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VERTICAL TRACK MODULUS
Test Results and Comparison of Analysis Techniques

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INTERIM REPORT

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16. Abstract

A vertical track response test was conducted at the AAR Track Structures Dynamic Test Facility. This test was intended to characterize the behavior of the track structure under increasing vertical loads. It was also intended to evaluate the different techniques available for the calculation of the vertical track modulus.

The response of the track was obtained by monitoring track deflection under increasing vertical loads. This load and deflection data was then used to calculate vertical track modulus, track stiffness and track compliance: Three widely used techniques were utilized to calculate the vertical modulus.

The results of the test indicate that the modulus of the track is related to the level of loading; thus identical track can give different modulus values for different load levels. Of the three different techniques used to calculate track modulus, the beam-on-elastic-foundation technique required only one deflection value, whereas the others required considerably more data.

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PREFACE

This report is the result of work done under Task 3, "Laboratory Testing" of the Track Structures Research Program, Contract DOT-FR-30038, sponsored by the U.S. Department of Transportation, Federal Railroad Administration.

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1. INTRODUCTION

Since the early days of the railroad industry, when track constructed with longitudinal steel rails and tranverse wooden crossties was introduced, track engineers have needed a reliable method to quantify the response of the track structure to given loads. The ability to specify the load-carrying capacity of track, and to determine the resulting rail stresses is considered to be essential to proper track design and maintenance.

Winkler (1) first proposed the use of an elastic beam theory to analyze rail stresses. His method assumed the rail to behave like a beam that was continuously supported on a distributed elastic foundation. He proposed the calculation of a fundamental parameter, called the track modulus, which was related to both the applied load and the resulting track deflection, measured at one location relative to the loading point. As more modern track structures evolved, using decreased tie spacings and heavier wheel loads, Winkler's original theory was shown to be justified.

Other investigators, including Gough (2), Czitary (3) and Wasiutynski (4), independently analyzed a track structure by two different methods, assuming: (1) a beam on discrete supports, and (2) a beam on a distributed elastic (Winkler) foundation. Both methods produce similar results, although the Winkler method involves simpler calculations, and has gradually become accepted by the railroad industry for use in track design. More recent investigators using the method include Timoshenko (5) and the ASCE - AREA Special Committee on Stresses in Railroad Track (6).

After the validity of the Winkler method had been established, track moduli calculations became very important. In the orginal Winkler model (1), the foundation was assumed to behave like a continuous linear spring, and the calculated modulus was a measure of the spring's stiffness. This method, however, failed to account for interactions among soil particles in the foundation. In an attempt to correct this deficiency, many early investigators either modified the Winkler model, or tried to develop new models that could more accuratly describe an actual track foundation's behavior under various applied loads. Reference (7) describes some of these alternate foundation models attributed to Filonenko-Borodich, Hetenyi, Pasternak, Vlasov and Reissner.

Although many mathematical track foundation models have been developed, little was done to determine track moduli from experimental data. The first attempt to do so was undertaken by the ASCE - AREA Special Committee on Stresses in Railroad Track (6). Under the leadership of A. N. Talbot, tests were conducted on Illinois Central Railroad trackage near Champaign, Illinois. From the test results Talbot determined the track moduli for various combinations of rail size, tie size and spacing, and ballast depth and consolidation. Talbot's method assumed that the modulus was proportional to the applied load divided by the area under the track section's deflection curve. Since deflections were measured over the entire length of the depressed section caused by the load, both soil particle interactions and load distribution by beam action of the rails were taken into account. A major advantage of this method is the averaging effect acting over the entire length of the depressed area, which compensates for any track discontinuities that may be present.

This method, however, has three distinct disadvantages, namely (1) a very large number of deflection measurements are needed on both sides of the applied loading point in order to accurately determine the shape of the deflection curve (2) since the foundation experiences compression only, any slack in the track is not taken into account, and (3) the effects of differing rail sizes are not taken into account.

To correct for the slack in the track, this method was modified, such that the modulus became equal to the difference between a light and heavy load, divided by the net area between the load deflection curves. Although eliminating the effects of slack, twice the number of deflections have to be measured.

A third method for determining track modulus from experimental data is to use a modified version of the beam-on-an-elastic-foundation theory. This method, which does account for differences in rail size, uses Winkler's equation to calculate the track modulus. The advantages of this method are:
(1) measurements are required at only one deflection point, and
(2) by taking rail stiffness into account, there is an averaging effect over the entire length of the depressed track section.
This method for the determination of track modulus from measured data appears to be the easiest to use, but to date little experimental work has been done to justify its use.

In order to study these three methods and compare their results, obtained under identical track configuration and loading conditions, tests were conducted at the Association of American Railroads's Track Structures Dynamic Test Facility in Chicago, Illinois. This report presents the test objectives, instrumentation, procedures and results. Theoretical track modulus values were calculated for track loadings ranging from zero to 50 Kips. Other related variables, such as track stiffness and compliance, track deflections and rail bending stresses were also obtained. This report also discusses the three different methods, and compares the results with each other and with previously-published data.

2. TEST PROCEDURES

The test was conducted at the Association of American Railroads' Track Structures Dynamic Test Facility, in Chicago, Illinois. The test area as shown in figure 1 consisted of a test pit 45 ft. x 26 ft. in which the test track was constructed. A cross section of the test track (figure 2) consisted of the following components:

136 RE - Rail

AREA #12 tie plates with two cut spikes per plate

7 in. x 9 in. x 9 ft. hardwood cross ties spaced at 19.5 in.

12 in. AREA No. 4 limestone ballast with 12 in. shoulders

6 in. limestone subballast, Illinois state specifications CA-8

Subgrade material (SP) poorly graded sand classified under the Unified Soil Classification System

Vertical load was applied through a loading bolster at the midsection of the test area with hydraulic loading jacks. Four jacks were used to apply the load. Two were used for the major part of the test, to represent single axle loading, and the other two were used when tests required simulated truck loading. The jacks were powered by an Amsler hydraulic power plant. The loading system was capable of applying 50 kips per loading jack. The loading bolster, (figure 3) was designed to approximate a freight car truck with 36 in. wheels. The contact between bolster and rail is made with four 36 in. wheel segments of sufficient size to represent true wheel - rail contact geometry.

