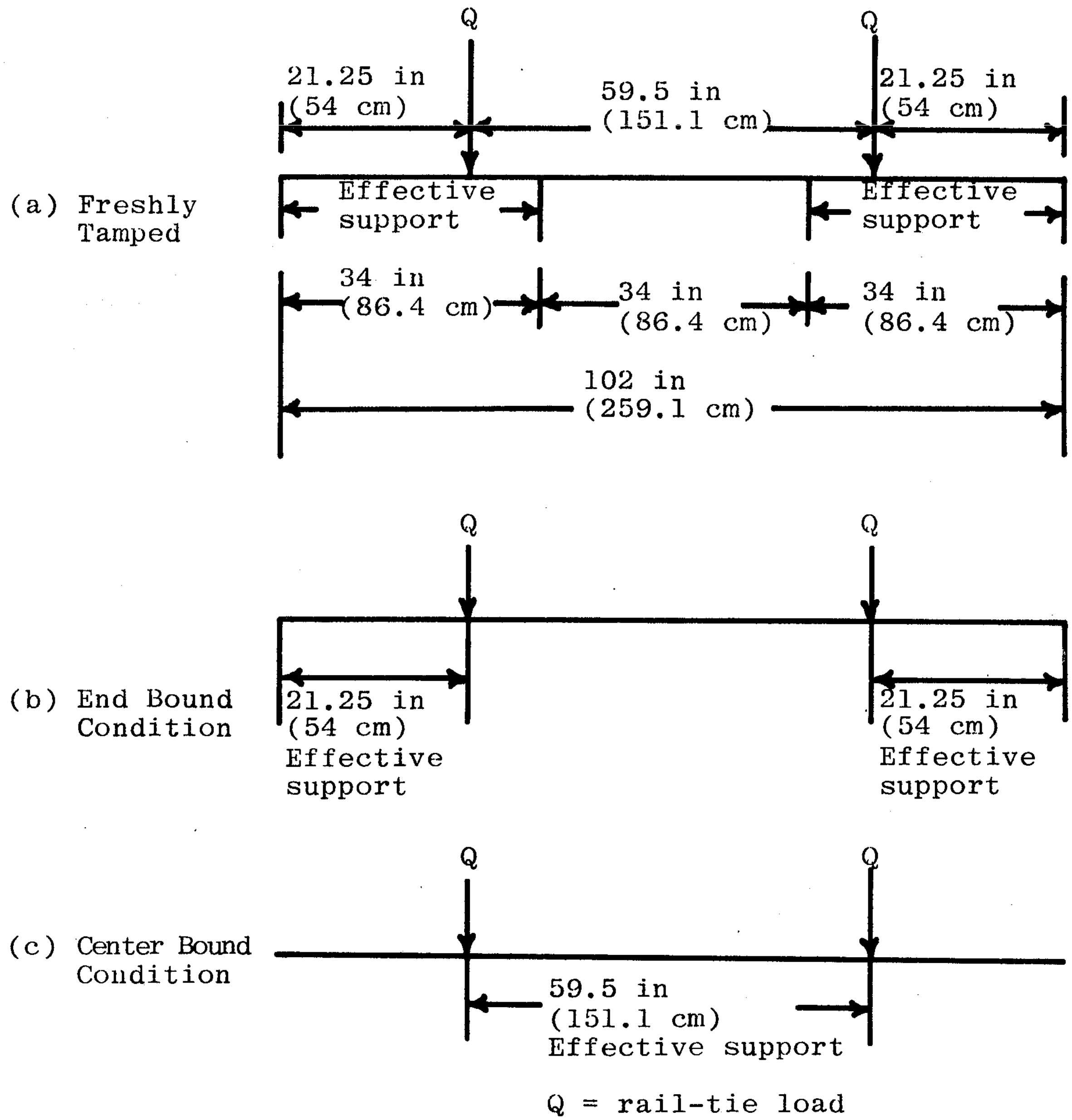


Figure 5.1. Tie Support Conditions

rotational stiffnesses about the vertical, longitudinal, and transverse rail axes are not available. The upper and lower bounds for fastener rotational stiffnesses are arbitrarily assumed in this investigation.

The design charts include a range of ballast depths from 6 in (15.2 cm) to 24 in (61.0 cm). Young's modulus of subgrade is varied from 2,000 psi (13.79 MPa) to 8,000 psi (55.16 MPa) with 2,000 psi (13.79 MPa) increments. For 20 in (50.8 cm) tie spacing and 12 in (30.5 cm) ballast depth, these values correspond to vertical tie-foundation stiffnesses ranging from 1,020 lb/in/in (71.7 kg/cm/cm) to 3,960 lb/in/in (278.4 kg/cm/cm) as obtained from the models used here. These values represent subgrades ranging from soft to stiff. Young's modulus of ballast is relatively constant for the ballast materials used in track design. As such, Young's modulus of ballast is maintained at 35,000 psi (241.3 MPa) for this investigation.

Precise test data is not available for lateral tie-foundation stiffness of track under vertical loading. It is estimated to range from 500 to 1,500 lb/in/in (35.2 to 105.5 kg/cm/cm) for track loaded with 100 ton (90,720 kg) cars and wood ties spaced at 20 in (50.8 cm).

A summary of the parameter variations is given in Table 5.1. A standard reference track is established in the next section. Each parameter is varied individually with the other parameters maintained in the reference configuration.

5.3 The Reference Track

The 132 RE (65.5 kg/m) rail size is chosen for the reference track. Ties are 7 in x 9 in x 8.5 ft (17.8 cm x 22.9 cm x 2.59 m) wood ties spaced at 20 in (50.8 cm), and have vertical flexural stiffness of 517.1×10^6 lb.in^2 (151.3×10^3 kg.m^2). Rail fasteners are selected with rotational stiffness about the vertical rail axis of

Table 5.1. Parameter Variation

Parameter	Units	Values				
Rail Size	RE Rail Size kg/m	115 57.0	119 59.0	132 65.5	136 67.5	140 69.4
Tie Spacing	in cm	20 50.8	22 55.9	24 61.0	26 66.0	28 71.1
Tie Support Condition		End Bound, Freshly Tamped, Center Bound				
Fastener Rotational Stiffness						
About Vertical Axis of Rail	lb.in/radian x 10^6 kN.m/radian	3 339	6 678	9 1017	12 1356	15 1695
About Longitudinal Axis of Rail	lb.in/radian x 10^6 kN.m/radian	1 113	1.5 169	2 226	2.5 283	3 339
About Transverse Axis of Rail	lb.in/radian x 10^6 kN.m/radian	2 226	3 339	4 452	5 565	6 678
Ballast Depth	in cm	6 15.2	12 30.5	18 45.7	24 61.0	
Young's Modulus of Subgrade	psi MPa	2000 13.79	4000 27.58	6000 41.37	8000 55.16	
Lateral Tie-Foundation Stiffness	lb/in/in kg/cm/cm	500 35.2	750 52.7	1000 70.3	1250 87.9	1500 105.5

9×10^6 lb.in/radian (10.17×10^5 N.m/radian), about the transverse rail axis of 4×10^6 lb.in/radian (4.52×10^5 N.m/radian), and about the longitudinal rail axis of 2×10^6 lb.in/radian (2.26×10^5 N.m/radian). The ties are assumed to be supported by the ballast for one third of their length under each end of the ties. This condition can be considered a reasonable approximation for freshly tamped track on the basis of the maintenance procedures of the railroads. The ballast depth is chosen to be 12 in (30.5 cm). Lateral tie-foundation stiffness is chosen as 1,000 lb/in/in (70.3 kg/cm/cm) and Young's modulus of the subgrade as 4,000 psi (27.58 MPa).

CHAPTER 6
DESIGN CHARTS

6.1 General

Design charts for the following track components are presented in this chapter:

a) rail bending moments in the vertical and lateral planes

b) tie bending moments due to vertical loading

c) rail fastener moments

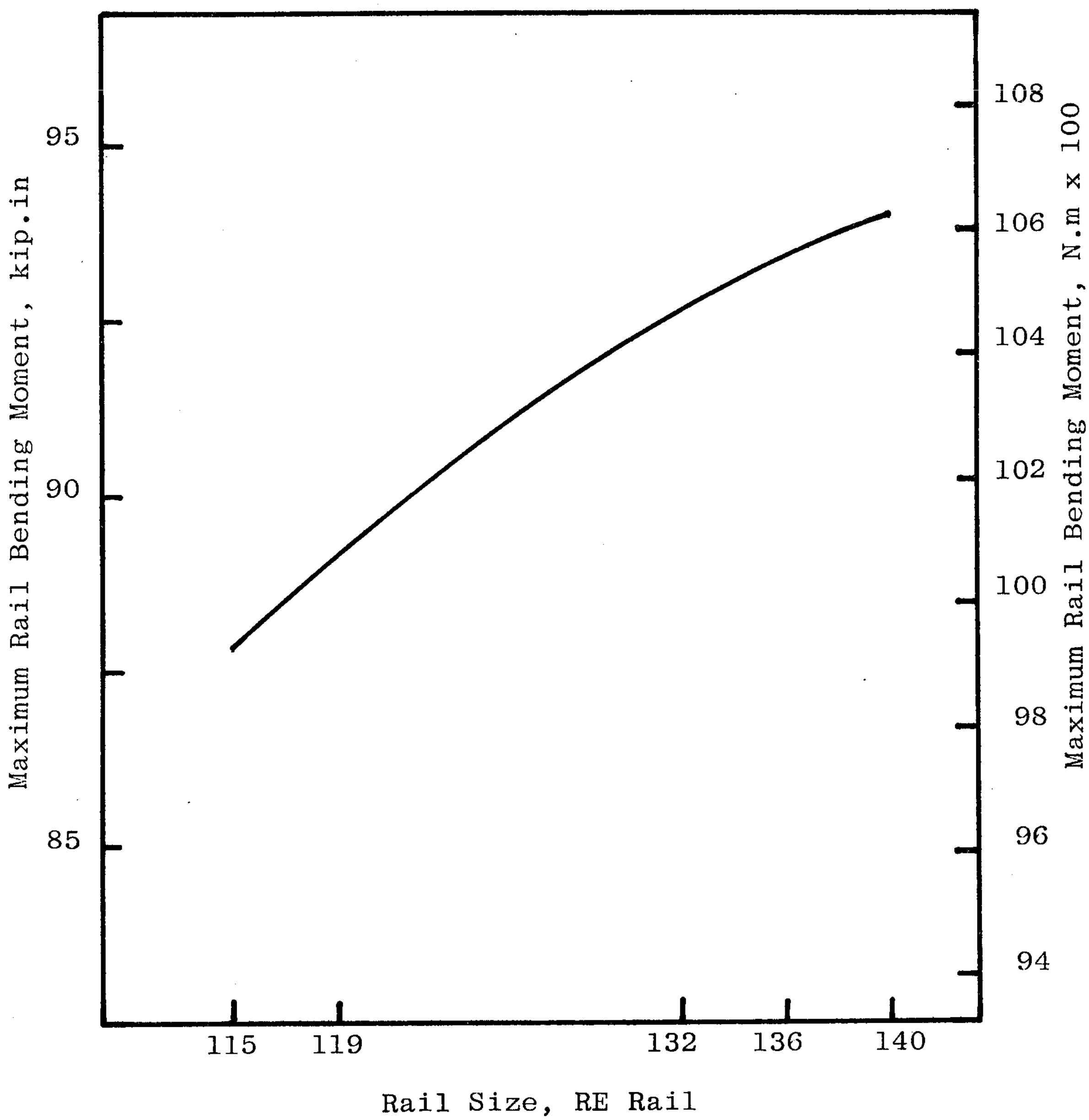
d) maximum shearing stresses and vertical normal stresses in subgrade due to vertical loads

e) lateral tie-ballast loading

The design charts do not consider stresses in ballast due to vertical loading, stresses in subgrade due to lateral loading, and tie bending moments due to lateral loading since they are rarely critical. Brief discussions of the design charts appear in the following sections. It should be noted that the comments in the discussions apply only to the ranges of parameters in the study.