Figure 1   Test track used for determining vertical track modulus

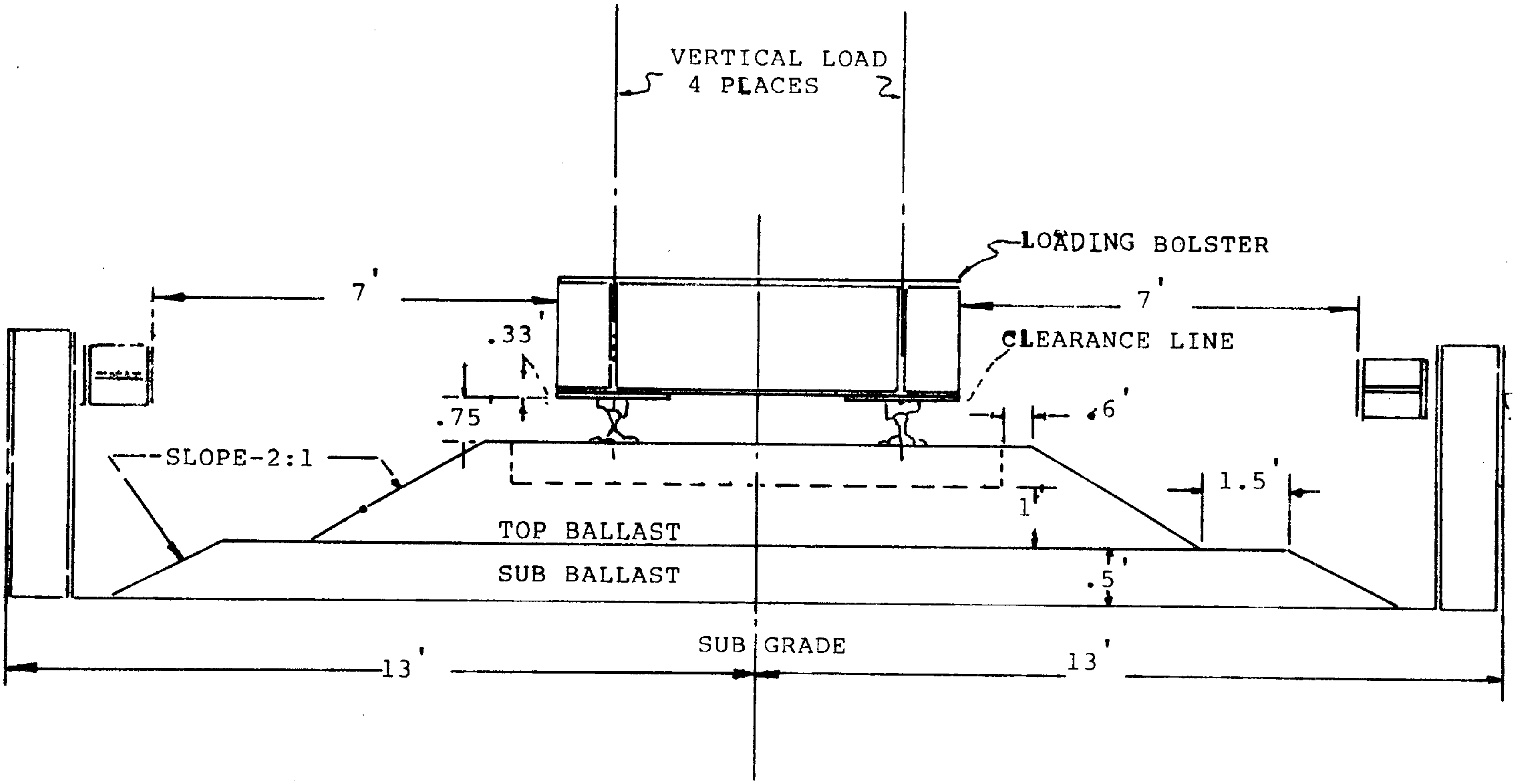


Figure 2.- General Arrangement of Test
Track Area - End View

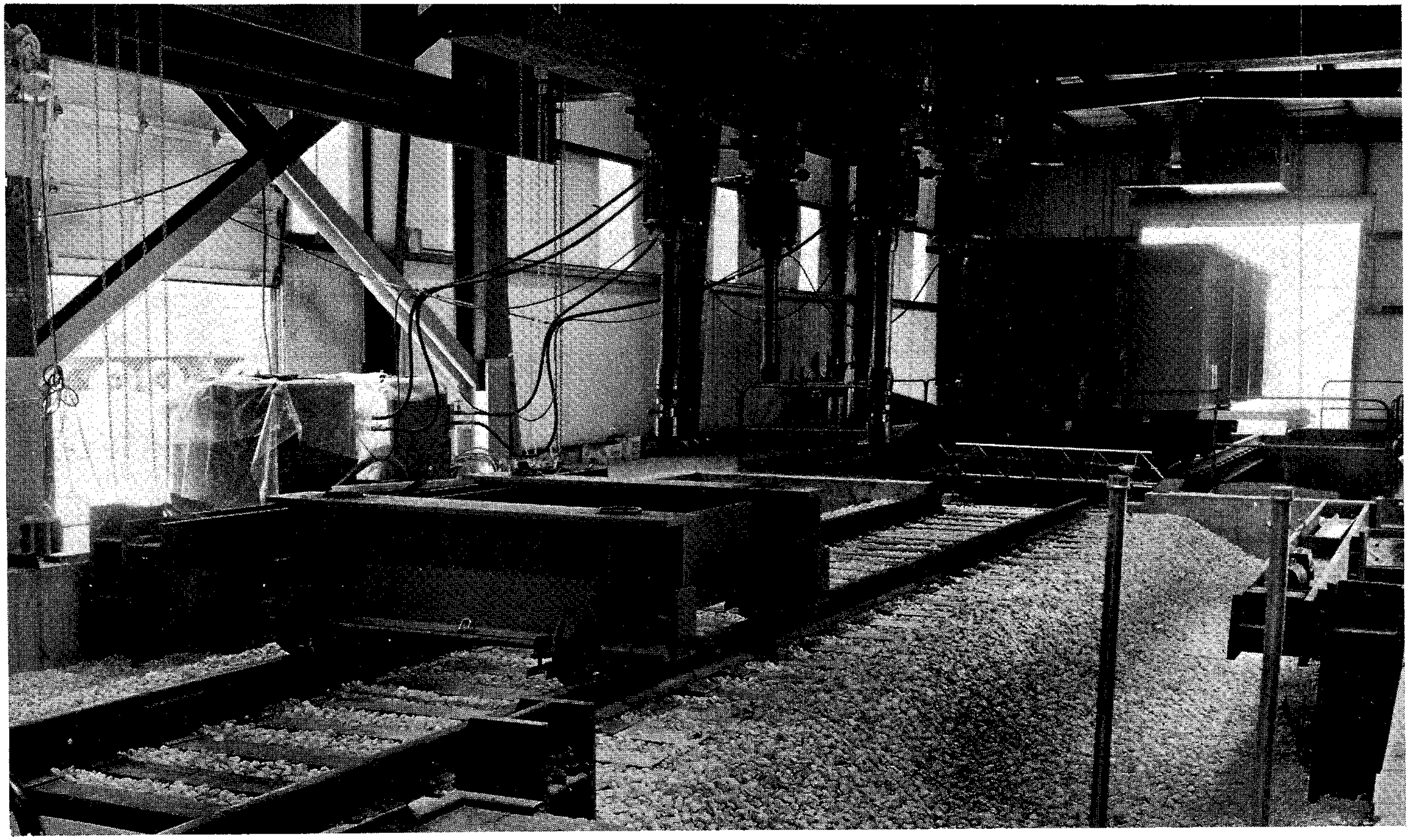


Figure 3 Loading bolster with retractable wheels used to apply the vertical load to the track

The test consisted of the application of three loading sequences (table 1) in which a simulated axle load was applied through the loading bolster and measurements taken of track deflection and rail bending strain. In the first sequence, the load was applied in increasing increments from 0 to 50 kips and data was recorded after each load increment. The second sequence was the unloading sequence and the data was taken after each decreasing increment of loading. At no time was the load returned to zero during the increasing or decreasing sequence. In the third sequence the load was released to zero after each test shown in column three of Table 1.

All loads were measured using a pressure transducer in the hydraulic line of the Amsler jack.

Deflections were measured at three locations using linear variable displacement transducers (LVDT) and at twenty-one locations using a surveyor's level. The deflections measured with LVDTs were read after each loading increment whereas the deflections measured with the level were read at each increments in table 2. All deflections measured were absolute, i.e. relative to a fixed zero point constant for all tests. To achieve this the LVDTs were mounted on a reference frame supported at the concrete walls of the test pit, (figure 4). A triangular aluminum truss section was used for the reference frame. A cantilever beam extending from the reference frame to the rail provided the transducers support at each station. A silicone adhesive was used to connect the instrument to the base of the rail. The level readings were taken with the level outside the test pit and using a one-hundreth of an inch graduated scale held at the measurement point.

TABLE 1: WHEEL LOADING SEQUENCE

Increasing Loads (Kips)	Decreasing Loads (Kips)	Incremented loads (Kips)
0.00	50.00	5.00
0.25	45.00	0.00
0.50	40.00	10.00
0.75	37.50	0.00
1.00	35.00	20.00
1.50	32.50	0.00
2.00	30.00	30.00
3.00	27.50	0.00
4.00	25.00	40.00
5.00	22.50	0.00
6.00	20.00	50.00
7.00	17.50	0.00
8.00	15.00	
9.00	12.50	
10.00	10.00	
12.50	9.00	
15.00	8.00	
17.50	7.00	
20.00	6.00	
22.50	5.00	
25.00	4.00	
27.50	3.00	
30.00	2.00	
32.50	1.50	
35.00	1.00	
37.50	0.75	
40.00	0.50	
45.00	0.25	
50.00	0.00	

TABLE 2: LOADS AT WHICH OPTICAL MEASUREMENTS WERE TAKEN

Increasing Loads (Kips)	Decreasing Loads (Kips)	Incremented loads (Kips)
0.00	50.00	0.00
2.25	37.50	40.00
5.50	5.50	50.00
37.50		
50.00		

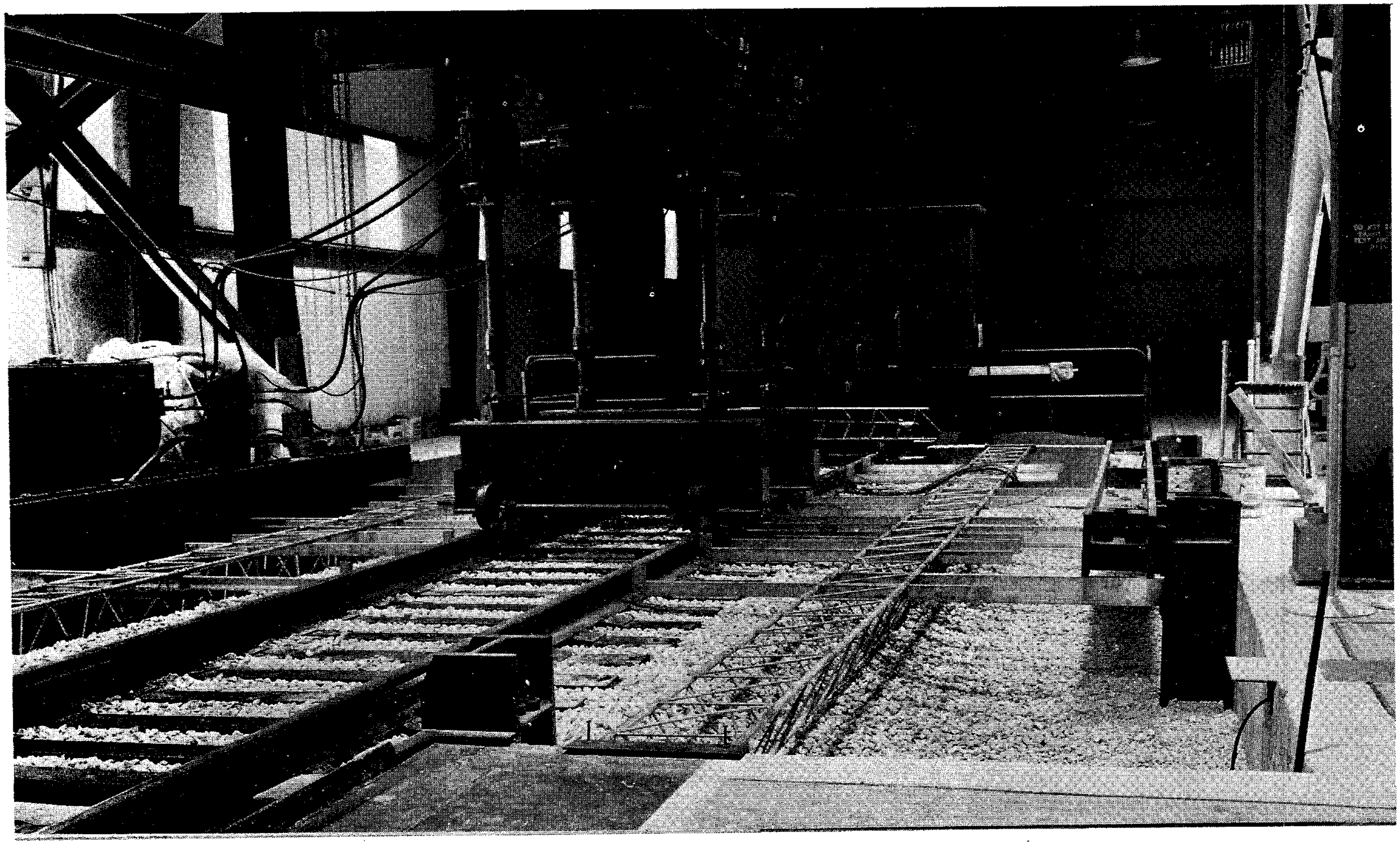


Figure 4 Reference frame used to measure absolute deflections

Readings were taken with the track under load. Using the LVDTs, deflections were measured at points under the load, at 66.75 in. and 104 in. from the load. Deflections, with the level, were measured under the load and at the centerlines of ties for ten ties on each side of the load. All deflection measurements were made on the west rail with the exception of level reading under the load on the east rail.