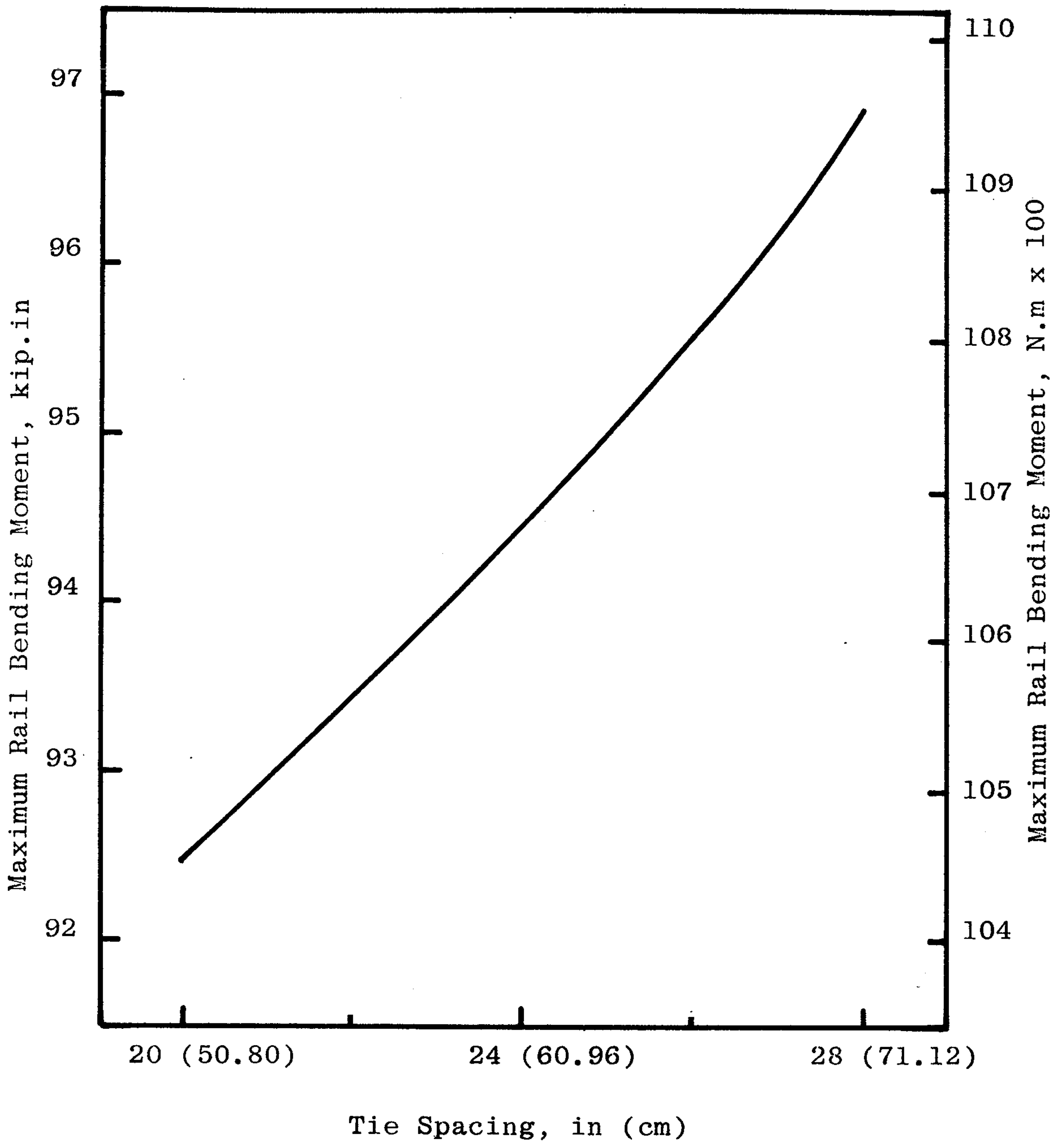
6.2 Rail Bending Moments

Figures 6.1 through 6.3 present the effects of variations in rail size, tie spacing, and subgrade modulus of elasticity on maximum rail bending moments in the vertical plane. Variations in rail size from 115 RE (57.0 kg/m) to 140 RE (69.4 kg/m) rail and tie spacing from 20 in (50.8 cm) to 28 in (71.1 cm) have slight effects on rail bending moments. Variation in subgrade modulus of elasticity, however, has a significant effect. Rail bending moments are 27% higher for track with Young's modulus of subgrade of 2,000 psi (13.79 MPa) versus 8,000 psi (55.16 MPa). Variations in tie support condition and ballast depth do not significantly affect rail bending moments in the vertical plane.



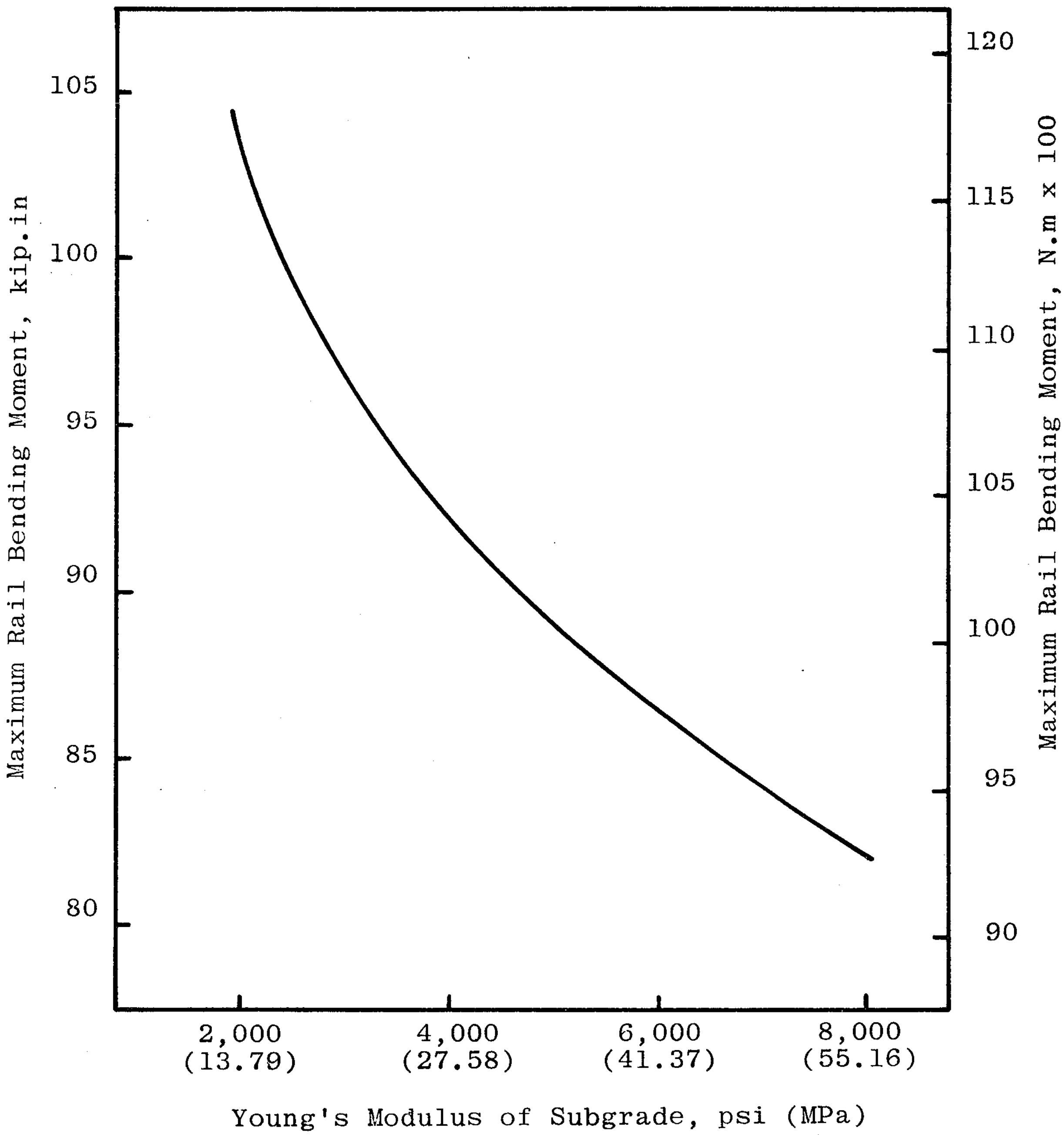
Note: frequency of occurrence = frequency of wheel load;
minimum moment - (50% maximum moment)

Figure 6.1. Maximum Vertical Rail Bending Moment versus Rail Size



Note: frequency of occurrence = frequency of wheel load; minimum moment - (50% maximum moment)

Figure 6.2. Maximum Vertical Rail Bending Moment versus Tie Spacing



Note: frequency of occurrence = frequency of wheel load; minimum moment - (50% maximum moment)

Figure 6.3. Maximum Vertical Rail Bending Moment versus Young's Modulus of Subgrade

Table 6.1 and Figures 6.4 through 6.6 show variations in maximum lateral rail bending moment as functions of rail size, tie spacing, lateral tie-foundation stiffness, and fastener rotational stiffness about the vertical rail axis. Rail size is varied from 115 RE (57.0 kg/m) to 136 RE (67.5 kg/m) rail. The maximum lateral rail bending moment increases 8% over this range. Tie spacing is varied from 20 in (50.8 cm) to 28 in (71.1 cm), lateral tie-foundation stiffness from 500 lb/in/in (35 kg/cm/cm) to 1,500 lb/in/in (106 kg/cm/cm), and fastener stiffness from 3×10^6 lb.in/radian (3.39×10^5 N.m/radian) to 15×10^6 lb.in/radian (16.95×10^5 N.m/radian). Lateral rail bending moments vary 12 to 15% for these parameter variations.

Superposition of rail stresses due to vertical and lateral loading will determine the total stress field in the rail for design purposes. However, statistical data on the occurrence of combined vertical and lateral loading are needed for meaningful superposition.