In addition to displacements, strains were monitored in the rails at five points. At two locations, strain gauge arrays were used to measure the applied load on each rail. This was done to provide a check on the load applied to the track. The other three arrays measured bending stresses in the rail head, (figure 5). These were measured at the load point, at 28.5 in. and 66.5 in. away from the load. Subsequently, another array was added to measure bending stress at the base of the rail under the load point. This was required to eliminate the difference between wheel and rail.

Figure 6 shows the approximate location of the deflection and strain instruments. Table 3 gives the exact instrument location based on a three dimension reference frame with zero at the longitudinal centerline of the east rail, the neutral axis of the rail and the load line.

For all the loading sequences, data were recorded on both magnetic and paper tape. The data were reduced according to the techniques defined in Appendix A of Reference 8.

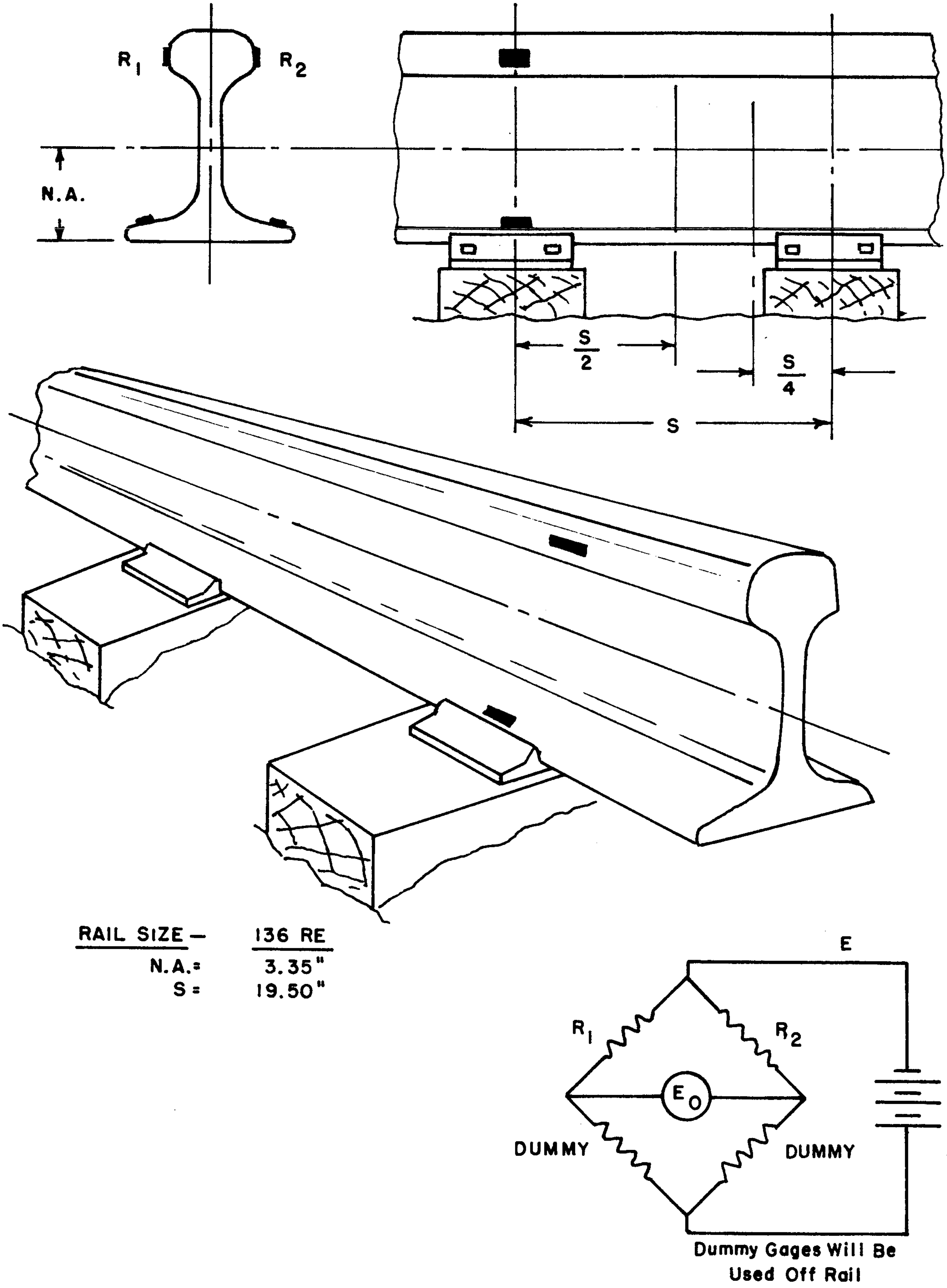


Figure 5. Strain gage placement vertical rail bending

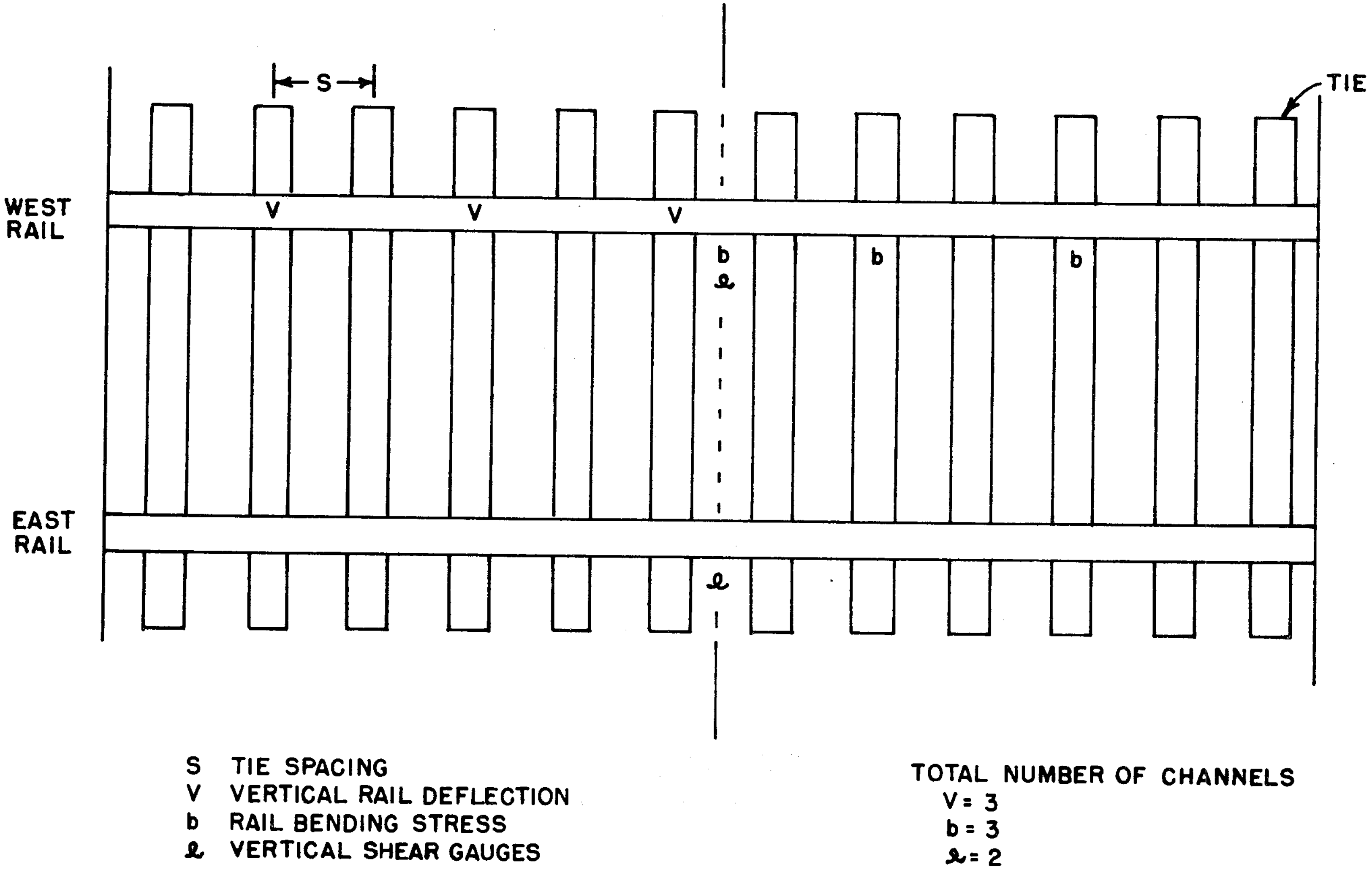
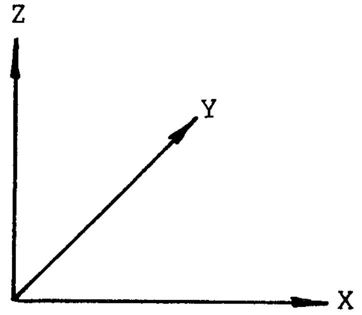


Figure 6. Instrument location in the test area

TABLE 3: GLOBAL COORDINATES OF THE INSTRUMENTS

CHAN. NO.	CO-ORDINATES X (IN.)	Y (IN.)	Z (IN.)
0	0.00	0.00	3.97
1			
2	0.00	59.44	2.53
3	28.50	59.44	2.53
4	66.50	59.44	2.53
5	0.00	59.44	-2.28
6	0.00	0.00	-2.28
7	0.00	59.44	0.00
8	0.00	0.00	0.00
9			
10	-87.50	61.94	-2.25
11	-29.81	61.94	-2.25
12	0.00	61.94	-2.25



Global co-ordinates based on the center line, the neutral axis of the east rail, and the loadline.

3. DISCUSSION

Three different analytical techniques were used to reduce the data with each method assuming a different definition of track modulus and also utilizing a separate procedure for calculating. The three methods are:

1. Deflection curve
2. Heavy-light wheel load deflection curve
3. Beam on elastic foundation

1. Deflection curve

This method was used by the ASCE-AREA Special Committee (6) under the leadership of Talbot. The basic assumption of this method is that the applied wheel load divided by the area under the load deflection curve is the track modulus.

$$k = \frac{P}{s \sum_{i}^{n} y_i} \qquad \text{------------------------} \quad (1)$$

Where k is the track modulus (lb/in^2)

P is the applied wheel load (lb.)

y is the deflection of the ith tie (inches)

s is the tie spacing (inches)

n is the number of depressed ties

Using this method, the modulus of the track was found to be 4,712 lb/in^2 for a load of 39,566 lb and 4,796 lb/in^2 for a load of 50,327 lb. Figure 7 shows the deflection curve under the two loads. (Deflection data based on level measurements)

2. Heavy-light wheel load deflection curve

This method differs with the previous one in the way the applied load is taken into account. This method assumes that the track modulus is the difference of a heavy and a light wheel load

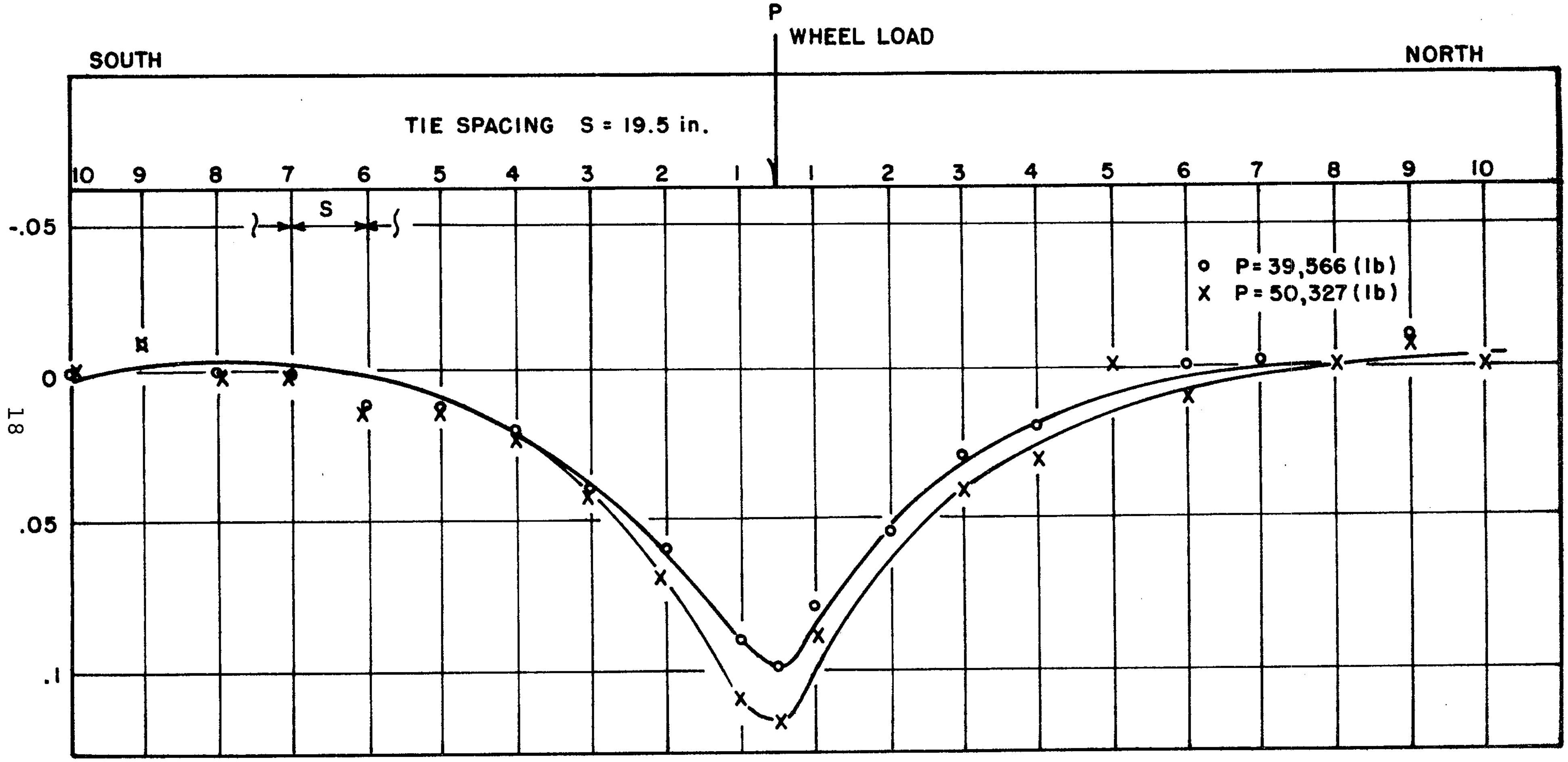


Figure 7. Load-deflection curve for heavy wheel loading

divided by the area under the load deflection curve.

$$k = \frac{P-p}{s \sum_{i}^{n} (y-y_1)} \quad \text{.....................(2)}$$

Where k is the track modulus (lb/in^2)

P is heavy wheel load (lb.)

p is light wheel load (lb.)

s is tie spacing (inches)

y is the individual tie depression (inches) under P

y_1 is the individual tie depression (in) under p

n is the number of depressed ties

Using this method, the modulus of the track was found to be 5,016.5 lb/in^2 for a heavy load of 37,536 lb and a light load of 5,695 lb. Figure 8 shows the deflection curve under the heavy and light load. (Level measurement data). It is evident from the data that it requires approximately 2,000 lb. before the slack in the system is removed.

In methods one and two, the level measurements were used in determining the area under the load-deflection curve. The number of deflection points measured with the LVDT's were insufficient to establish a valid deflection curve under the given load.

3. Beam on elastic foundation theory

This method differs from the previous two in the following manner: in both previous methods, the stiffness of the rail was taken into account indirectly and only the load and deflection was used to determine the modulus. Also the other two methods required the deflection at every tie for a significant distance on both sides of the load. This method requires only one deflection measurement