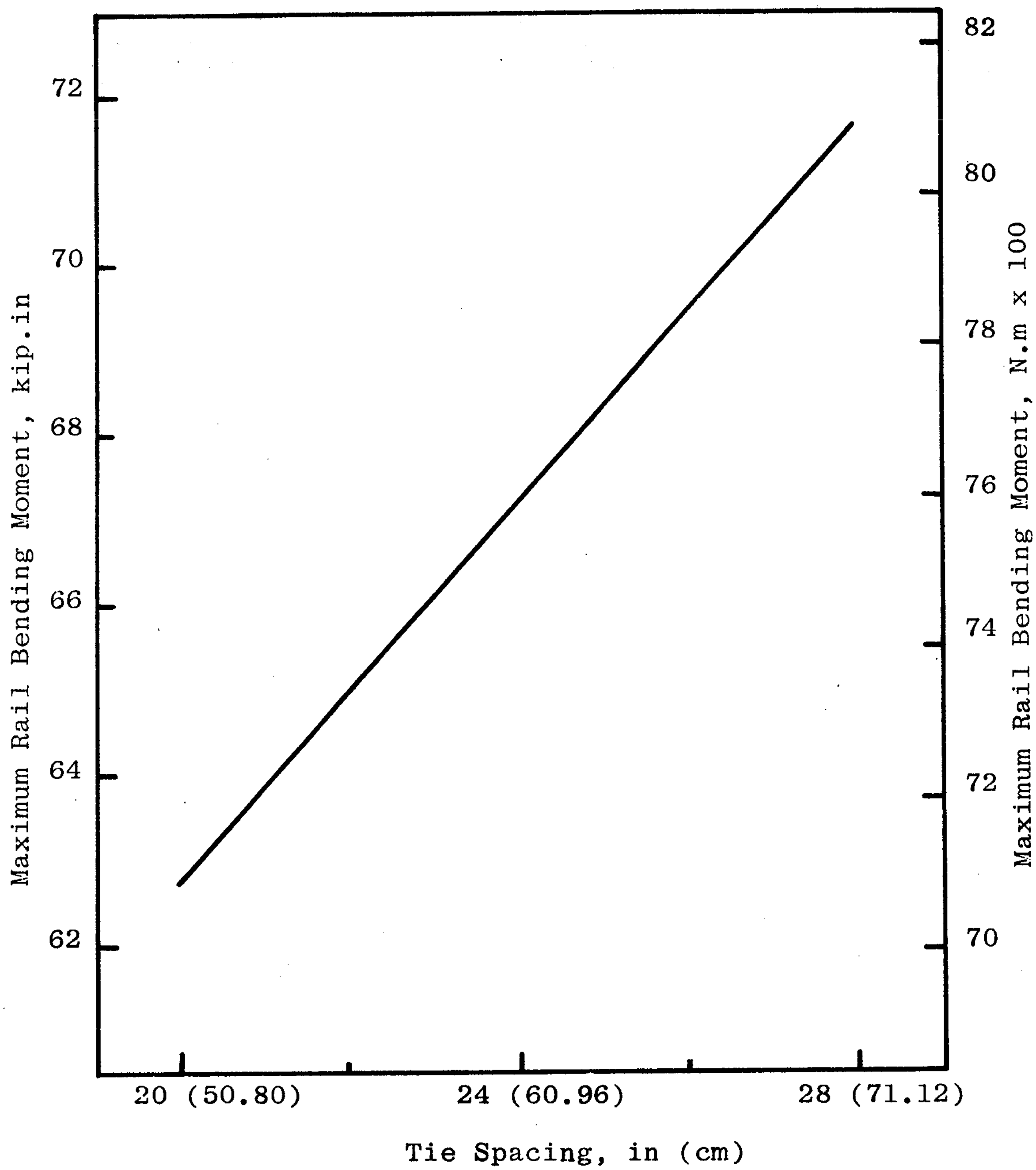
6.3 Tie Bending Moments

Tie bending moments are investigated for various track conditions subjected to the loading defined in Section 4.2--Vertical Wheel-Rail Loading. Figures 6.7 and 6.8 show the variation in maximum tie bending moment as rail size is varied from 115 RE (57.0 kg/m) to 140 RE (69.4 kg/m) and tie spacing from 20 in (50.8 cm) to 28 in (71.1 cm). The maximum bending moment is not appreciably affected by the changes in rail size, but increases 36% over the range of increase in tie spacing. Table 6.2 shows maximum tie bending moments for three tie support conditions: normal (representing freshly tamped track), end-bound, and center-bound. For the end-bound condition, maximum tie bending moments are increased 74% and, for the center-bound, 140% over normal. Figure 6.9 shows that the

Table 6.1. Maximum Lateral Rail Bending Moment versus Rail Size

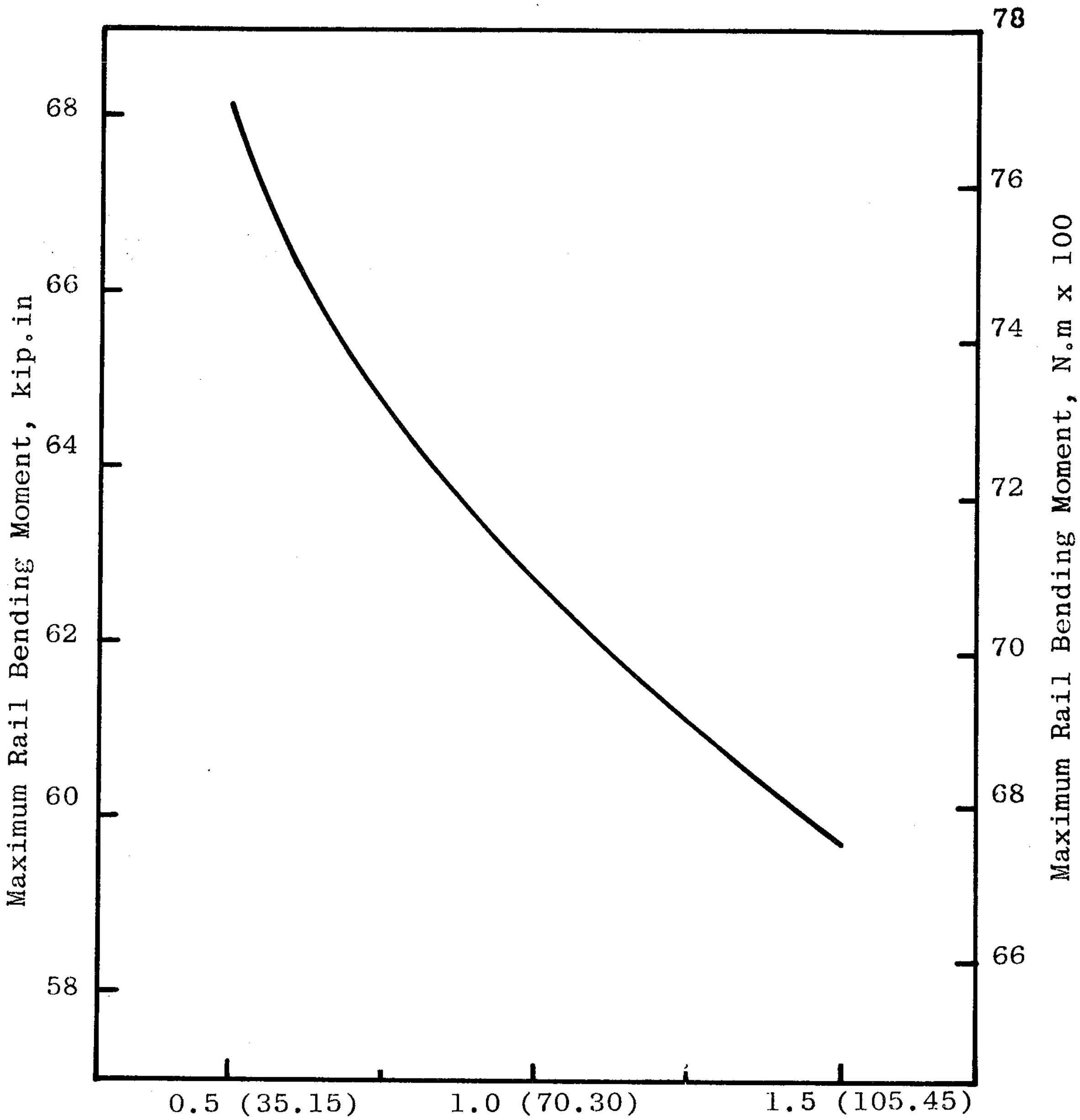
Rail Size		Maximum Lateral Rail Bending Moment	
RE rail	kg/m	kip.in	N.m x 100
115	57.0	58.1	65.6
119	59.0	58.3	65.9
132	65.5	62.8	71.0
136	67.5	62.9	71.1

Note: frequency of occurrence = frequency of wheel load; minimum moment - (10% maximum moment)



Note: frequency of occurrence = frequency of wheel load;
minimum moment - (10% maximum moment)

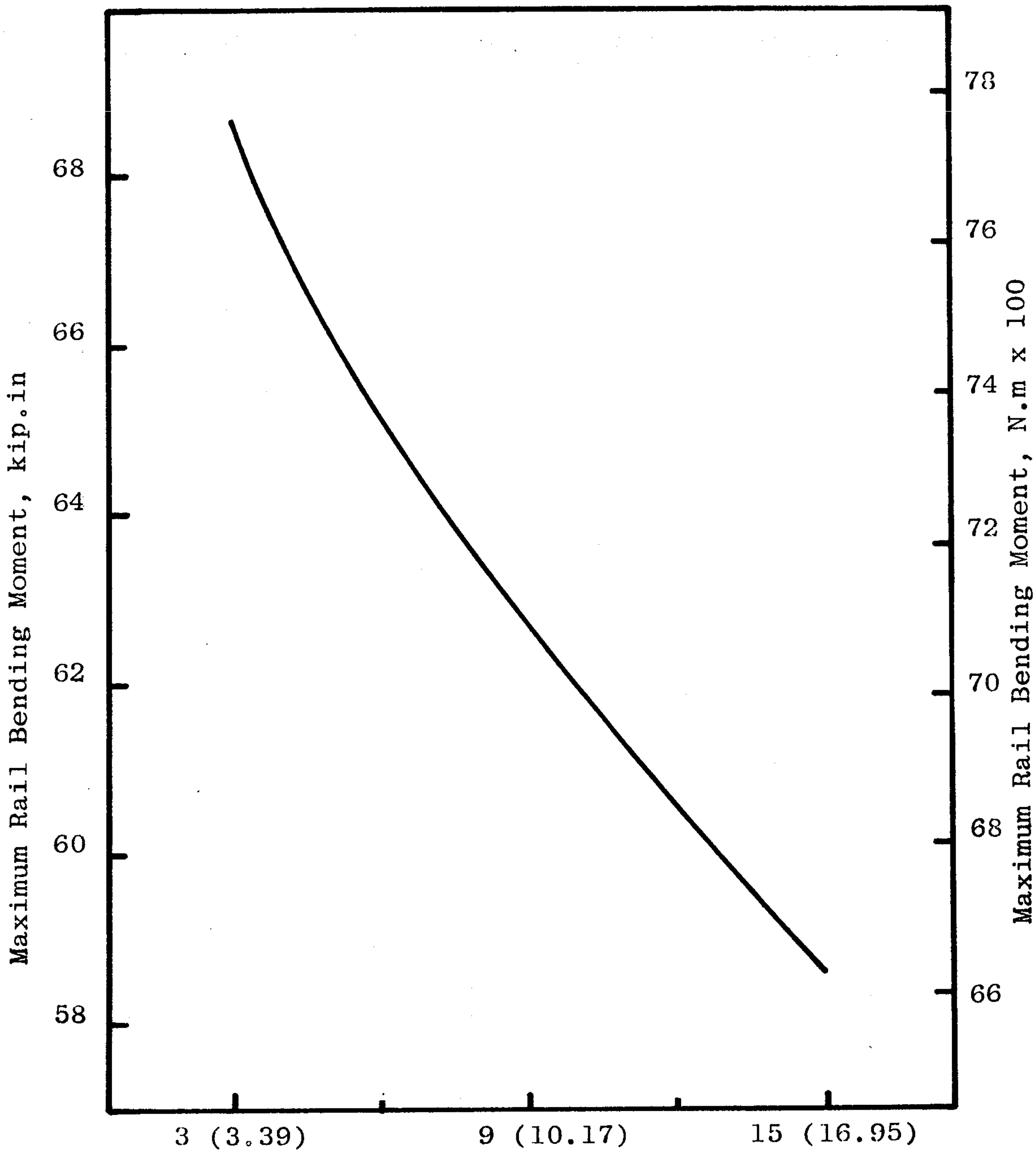
Figure 6.4. Maximum Lateral Rail Bending Moment versus Tie Spacing



Lateral Tie-Foundation Stiffness, lb/in/in x 1,000 (kg/cm/cm)

Note: frequency of occurrence = frequency of wheel load;
minimum moment - (10% maximum moment)

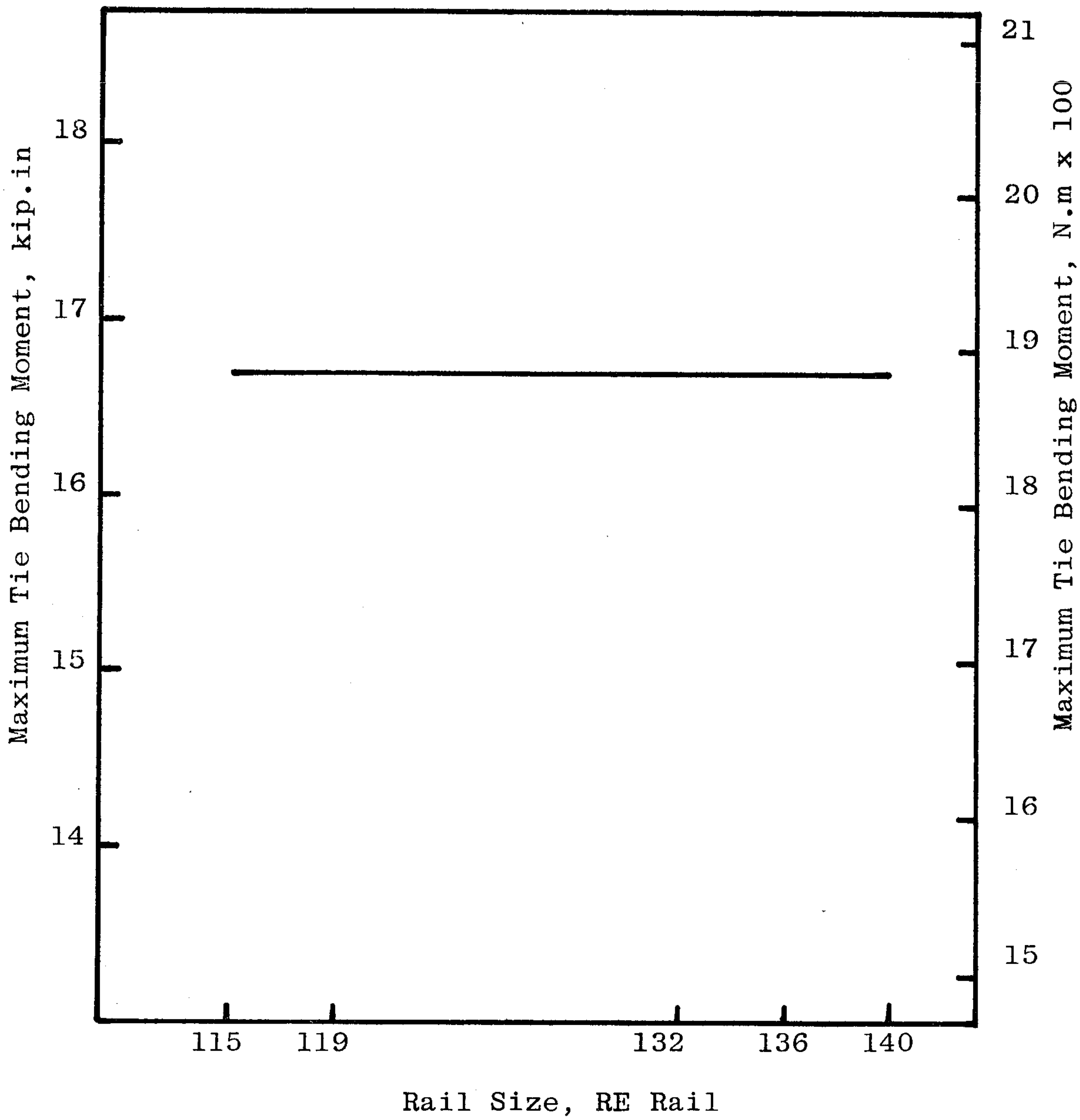
Figure 6.5. Maximum Lateral Rail Bending Moment versus Lateral Tie-Foundation Stiffness



Fastener Stiffness, lb.in/radian x 10^6 (N.m/radian x 10^5)

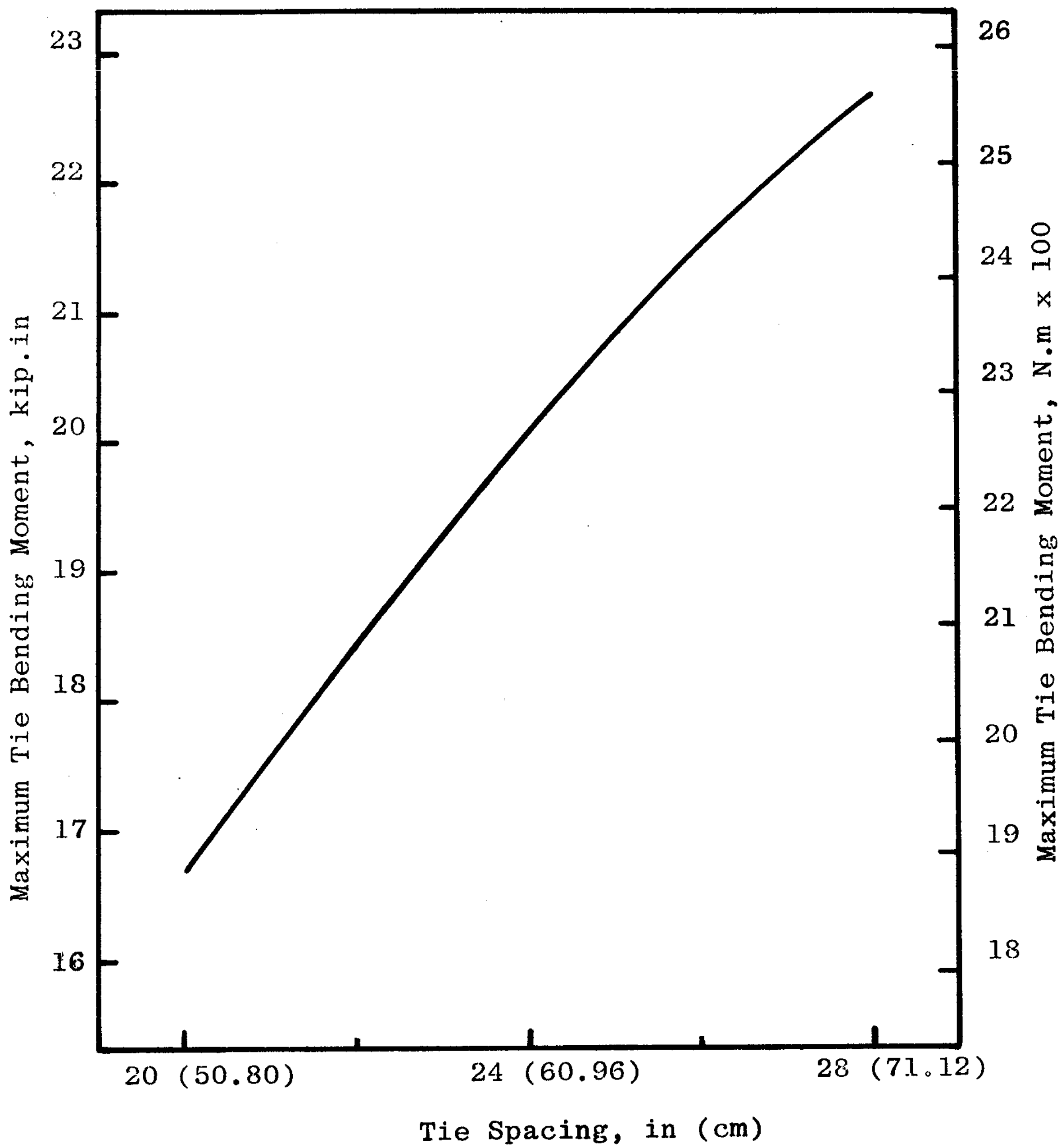
Note: frequency of occurrence = frequency of wheel load;
minimum moment - (10% maximum moment)

Figure 6.6. Maximum Lateral Rail Bending Moment versus Fastener Rotational Stiffness about Vertical Rail Axis