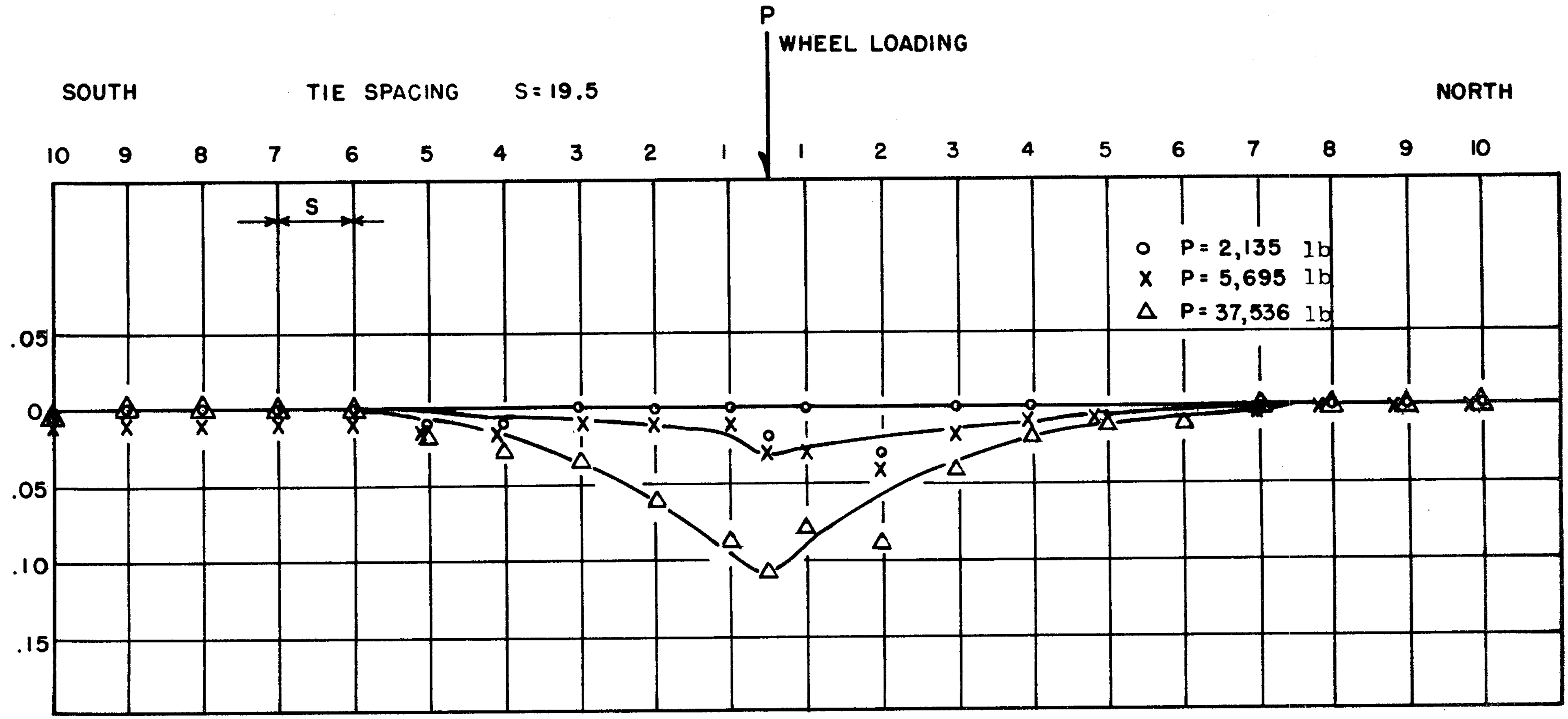


Figure 8. Load-deflection curve for heavy and light wheel loads

at a known location relative to the load. It is based on the beam on elastic foundation equation, developed by Winkler (1), which relates the deflection of the track, the applied load and the track modulus. If the track deflection is measured under the load, by direct substitution the track modulus can be determined by

$$k = \sqrt[3]{\frac{P^4}{64EIy^4}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

Where k is the track modulus (lb/in^2)

EI is the stiffness of the rail (lb-in^2)

P is the applied wheel load (lb)

y is the deflection under the load (inches)

Evaluation of the test data showed that the track modulus varied with the applied load. Figure 9 shows the track modulus vs the load for the loading and unloading sequence. It can be seen that for loads above 5,000 lb. and up to 50,000 lb. the modulus varied linearly with respect to the load for increasing loads. For decreasing loads it varied linearly from 50,000 lb. to 10,000 lb. At loads less than 10,000 lb. on the decreasing sequence, and less than 5,000 lb. on the increasing sequence, the modulus variation was quite non-linear. Table 4 shows the modulus values for the increasing and decreasing load sequences. Table 5 shows the modulus determined in the above manner for the third loading sequence.

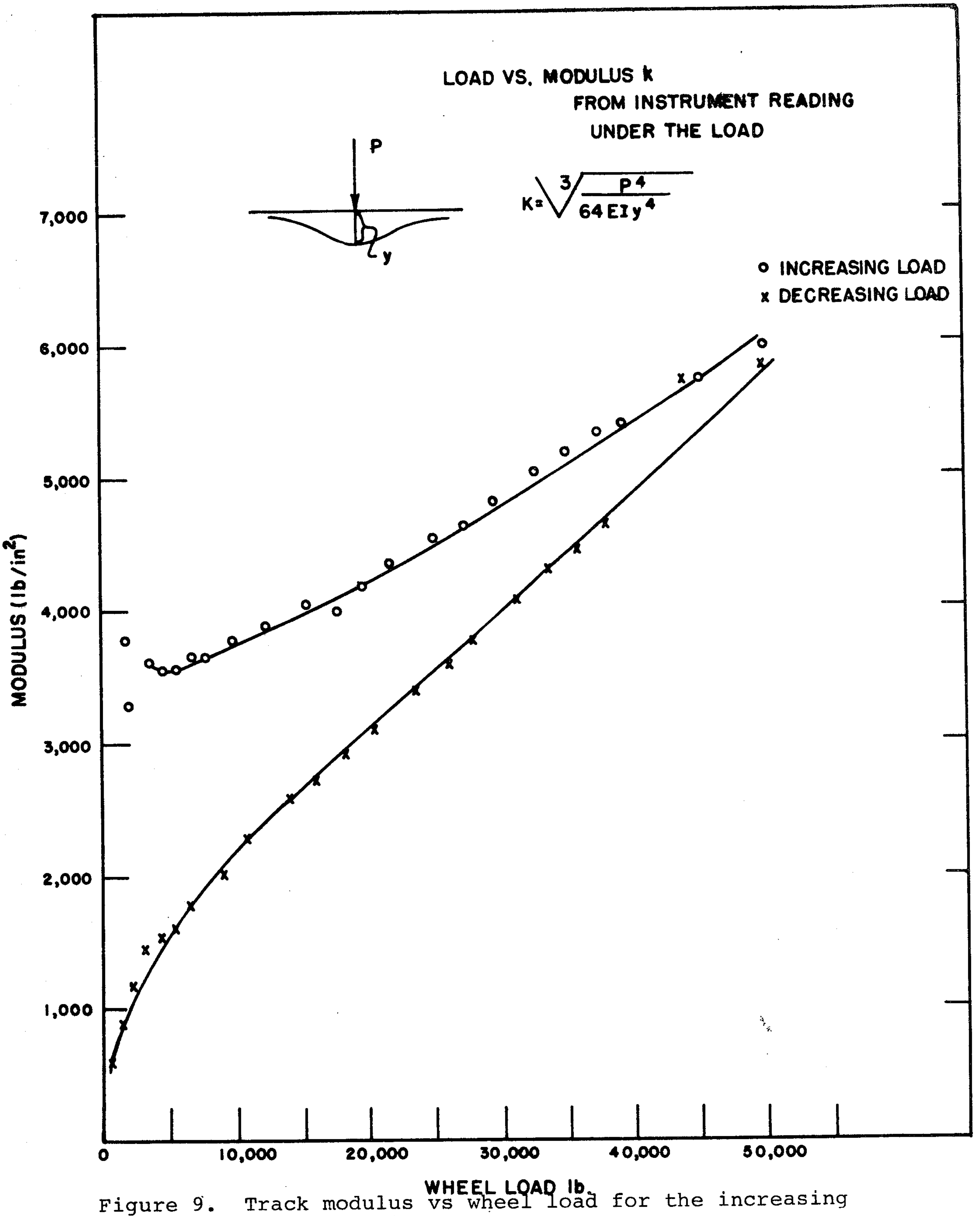


Figure 9. Track modulus vs wheel load for the increasing and decreasing load sequence

TABLE 4: SUMMARY OF RESULTS FOR THE LOADING AND UNLOADING SEQUENCE

LOAD (LB)	DEFLECTION (IN)	TRACK MODULUS (LB/IN x IN)	TRACK STIFFNESS (LB/IN)	TRACK COMPLIANCE (IN/LB)
		INCREASING LOADS		
641.9	.0013	6967.50	0.49379E+06	0.20252E-05
1867.4	.0060	3765.39	0.31124E+06	0.32130E-05
2135.9	.0076	3286.26	0.28104E+06	0.35583E-05
3583.1	.0119	3602.80	0.30110E+06	0.33211E-05
4586.9	.0154	3550.97	0.29785E+06	0.33574E-05
5695.6	.0191	3556.59	0.29820E+06	0.33534E-05
6804.4	.0224	3645.40	0.30377E+06	0.32920E-05
7808.2	.0257	3646.21	0.30382E+06	0.32914E-05
9990.7	.0321	3765.39	0.31124E+06	0.32130E-05
12185.0	.0384	3863.76	0.31732E+06	0.31514E-05
15476.3	.0472	4036.33	0.32789E+06	0.30498E-05
17588.8	.0541	3990.92	0.32512E+06	0.30758E-05
19771.4	.0588	4174.13	0.33625E+06	0.29740E-05
21930.6	.0636	4316.62	0.34482E+06	0.29001E-05
25151.9	.0704	4525.68	0.35727E+06	0.27990E-05
27486.2	.0755	4640.63	0.36406E+06	0.27468E-05
29738.8	.0797	4795.56	0.37313E+06	0.26800E-05
32936.7	.0854	5011.68	0.38568E+06	0.25929E-05
35247.7	.0894	5161.12	0.39427E+06	0.25363E-05
37535.3	.0933	5301.89	0.40231E+06	0.24857E-05
39566.1	.0975	5363.46	0.40581E+06	0.24642E-05
45285.1	.1063	5722.46	0.42601E+06	0.23474E-05
50268.8	.1144	5963.71	0.43941E+06	0.22758E-05
		DECREASING LOADS		
50222.1	.1166	5806.95	0.43072E+06	0.23217E-05
44246.3	.1111	5230.83	0.39826E+06	0.25109E-05
38188.9	.1048	4646.44	0.36440E+06	0.27443E-05
35983.0	.1019	4455.71	0.35312E+06	0.28319E-05
33835.4	.0987	4283.11	0.34281E+06	0.29171E-05
31524.5	.0954	4078.37	0.33045E+06	0.30262E-05
28128.1	.0899	3791.94	0.31288E+06	0.31961E-05
26097.3	.0868	3595.73	0.30066E+06	0.33260E-05
23751.3	.0825	3393.64	0.28789E+06	0.34735E-05
20553.4	.0759	3127.58	0.27080E+06	0.36928E-05
18382.5	.0714	2923.89	0.25746E+06	0.38841E-05
16153.2	.0659	2738.54	0.24512E+06	0.40797E-05
13982.3	.0597	2577.28	0.23421E+06	0.42697E-05
10807.7	.0507	2273.29	0.21317E+06	0.46911E-05
8496.8	.0435	2023.21	0.19533E+06	0.51196E-05
6501.0	.0366	1782.45	0.17762E+06	0.56299E-05
5333.8	.0320	1637.59	0.16668E+06	0.59994E-05
4423.5	.0283	1503.10	0.15631E+06	0.63977E-05
3443.1	.0224	1469.88	0.15371E+06	0.65058E-05
2205.9	.0170	1172.74	0.12976E+06	0.77066E-05
1283.9	.0117	937.84	0.10973E+06	0.91132E-05
548.6	.0072	576.60	0.76188E+05	0.13125E-04
175.1	.0047	222.08	0.37249E+05	0.26846E-04