Note: frequency of occurrence = frequency of car; minimum moment = 0

Figure 6.7. Maximum Tie Bending Moment versus Rail Size

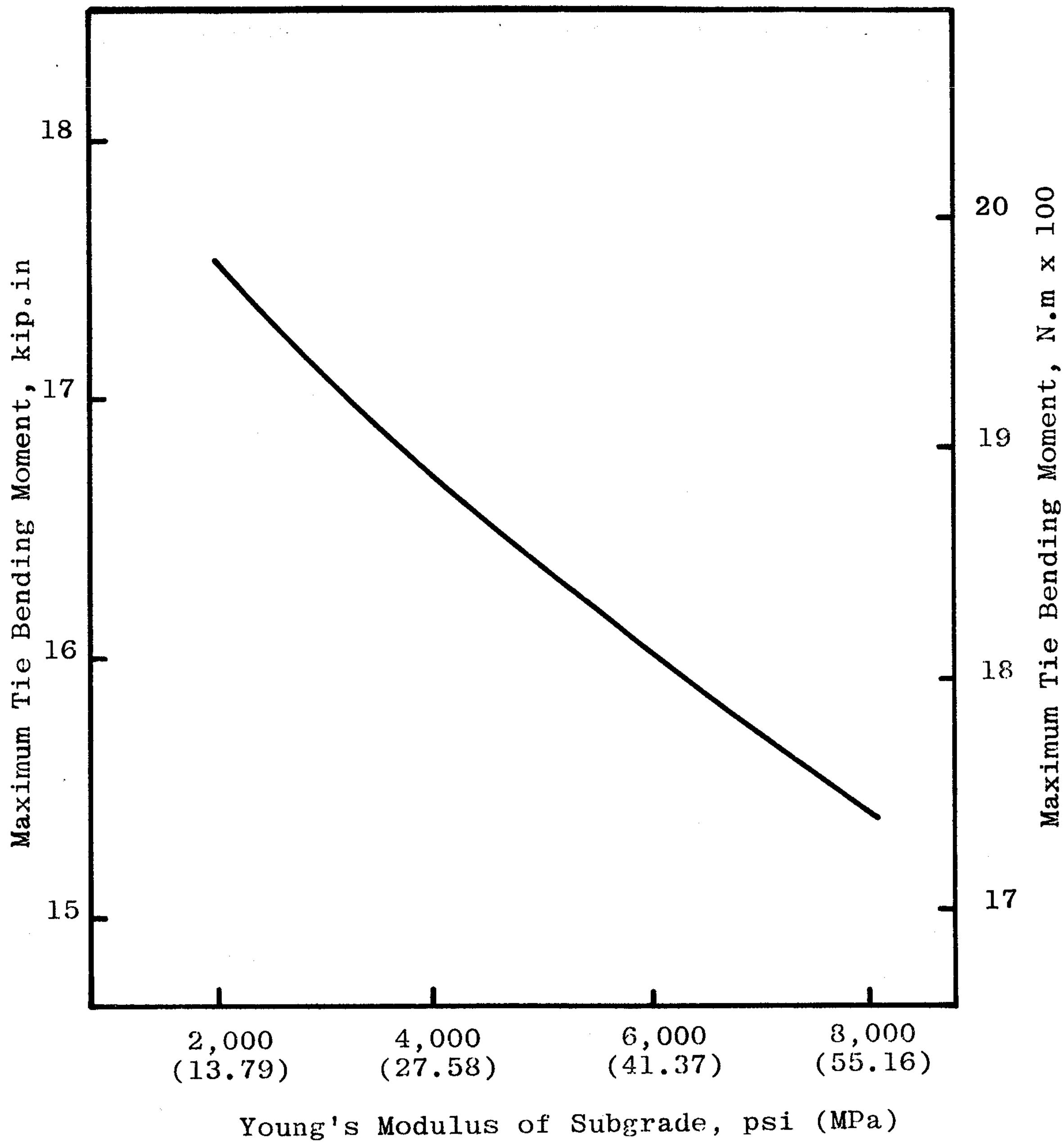


Note: frequency of occurrence = frequency of car;
minimum moment = 0

Figure 6.8. **Maximum Tie Bending Moment versus Tie Spacing**

Table 6.2. Maximum Tie Bending Moment versus Tie Support

Tie Support	Maximum Tie Bending Moment	
	kip.in	N.m x 100
Normal Condition (Freshly Tamped Track)	16.7	18.9
End-Bound Condition	28.9	32.7
Center-Bound Condition	40.1	45.3



Note: frequency of occurrence = frequency of car; minimum moment = 0

Figure 6.9. Maximum Tie Bending Moment versus Young's Modulus of Subgrade

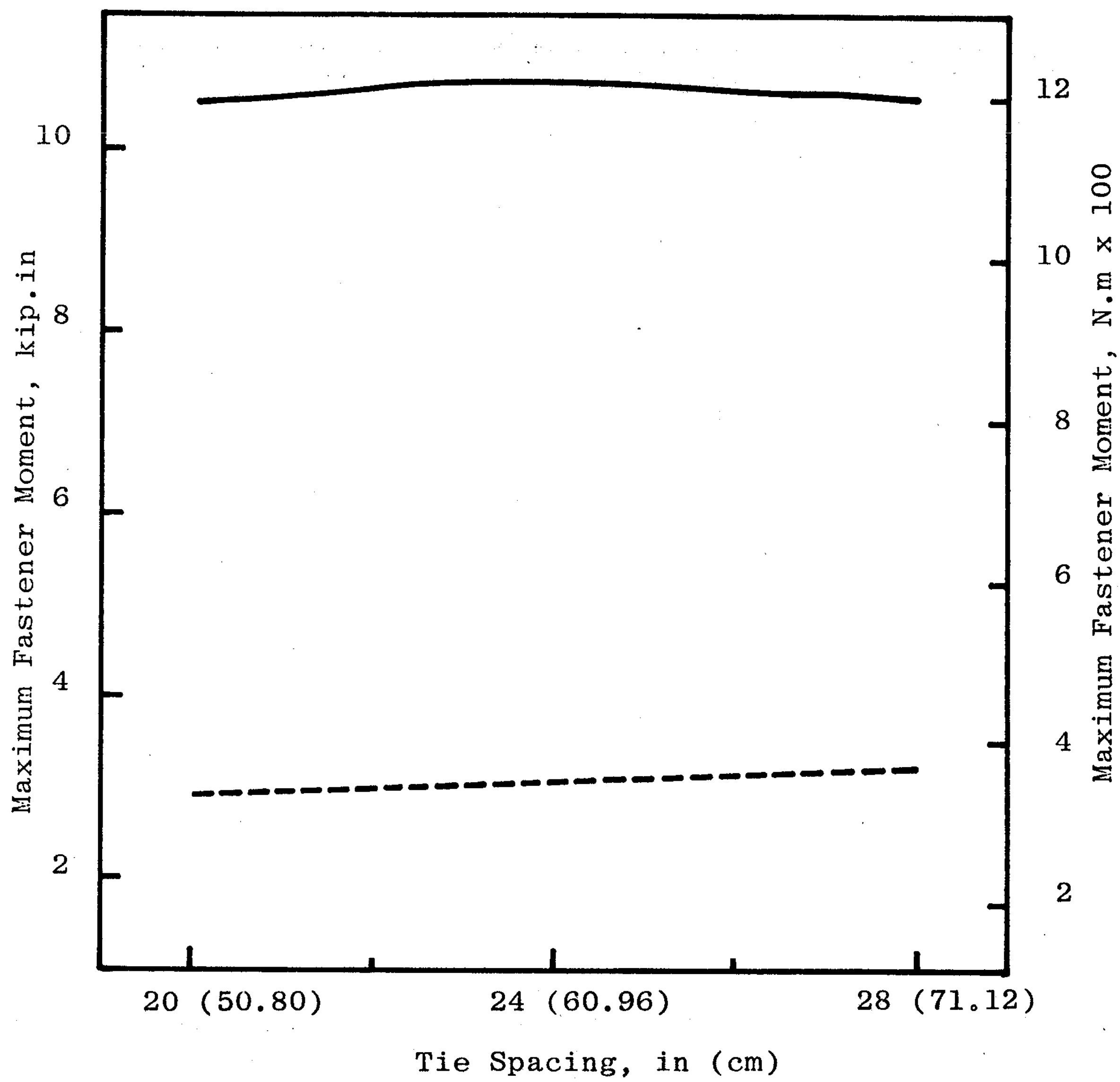
maximum tie bending moment decreases 12% as the subgrade modulus of elasticity is increased from 2,000 psi (13.79 MPa) to 8,000 psi (55.16 MPa). Ballast depth has no effect.

6.4 Rail Fastener Moments

The loadings used in developing fastener moments are defined in Section 4.4--Rail Fastener Loading. Fastener moments are investigated as functions of tie spacing and fastener rotational stiffnesses about the vertical and longitudinal axes of rail. See Figures 6.10 - 6.12. Fastener moments about the longitudinal and transverse rail axes, due to lateral loads applied at the base of one rail, and about the vertical and transverse rail axes, due to moments applied about the longitudinal axis of one rail, are insignificant and are not presented in the figures. For the other conditions, increasing tie spacing from 20 in (50.8 cm) to 28 in (71.1 cm) produces 9% increase in fastener moments about the longitudinal rail axis. Increasing fastener stiffness about the vertical rail axis from 3,000 kip.in/radian (339 kN.m/radian) to 15,000 kip.in/radian (1,695 kN.m/radian) increases fastener moments about the vertical rail axis from 4.37 kip.in (494 N.m) to 14.66 kip.in (1,656 N.m). Over the range of values for fastener stiffness about the longitudinal rail axis, fastener moments about the longitudinal rail axis vary approximately 38%.

6.5 Subgrade Stresses Due to Vertical Loading

Maximum shearing stresses and vertical normal stresses in the subgrade due to vertical track loading are investigated as functions of tie spacing, ballast depth, and subgrade modulus of elasticity. The results are presented in Figures 6.13 - 6.15. Rail size and tie support condition have insignificant effects on subgrade stresses and are not presented in the design charts. Subgrade stresses are found to be moderately affected by

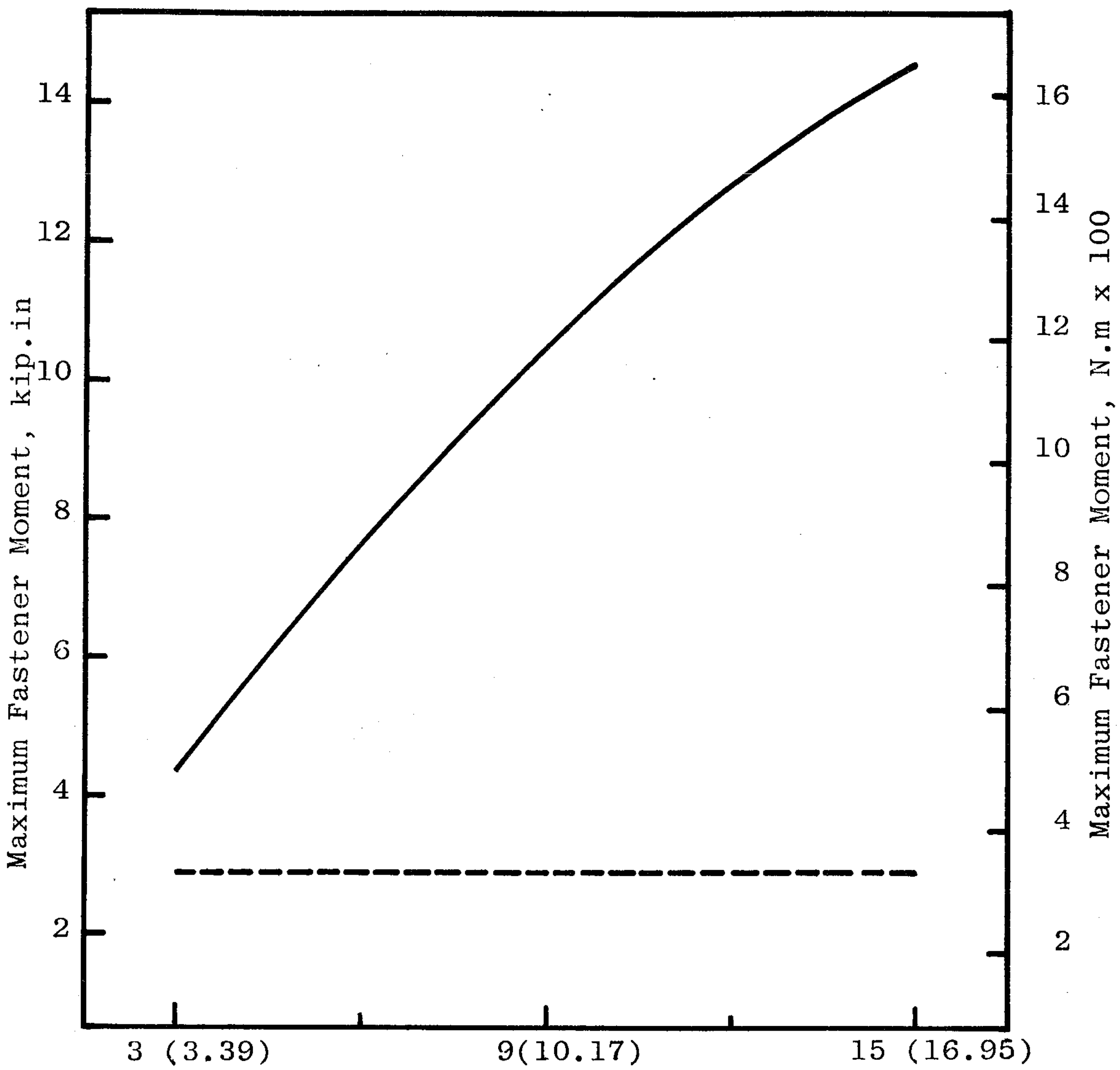


——Fastener Moment about Vertical Rail Axis for 10,000 lb (4,536 kg) Lateral Load at Rail Base