TABLE 5: SUMMARY OF RESULTS FOR THE THIRD LOADING SEQUENCE

LOAD (LB)	DEFLECTION (IN)	TRACK MODULUS (LB/IN x IN)	TRACK STIFFNESS (LB/IN)	TRACK COMPLIANCE (IN/LB)
	INCREMENTED LOADS			
5940.7	.0230	2936.55	0.25829E+06	0.38716E-05
10037.4	.0359	3263.80	0.27959E+06	0.35766E-05
19794.7	.0596	4106.05	0.33213E+06	0.30109E-05
29937.2	.0795	4854.50	0.37657E+06	0.26556E-05
39659.5	.0963	5469.92	0.41183E+06	0.24282E-05
50420.5	.1142	6001.71	0.44151E+06	0.22650E-05

Like the two previous methods, multiple wheel loads can be used with the beam-on elastic-foundation theory. When more than one load is used, an iteration method has to be used to determine the modulus. The equation for deflection of the track is derived in reference (9) and given in equation (4).

$$y(x) = \frac{P\beta}{2k} \quad \eta(x) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

Where y(x) is the track deflection at x (inches)

x is the distance from the load (inches)

P is the applied load (lb.)

k is the track modulus (lb./in^2)

$$\eta(x) = e^{\beta x} \quad (\cos \beta x - \sin\beta x) \quad \ldots\ldots\ldots\ldots (5)$$

$$\beta = \left(\frac{k}{4EI}\right)^{\frac{1}{4}} \quad \ldots\ldots\ldots\ldots (6)$$

Where: EI is the stiffness of the rail.

Using superposition theory for multiple loads and solving for the track modulus, equation (4) becomes:

$$k = \frac{\beta}{2y} \sum_{i=1}^{n} P_i \quad \eta_i \quad \ldots\ldots\ldots\ldots (7)$$

Where n is the number of loads

i is the ith load.

Equation (7) is a transcendental equation and has to be solved by an iteration method. Rewriting equation (7)

$$\left| \; k - \frac{\beta}{2y} \sum_{i=1}^{n} P_i \; \eta_i \; \right| \leq \varepsilon \quad \ldots\ldots (8)$$

a solution can be obtained by choosing a value of k and systemetically changing it until equation (8) is satisfied, for an arbitrary preassigned accuracy.

The track stiffness and track compliance values were also calculated from the test data and are given in Tables 4 and 5. The track stiffness was determined by dividing the load by the deflection of the track under the load.

$$K = \frac{P}{y} \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (9)$$

Where K is the track stiffness (lb/in)

P is the applied wheel load (lb)

y is the deflection under the load (inches)

The track compliance is defined as the inverse of the stiffness:

$$C = \frac{1}{K} \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (10)$$

Where K is the track stiffness (lb/in)

C is the track compliance (in/lb)

The load deflection curves for the loading and unloading sequence and the incremented loading sequence are given in Figures 10 and 11, respectively. Figure 12 shows the comparison between track modulus and track stiffness.

Utilizing the results of the bending strain gauge arrays, the bending stresses were examined. The results are plotted in Figure 13 against the moment influence line predicted by beam on elastic foundation theory, given in (9). Note the excellent agreement between the test data and the theory.

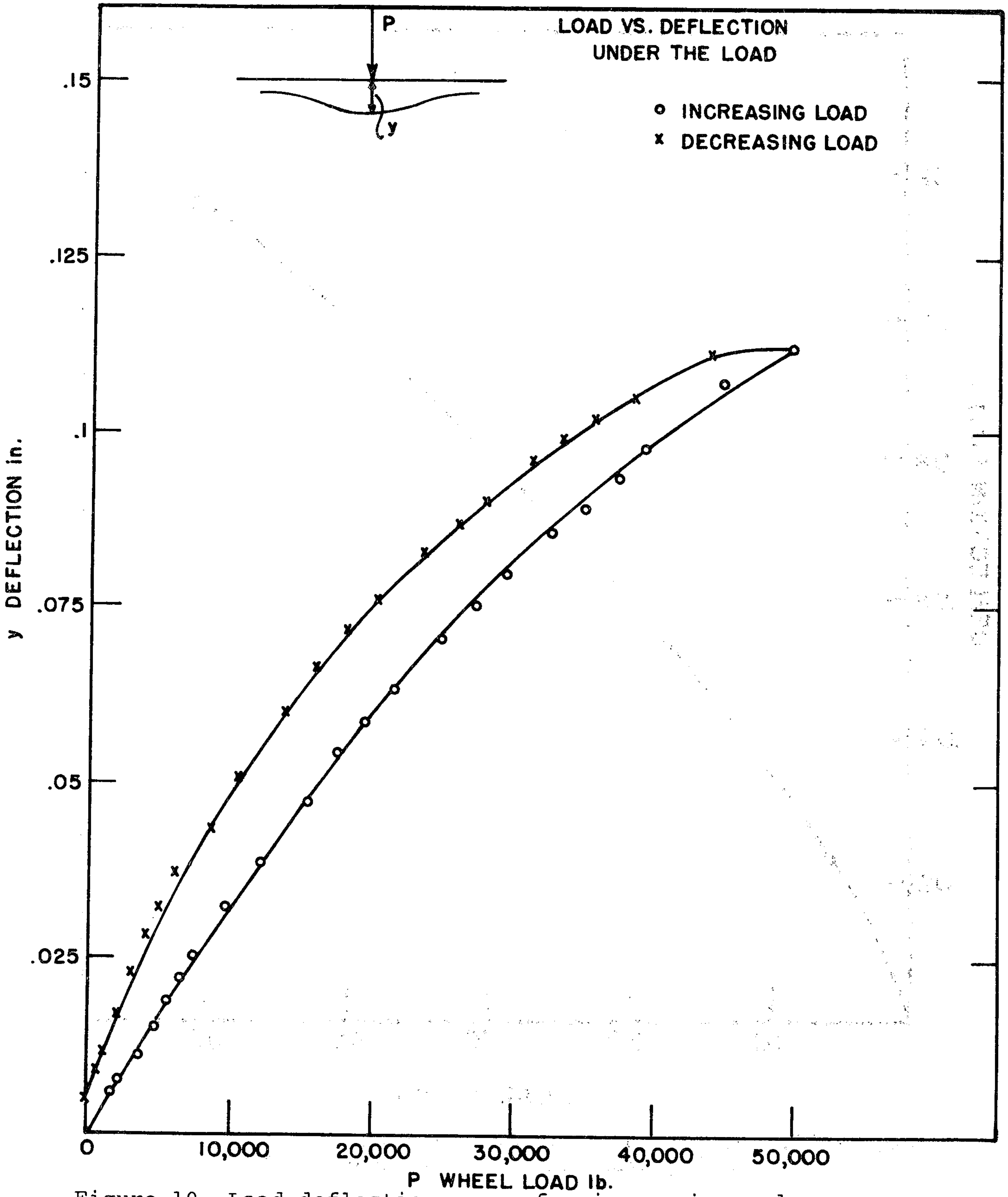


Figure 10. Load-deflection curve for increasing and decreasing load sequence.

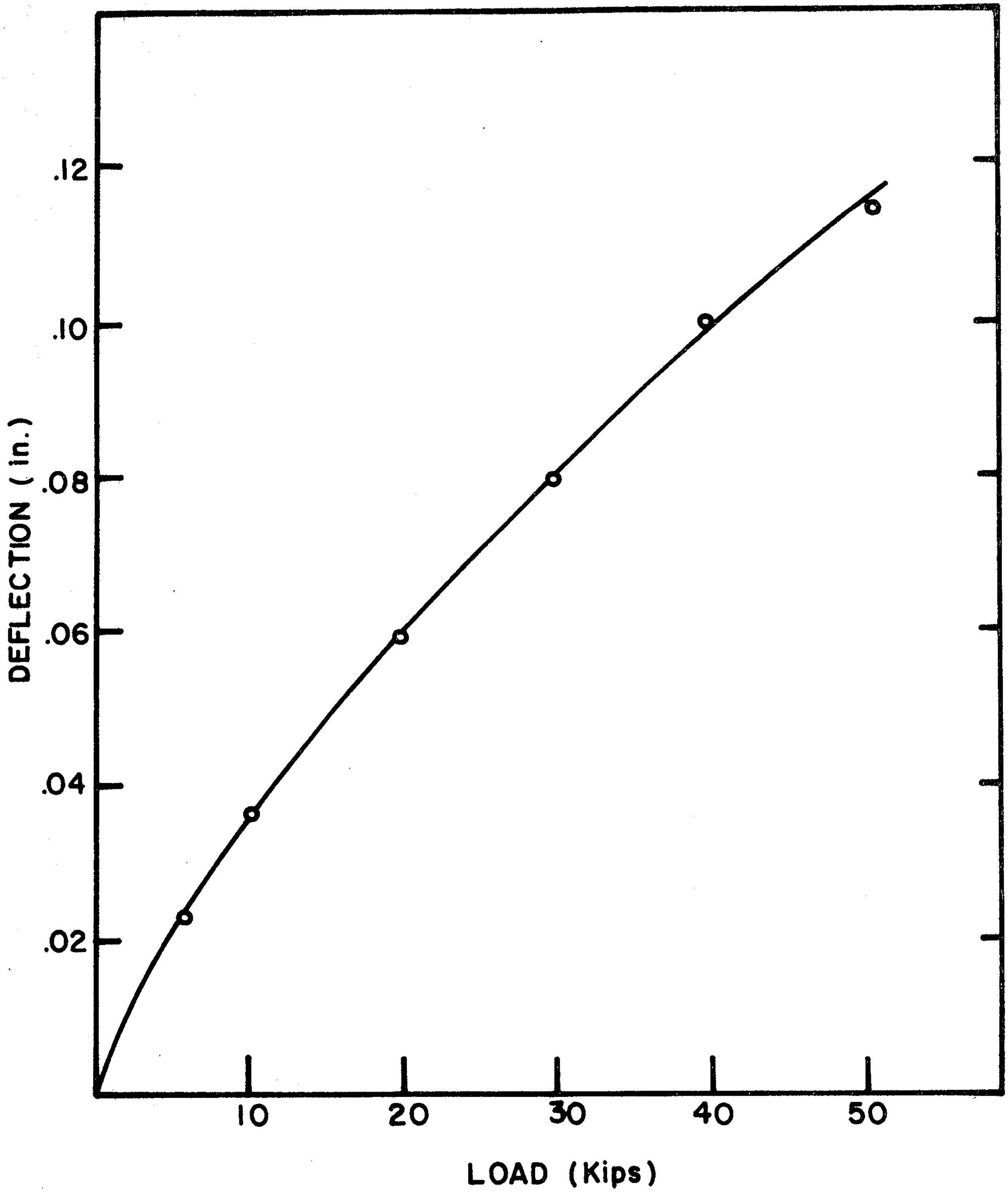


Figure 11. Envelope of load vs. deflection for loading sequence #3 incremented load

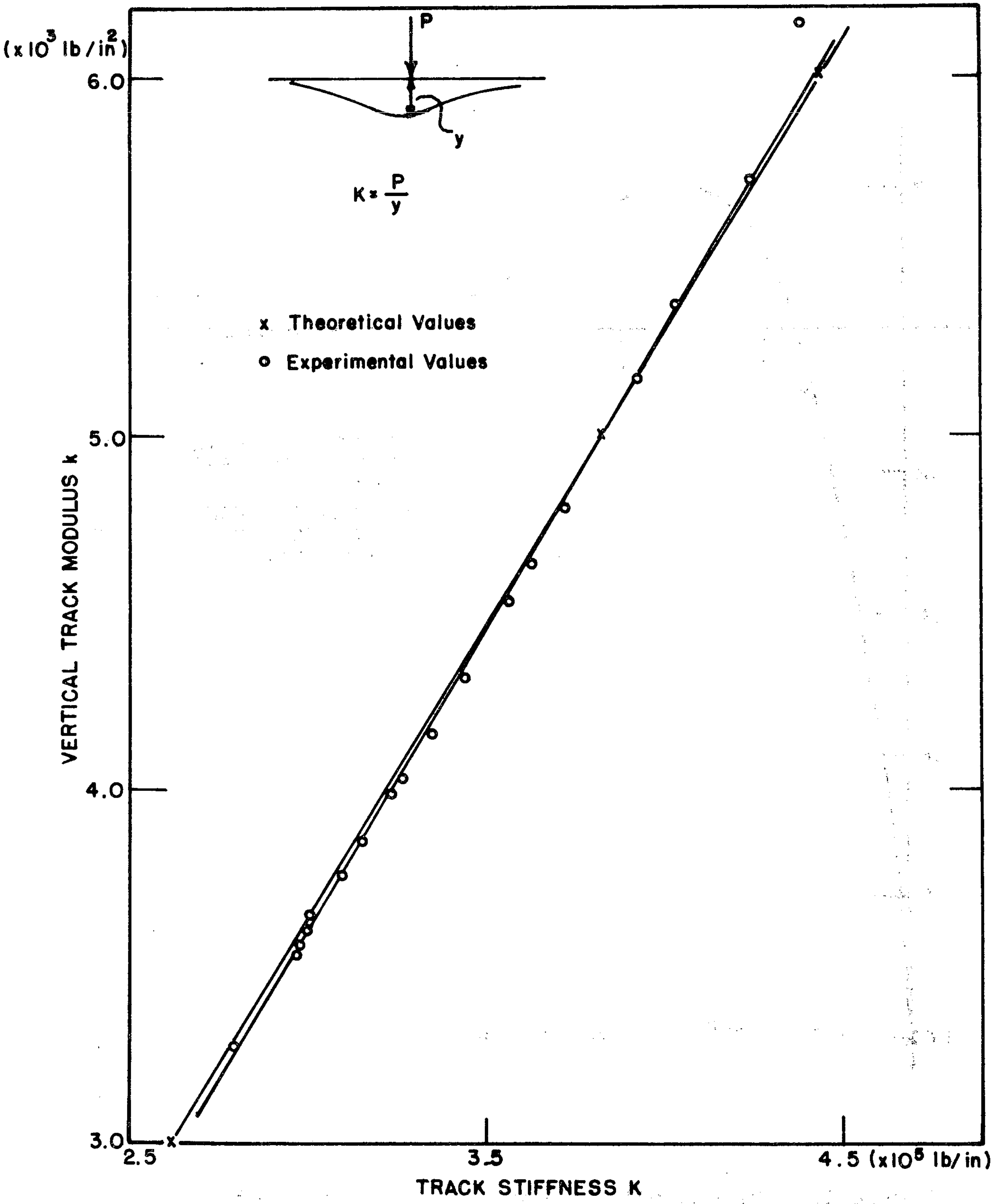


Figure 12. Comparison of Theoretical with experimental results, track stiffness and track modulus

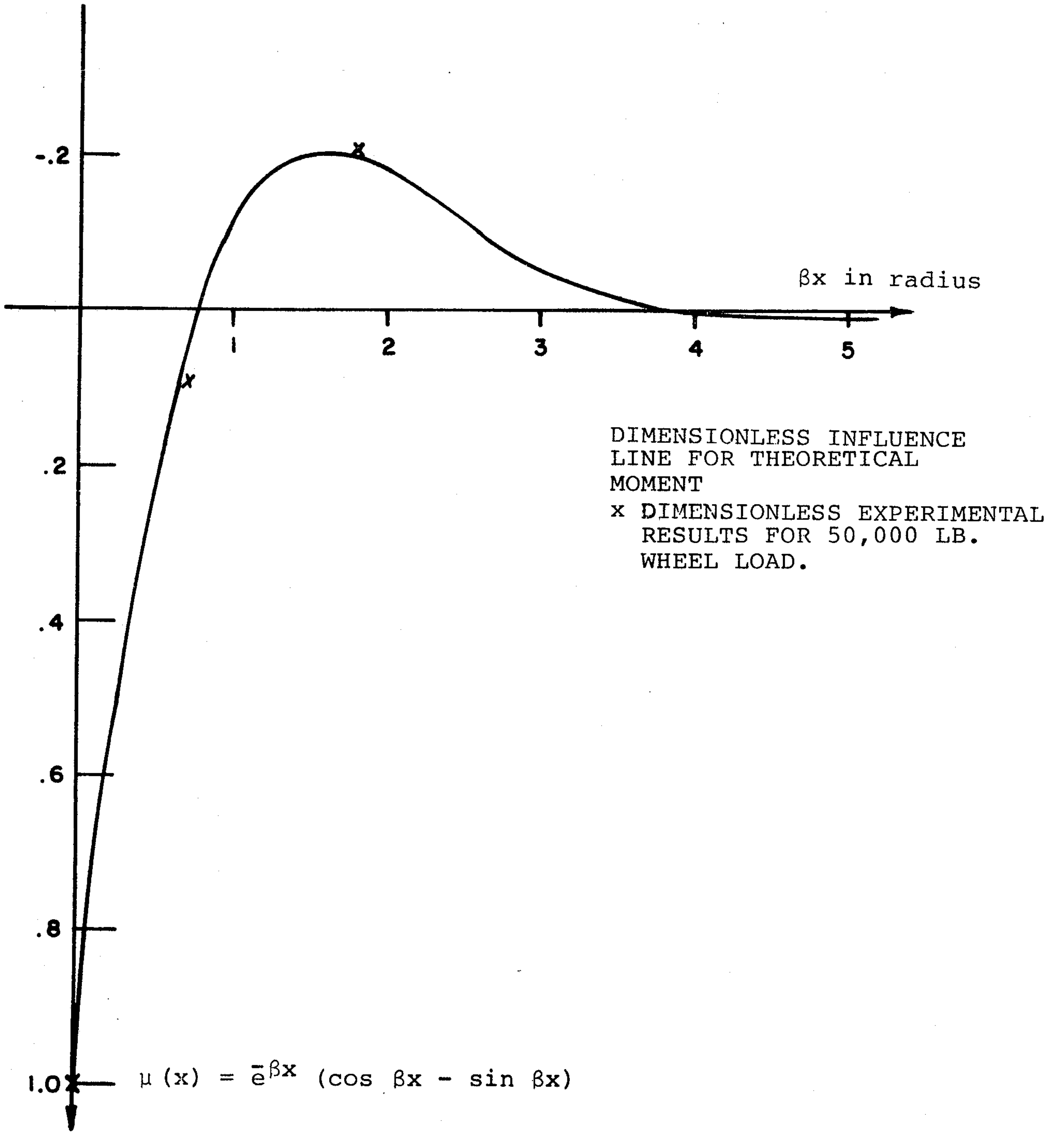


Figure 13. Dimensionless comparison of predicted results and experimental values

4. RESULTS

In comparing the three methods with each other and with previous published results, one should concentrate not only on the results that best represent the track response but also on the technique that is easiest to use. The equations and the basic assumptions of each method used in this report have been given along with the tests results. These results and their signifance to track engineers will be discussed in this section.