---Fastener Moment about Longitudinal Rail Axis for 10,000 lb.in (1,129.8 N.m) Moment about Longitudinal Rail Axis

Note: frequency of occurrence = frequency of wheel load;
minimum moment about vertical axis = - maximum moment
minimum moment about longitudinal axis = 0

Figure 6.10. Maximum Fastener Moments versus Tie Spacing



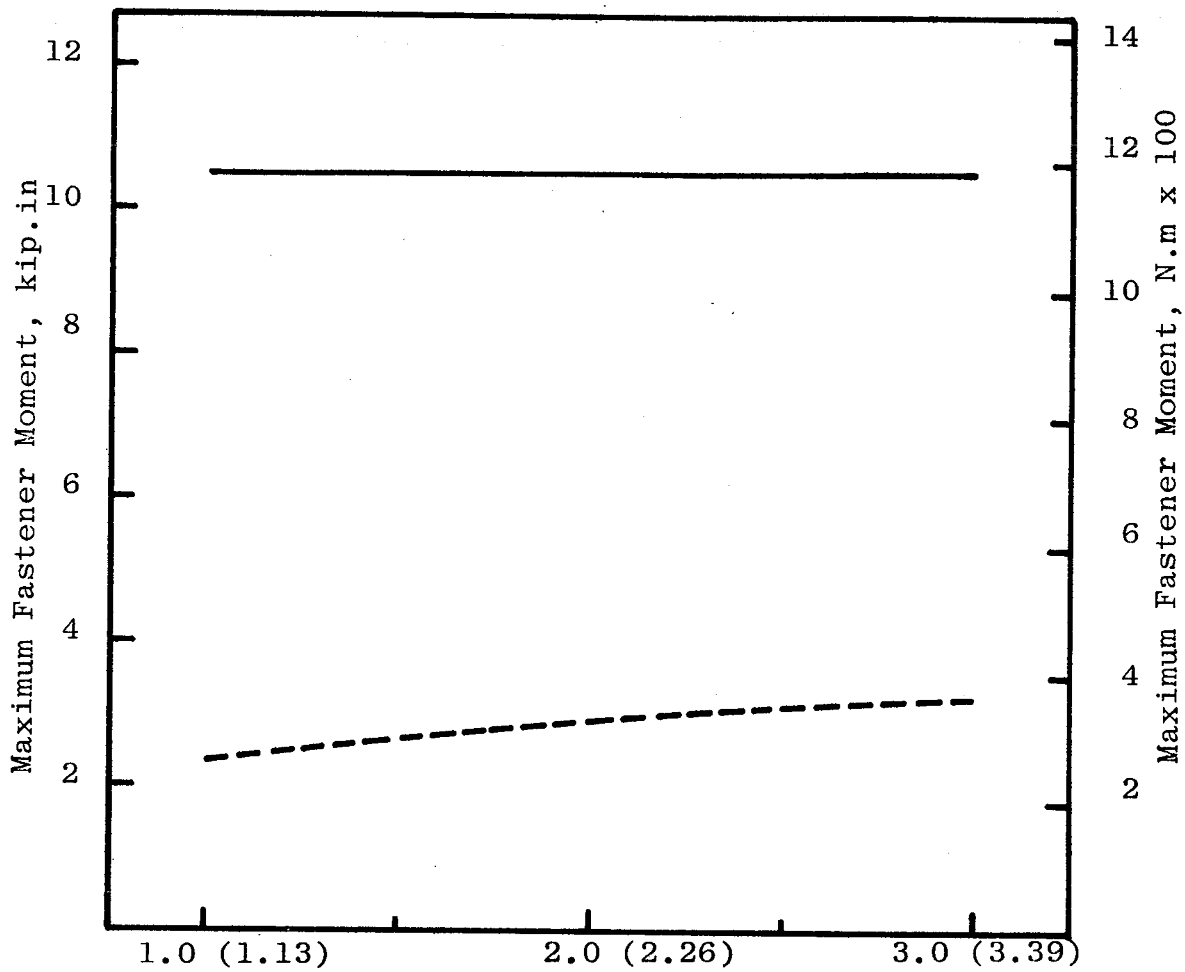
Fastener Stiffness, lb.in/radian x 10^6 (N.m/radian x 10^5)

——Fastener Moment about Vertical Rail Axis for 10,000 lb (4,536 kg) Lateral Load at Rail Base

---Fastener Moment about Longitudinal Rail Axis for 10,000 lb. in (1,129.8 N.m) Moment about Longitudinal Rail Axis

Note: frequency of occurrence = frequency of wheel load;
minimum moment about vertical axis = - (maximum moment)
minimum moment about longitudinal axis = 0

Figure 6.11. Maximum Fastener Moments versus Fastener Rotational Stiffness about Vertical Rail Axis



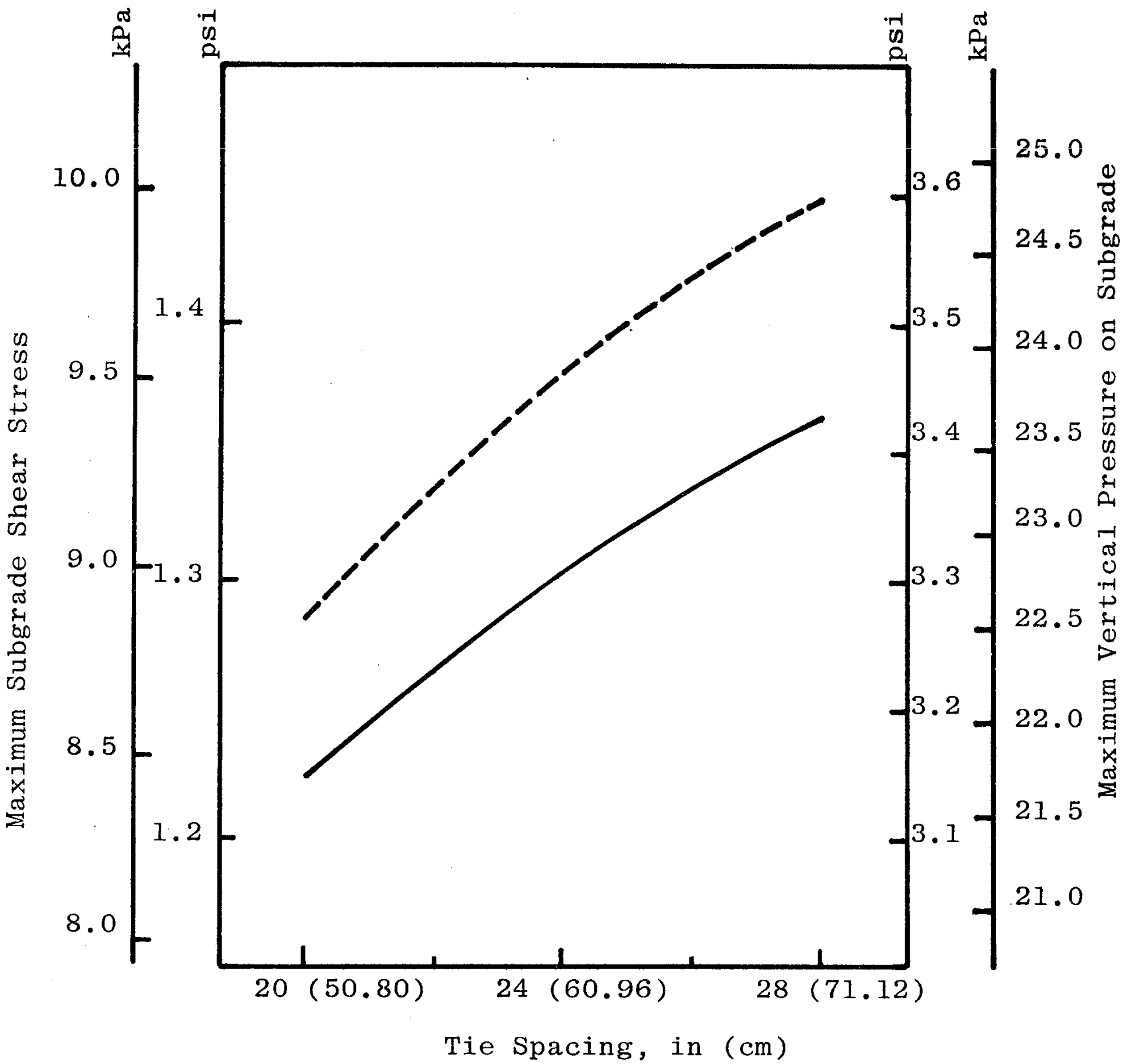
Fastener Stiffness, lb.in/radian x 10^6 (N.m/radian x 10^5)

——Fastener Moment about Vertical Rail Axis for 10,000 lb (4,536 kg) Lateral Load at Rail Base

---Fastener Moment about Longitudinal Rail Axis for 10,000 lb.in (1,129.8 N.m) Moment about Longitudinal Rail Axis

Note: frequency of occurrence = frequency of wheel load;
minimum moment about vertical axis = - (maximum moment)
minimum moment about longitudinal axis = 0

Figure 6.12. Maximum Fastener Moments versus Fastener Rotational Stiffness about longitudinal Rail Axis

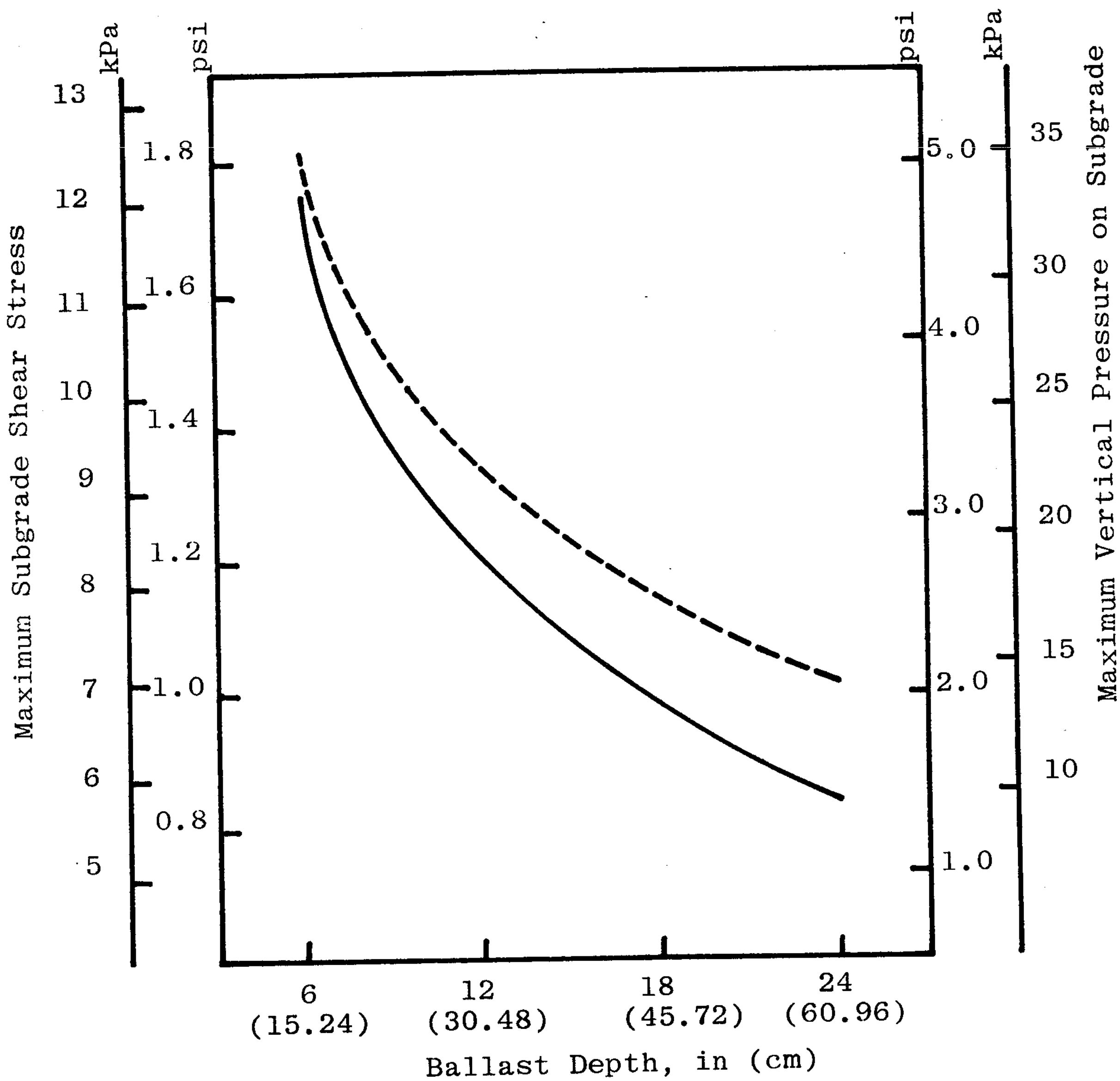


——Maximum Subgrade Shear Stress

---Maximum Subgrade Vertical Pressure

Note: frequency of occurrence = frequency of car; minimum stress and pressure = 0

Figure 6.13. Maximum Subgrade Stresses due to Vertical Loads versus Tie Spacing

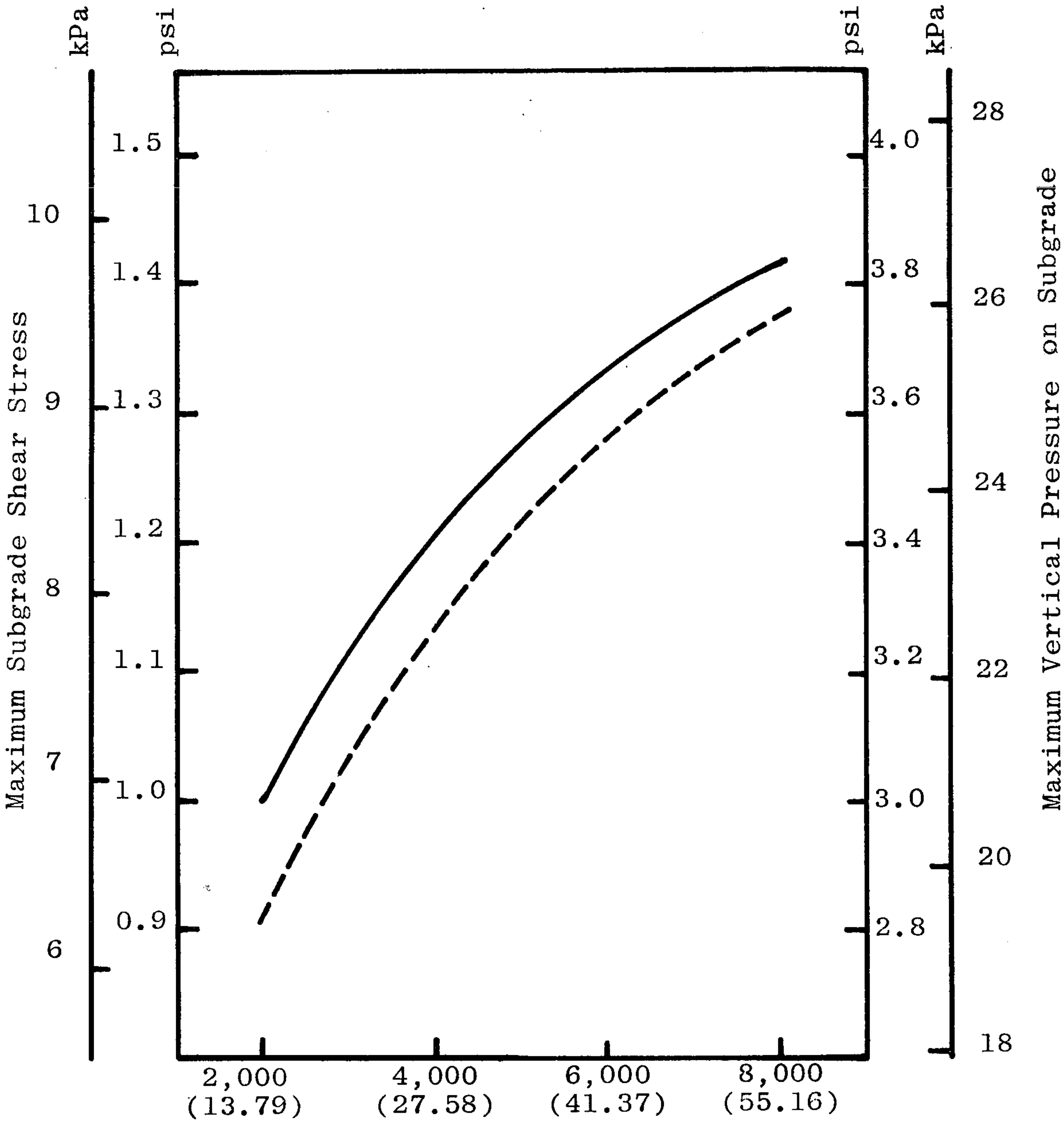


——Maximum Subgrade Shear Stress

---Maximum Subgrade Vertical Pressure

Note: frequency of occurrence = frequency of car; minimum stress and pressure = 0

Figure 6.14. Maximum Subgrade Stresses due to Vertical Loads versus Ballast Depth



Young's Modulus of Subgrade, psi (MPa)

——Maximum Subgrade Shear Stress

---Maximum Subgrade Vertical Pressure

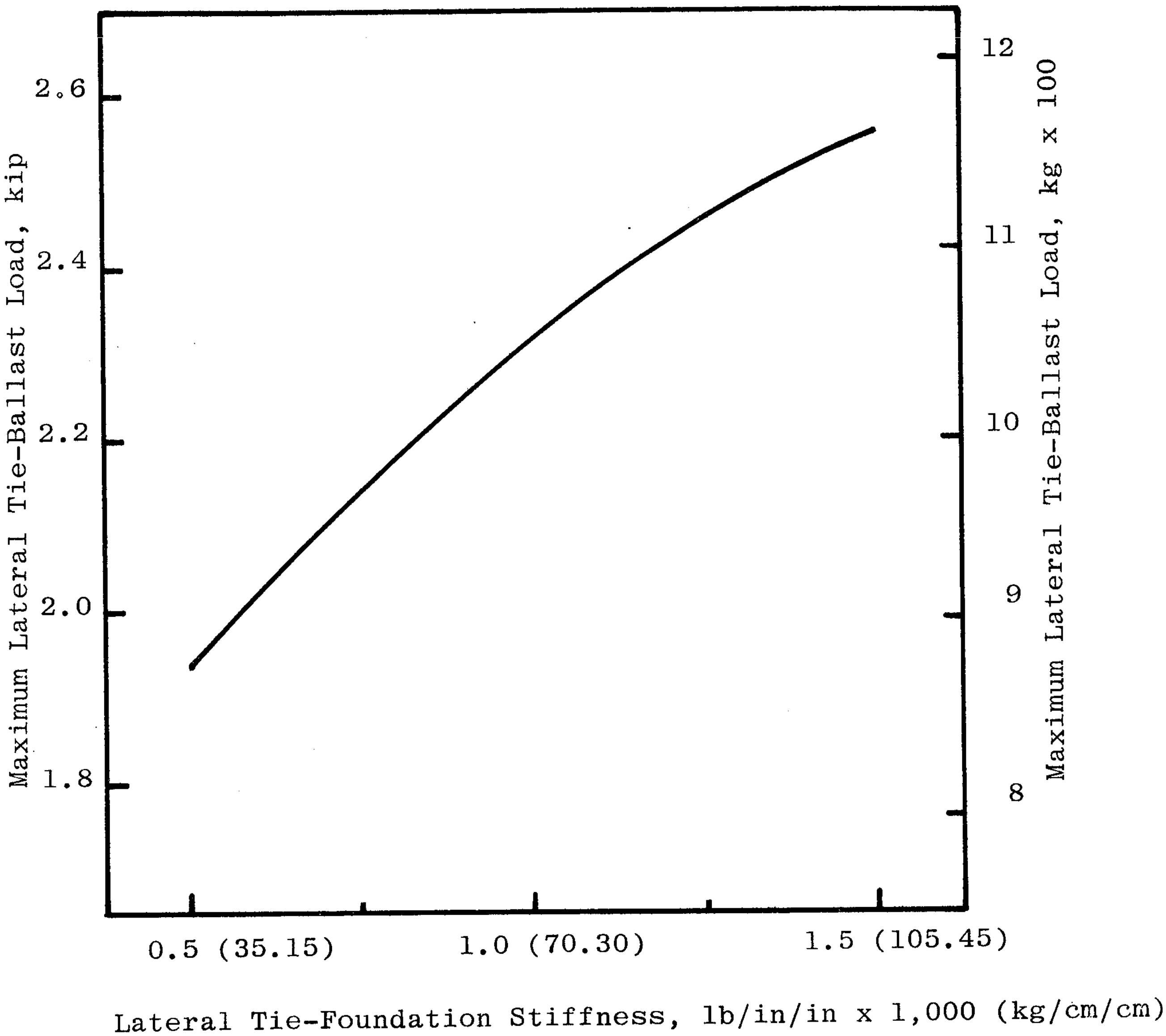
Note: frequency of occurrence = frequency of car; minimum stress and pressure = 0

Figure 6.15. Maximum Subgrade Stresses due to Vertical Loads versus Young's Modulus of Subgrade

tie spacing, a change in spacing from 20 in (50.8 cm) to 28 in (71.1 cm) increasing stresses approximately 10%. An increase in ballast depth significantly reduces subgrade stresses, although the effect becomes less pronounced for greater ballast depths. Subgrade stresses are found to increase 40% to 50% as the subgrade modulus of elasticity is varied from 2,000 psi (13.79 MPa) to 8,000 psi (55.16 MPa). Note however that an increase in subgrade modulus of elasticity will generally be accompanied by an increase in subgrade strength.