Referring to the results for a given load of 37,000 lbs, the track modulus of 4,464.5 lb/in^2 determined by the use of method 1 (deflection curve) differs by 13.6% from the track modulus of 5,167.3 lb/in^2 determined by method 2 (heavy-light wheel load deflection curve). This difference can be explained from the main assumption of the second method; the elimination of slack in the track. By this elimination, the deflected area of the track decreased, and since the area is inversely proportioned to the track modulus, the modulus increased as shown by the higher value obtained for this loading. Now comparing method 3 (beam on elastic foundation) it is shown that the track modulus determined by this method, 4,256.9 lb/in^2 is 4.6% lower than method 1 and 17.6% lower than method 2 for the same load. In methods 1 and 3 the initial slack in the track is not taken into account, and both methods appear to be in good agreement with each other. However method 2, which does eliminate the effects of initial slack, results in a value which is somewhat higher than by the other methods. Table 6 gives a summary of these results and shows the differences between each method. It can be seen in Table 6 that the track modulus determined by method 3 increases with increasing loads. It will be shown that this increase is linear. No conclusion to that effect can be stated for the other two methods. Method 1 does exhibit an increasing pattern but at the higher loads it is so slight that this could be due to experimental error, error in plotting the deflection curve, or instrument error. Note that all results are based on deflection data reduced from the level measurements.

The data read with the LVDTs were reduced using method 3 only since there was an insufficient number of points to determine the track deflection curve (Methods 1 and 2). Figure 10 is a plot of the wheel loads vs. the deflection under the load for the loading and unloading sequence. It can be seen that the track does not react linearly under load and that the unloading path is not identical to the loading path. This is a strong indication that there is permanent deformation in the track structure. For this test, settlements of .005 in. were noted. Analysis of Figure 10 shows that the load deflection curve is non-linear and consequently does not quite agree with the original Winkler hypothesis that the foundation acts as a linear spring. However, it appears that as a first order approximation, the Winkler hypothesis can be justified. Determining the modulus with method 3 (Figure 9), an examination of the data shows that the track modulus varies linearly with wheel load for loads above 10,000 lb and up to 50,000 lb for increasing and decreasing load. The difference in values for

TABLE 6: SUMMARY OF TRACK MODULUS (LEVEL READINGS DATA)

METHOD 1

Load (lb)	Modulus (lb/in/in)	% Difference Method 2	 3
37,536	4,464.5	15.7	-4.6
39,566	4,711.8	20.1	-1.0
50,327	4,769.0	15.9	17.5

METHOD 2

Load (lb)	Modulus (lb/in/in)	% Difference Method 1	 3
37,536	5,167.3	-13.6	-17.6
39,566	5,657.9	-16.7	-17.5
50,327	5,528.6	-13.7	1.4

METHOD 3

Load (lb)	Modulus (lb/in/in)	% Difference Method 1	 2
37,536	4,256.9	4.9	21.4
39,566	4,666.6	1.0	21.2
50,327	5,604.2	-14.9	-1.4

track modulus shown for the loading and unloading sequence is due to the permanent deformation in the track shown by the load deflection curve, Figure 10. The variability of the track modulus with load suggests that the track modulus should be measured as close as possible to the expected load environment of the given track. The difference between loading and unloading curves shown in Figure 9 would suggest that the two values of track modulus determined are dependent on the time duration of the load, and, if possible, should be considered when determining the track modulus with method 3.

Table 5 presents a summary of the results of this test using the LVDT deflection data. This table shows the loads, deflections, track modulus, track stiffness and track compliance. From these results a plot was made of track modulus vs track stiffness (Figure 12).Comparing these with the theoretical results, it is shown that the results are practically identical for the range of track modulus and rail size used. Figure 14 shows the theoretical relation between track modulus and track stiffness for a range of eight rail sizes from 70 RE to 155 PS. The theoretical results for the graph are supported by the experimental values from this test with 136 RE rail, where the modulus and stiffness were determined independently.

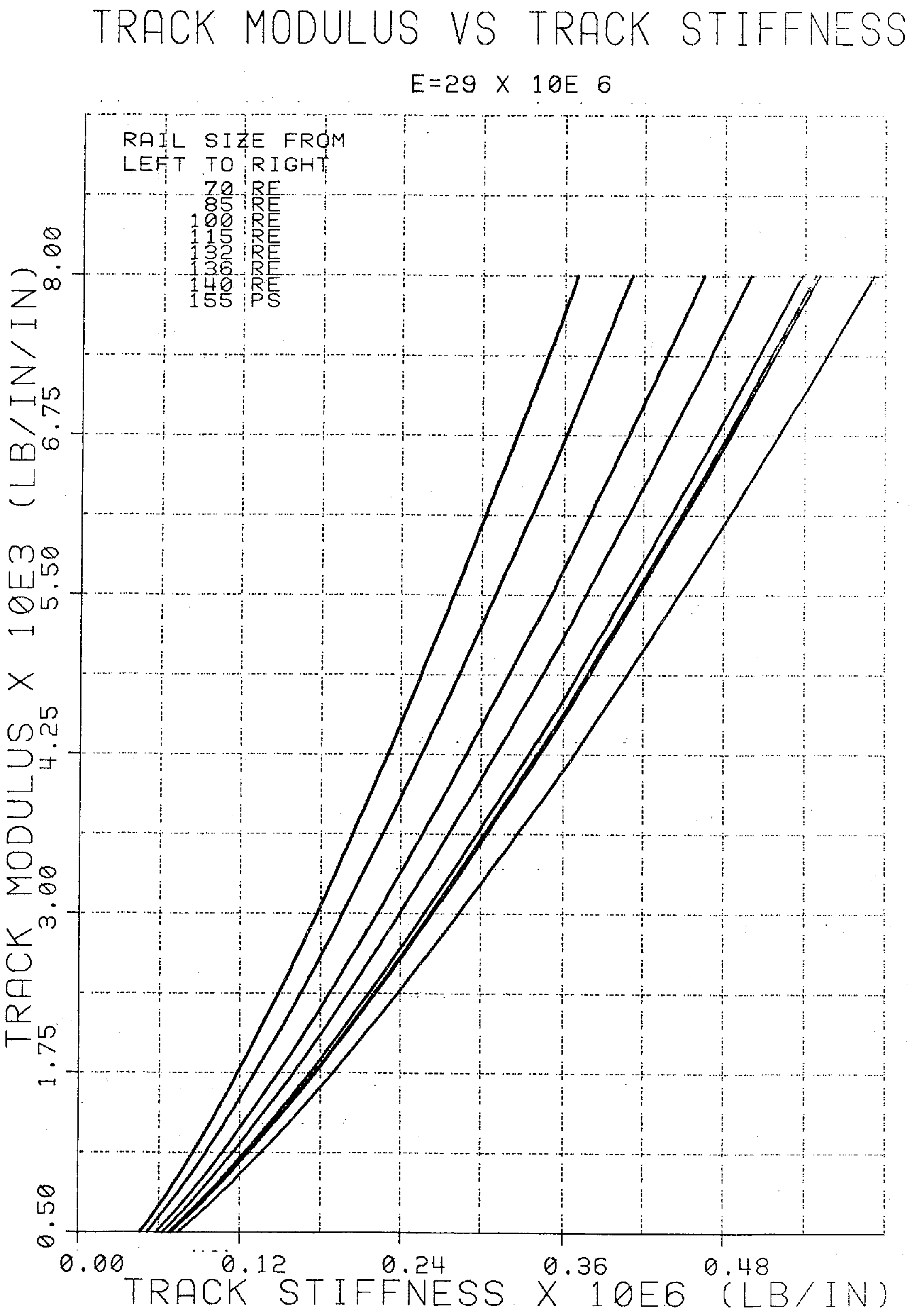


FIGURE 14. THEORETICAL RESULTS FOR VARIOUS SIZE OF RAILS, TRACK MODULUS VS TRACK STIFFNESS.

Before any conclusion can be made as to the "best" method for calculating track modulus, consideration should be given to the practical problem of collecting data for each. As mentioned in the discussion of each method, method 1 seem to be more accurate because it takes into account a large portion of the track, thus eliminating local effects. However, the use of this method, in the field could be cumbersome. Along with the load, at least six locations of absolute deflections would have to be measured on each side of the load. This is done either manually with a level or mechanically with displacement transducers, which does create a problem of pretest instrumentation or significant time delay in reading of the level. Additional time delay, especially in soft track, would give erroneous readings due to creeping under load. These disadvantages tend to outweigh the good accuracy obtained by using this method. Method 2 has the same disadvantages as method 1 but in this case they are worse since two deflection curves have to be determined, doubling the complication and time mentioned above. As far as the accuracy goes, it is believed that this method would always give a higher track modulus than what a vehicle in service would experience. This would lead to smaller deflections and higher bending stresses than what has been determined by using the track modulus from method 2. Method 3 appears to be the best method. It is substantially easier to collect data for this method, since it requires only one deflection point and the applied load value(s). The accuracy of this method compared to method 1 which is considered by many to be the correct method for determining track modulus is quite good. This could be outweighed by the ease in which this method can be used. It is therefore recommended that the beam-on-elastic-foundation theory be used where track modulus is required. Furthermore, it should be determined under a wheel load similar to that experienced by the track. Thus for track that sees 100 ton car traffic, a similar wheel load of approximately 33,000 lb should be used to calculate the track modulus. For track that sees lighter traffic, an appropriate lower load should be used. If a comparison of different tracks with varying support conditions is derived, a lighter wheel load, possibly 27,500 lb (70 ton car), should be used.

Once the track modulus is known, the stiffness and the track compliance can be readily determined from Figure 14, for the appropriate rail size. With this information, the track engineer is then in a better position to evaluate the condition of his track.

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