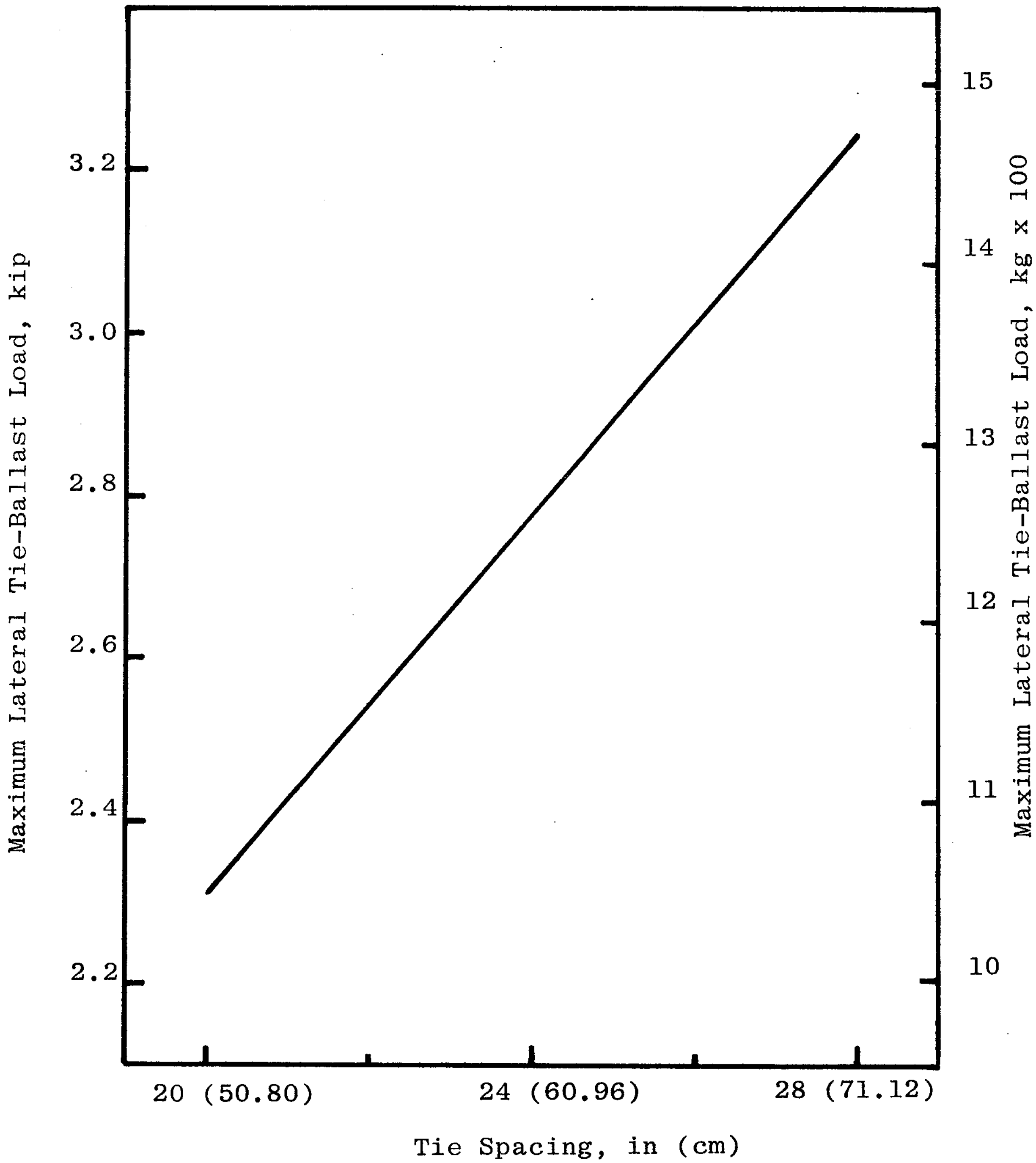
6.6 Lateral Tie-Ballast Loading

Maximum lateral tie-ballast loading increases 32% as lateral tie-foundation stiffness is increased from 500 lb/in/in (35.15 kg/cm/cm) to 1,500 lb/in/in (105.45 kg/cm/cm), as shown in Figure 6.16. An increase in tie spacing from 20 in (50.8 cm) to 28 in (71.1 cm) increases the maximum lateral tie-ballast load 41%. See Figure 6.17. Variations in rail size and fastener rotational stiffness do not significantly affect lateral tie-ballast loading.



Note: frequency of occurrence = frequency of wheel load; minimum load - (4% maximum load)

Figure 6.16. Maximum Lateral Tie-Ballast Load versus Lateral Tie-Foundation Stiffness



Note: frequency of occurrence = frequency of wheel load; minimum load - (4% maximum load)

Figure 6.17. Maximum Lateral Tie-Ballast Load versus Tie Spacing

CHAPTER 7
SUMMARY AND CONCLUSIONS

7.1 Summary

The objective of this study is to demonstrate the use of the Track Structure Models in combinations. The Track Structure Models are a family of mathematical models developed or modified from previous models for the Track Structures Research Program, Contract DOT-FR-30038. The emphasis here is on the development of design charts both for track structure design and the evaluation of existing track structures.

The finite element models are chosen from the Track Structure Models for use in developing the design charts because of their versatility. The only exception, Burmister's Multi-layer Elastic System used to simulate ballast and subgrade, is chosen because it requires less computational time compared with the finite element model available for simulating ballast and subgrade. The nonlinear capabilities of the finite element models are not utilized in the present limited investigation. The models represent different track components: rail, tie, rail fastener, ballast and subgrade. The models are used in combinations with the output of some models serving as input to other models. Force and displacement compatibility between models is achieved by adjusting model parameters in successive iterations.

The criterion for acceptable track design is that the strength of the track structure on a fatigue basis not be exceeded. Determination of stress levels in the track structure is for the following purposes: (a) determination of the loading environment for the ballast and subgrade for design against excessive vertical track settlement, (b) determination of the loading environment for the ballast for design against excessive lateral track displacement, (c) determination of the loading environment for rails for use in rail design, (d) determination of the loading environment for ties for use in tie design, and (e) determination of the loading environment for rail fasteners for use in rail fastener design.

The design charts are based on the following wheel-rail loading configurations:

(a) For vertical loading, the configuration consists of eight wheel loads and corresponds to that of two adjacent trucks of two coupled 100 ton (90,720 kg) cars. (Figure 4.1).

(b) For lateral loading, a single load is applied at the base of one rail.

(c) For rail fastener loading, two load components are input separately: a single lateral load applied at the base of one rail and a moment about the longitudinal axis of the same rail.

The design charts are based on the following arbitrarily chosen quasi-static wheel-rail load magnitudes: 10,000 lb (4,536 kg) vertical and lateral loads, and 10,000 lb.in (1,130 N.m) moments about the longitudinal rail axis. Since the relation between the applied wheel-rail loading and the resulting stresses in track components is linear for the models used in this study, the stresses in a component arising from any wheel-rail loading may be obtained by direct proportion.

Variations in the following track design parameters are incorporated in the design charts: rail size, tie spacing, rail fastener rotational stiffnesses, tie support conditions, ballast depth, lateral tie-foundation stiffness, and Young's modulus of subgrade. A standard reference track is established and each parameter is varied individually with the other parameters maintained in the reference configuration. The ranges of parameter variation are selected to reflect the range of existing track design practice.

Design charts are developed to enable the determination of the maximum stress level reached in a track component given the wheel-rail loading on the track and the track structure design. Charts are presented in Chapter 6 for: (a) rail bending moments in the vertical and lateral planes, (b) tie bending moments due to vertical loading, (c) rail fastener moments, (d) maximum shearing stresses and vertical normal stresses in subgrade due to vertical loads, and (e) lateral tie-ballast loading.

7.2 Conclusions

This study has demonstrated the use of the Track Structure Models in combinations for the development of design charts. While it is only a limited demonstration, the usefulness of design charts developed by means of mathematical models can be seen. Track structure design is now mainly based on standards that have resulted from experience. Maintenance level on the track structure is principally based on the judgement of the field engineer. Design charts developed with mathematical models can predict the structural performance of the track. Hence, design charts can provide a more rational approach to track design and maintenance than the current practice.

The finite element models employed in this study are particularly suitable for the generation of design charts because of their versatility and efficiency. Nonlinear foundation responses, multiple wheel loads, unequal wheel loads from a wheel-axle set, and track irregularities such as ineffective ties, ineffective fasteners, and ineffective rail joints are important considerations in track design and maintenance. The models have the capabilities for these simulations. The models are in a form suitable for solution on a digital computer and the modelling approach utilizes a series of models to simulate various aspects of the track structure, thus keeping each model from becoming too cumbersome.

It should be recognized that mathematical models cannot fully represent all the variables in track structure design. The models should be considered to represent only some aspects of the behavior of a system. Reasonable judgement must be exercised in interpreting the design charts and incorporating them with experience.

The limitations of the Track Structure Models used in this study should also be noted. The models assume the existence of data on the wheel-rail loading environment and fatigue failure of track components. The study neglects longitudinal rail loads. The models are quasi-static, i.e., they assume the stresses in the track structure can be accurately predicted from the dynamic wheel loads without considering inertia effects of the track system. The models do not consider design involving special trackwork, nor do they consider sudden instabilities such as lateral track buckling. Economics and vehicle performance are not considered.

To limit the scope of the present design charts, unequal wheel loads from a wheel-axle set and track irregularities are not considered, nor are the nonlinear versions of the mathematical models utilized. Only the minimal parameter variations and loading conditions are included in the design charts.

Before extensive use, the present design charts need to be expanded to include more complete ranges of parameter variations and loading conditions, nonlinear track responses, track irregularities and unequal wheel loads from a wheel-axle set. Laboratory and field tests will be required to validate the models and to provide data on wheel-rail loading environment, structural characteristics and fatigue failure characteristics of track components.

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