REPORT NO. FRA/ORD-77/28

A MATHEMATICAL-COMPUTER SIMULATION OF THE DYNAMICS OF A FREIGHT ELEMENT IN A RAILROAD FREIGHT CAR

JULY 1977
SUMMARY REPORT

Prepared for
U.S. DEPARTMENT OF TRANSPORTATION
FEDERAL RAILROAD ADMINISTRATION
Office of Research and Development
Washington, D.C. 20590

NOTICE

The United States Government does not endorse products or manufacturers. Trade or manufacturers' names appear herein solely because they are considered essential to the object of this report.

TECHNICAL REPORT STANDARD TITLE PAGE

1. Report No.	2. Government Accession No.	3. Recipient's Catalog No.
I.I.T.-TRANS-75-2		

4. Title and Subtitle
A MATHEMATICAL-COMPUTER SIMULATION OF THE DYNAMICS OF A FREIGHT ELEMENT IN A RAILROAD FREIGHT CAR

5. Report Date
May, 1975

6. Performing Organization Code

7. Author(s)
KWOK L. SHUM and TERRENCE WILLIS

8. Performing Organization Report No.
I.I.T. TRANS-75-2

9. Performing Organization Name and Address
ILLINOIS INSTITUTE OF TECHNOLOGY
Department of Mechanics and Mechanical and Aerospace Engineering
CHICAGO, ILLINOIS 60616

10. Work Unit No.

11. Contract or Grant No.
DOT-OS-40103

12. Sponsoring Agency Name and Address
1. Office of the University Research, and Federal Railroad Administration, U.S. Department of Transportation
2. Association of American Railroads
3. ~~General Motors Electro-Motive Division~~

13. Type of Report and Period Covered
INTERIM TECHNICAL REPORT NO. 3

14. Sponsoring Agency Code

15. Supplementary Notes

This study was, also, partially supported by Illinois Institute of Technology, Chicago, Illinois, 60616.

16. Abstract

This research studies the dynamic response of a freight element, inside a typical freight box car under service conditions, by a computer-model simulation technique. A 27 degree of freedom mathematical model has been developed to represent the freight car, truck and freight element, with the car body as a single rigid mass. This model has been validated against published railroad research data. This model is a more detailed one than most previously published simulations, and has additional characteristics. One is the option of modeling dry friction dampers by either Coulomb friction or equivalent viscous damping. A second improvement is the facility to express the response of the system in either time or frequency domain. The computer simulation shows that the critical roll mode speed of a representative 70-ton box car is around 17.5 mph. The maximum car body roll angle is 11.4° peak to peak, the maximum wheel load is 69,000 lb/wheel, and wheel lift durations are 0.2-0.4 sec. For a specific freight element near the roof maximum lateral accelerations of 1.5 g peak to peak at 0.64 Hz were calculated. At 50 mph, this value becomes 0.2 g at 2 Hz.

Vertical acceleration of 0.1 g at 1.25 Hz is computed for freight near the car body center of gravity at 50 mph. The mathematical model can be used for parametric studies on designs of the car body and truck.

Cushioning requirements for freight/package systems subjected to vibrations inside a freight car can also be established.

17. Key Words	18. Distribution Statement
Vibrations Freight Dynamics Computer Simulation Railroad Car Vibrations	Document is available to the public through the National Technical Information Service, Springfield, Virginia, 22151

19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No. of Pages	22. Price
		127	A07-A01

Form DOT F 1700.7 (8-69)

PREFACE

The work reported here is part of a Research Program into various technical problems in Rail Transportation which is currently underway at Illinois Inst. of Tech., Chicago, Ill. The Program is under the joint sponsorship of the U.S. Department of Transportation, Association of American Railroads, General Motors Corp. (Electro-Motive Division), and Illinois Institute of Technology under Contract Number DOT-OS-40103.

The Program of work is being conducted in the Department of Mechanics, Mechanical and Aerospace Engineering at IIT, with Dr. Sudhir Kumar, Department Chairman, as Project Director. Three technical areas of Rail Transportation are being investigated in this research program, and these are:

1) Freight Damage
2) Wheel/rail Friction
3) Diesel Engine Noise

This report is a Technical Report on a part of the work completed on Freight Damage in this project.

The support of the sponsors is gratefully acknowledged, and we are indebted to a number of individuals for their assistance in the course of this work. Specifically, our gratitude is due to Dr. S. Kumar, Project Director, who has provided invaluable advice and assistance in this work, and to Messrs. L. Olson and T. Tse, of AAR, for their continued help and advice. Also, we appreciate the effort put into the project by Mr. S. Shah of IIT, during the summer of 1974. Gratitude is also due to Mr. R. Bullock, Senior Project Engineer of Standard Car Truck Co. of Chicago, for his valuable advice and support for this project.

K. S.

T. W.

TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

LIST OF TABLES

LIST OF ABBREVIATIONS AND SYMBOLS

Symbol	Description	Units
B_1	axle centers in each truck	inch
C	bolster center of gravity above that of the truck	inch
CF	freight cushion material damping coefficient	lb-sec/in
C_i	damping coefficient at point i	lb-sec/in
C_iL	lateral damping coefficient at point i	lb-sec/in
D	truck center distance	inch
d_1	horizontal distance of center plate from car body center of gravity	inch
d_2, d_3	longitudinal distance between center of gravity of the bolsters from suspension spring groups	inch
d_4, d_5	longitudinal distance between center of gravity of the trucks from suspension spring groups	inch
e	center plate radius	inch
g	acceleration due to gravity	in/sec^2
g_1	distance between centerline of bolster and sidebearings	inch
GAP	sidebearing clearance	inch
GIB	gib clearance	inch
H	half rail gauge	inch
h_2, h_3	lateral distance between bolster center of gravity and suspension groups	inch
I_{xi}	rotational inertia of mass i about x axis	$lb\text{-}in\text{-}sec^2$
I_{yi}	rotational inertia of mass i about y axis	$lb\text{-}in\text{-}sec^2$
I_{zi}	rotational inertia of mass i about z axis	$lb\text{-}in\text{-}sec^2$
KBOM	bottomed spring stiffness	lb/in
KF	freight spring stiffness	lb/in
K_i	vertical spring stiffness at point i	lb/in
K_iL	lateral spring stiffness at point i	lb/in
KP_{ij}	pitching spring stiffness between bodies i and j	lb/rad
KT_{ij}	torsional spring stiffness between bodies i and j	lb/rad
m_i	mass of body i	$lb\text{-}sec^2/in$
R	center of gravity of wheel sets above rails	inch

Symbol	Description	Units
R_9 to R_{16}	rail lateral inputs	inch
RL	rail length	inch
S	maximum rail surface variation	inch
TL	spring travel to solid	inch
V_1 to V_8	rail vertical inputs	inch
x_i	lateral displacement of mass i	inch
y_i	longitudinal displacement of mass i	inch
z_i	vertical displacement of mass i	inch
ψ_i	roll angle of mass i	radian
ϕ_i	pitch angle of mass i	radian
α_i	yaw angle of mass i	radian

1. INTRODUCTION TO FREIGHT DAMAGE ASPECTS

Freight loss and damage faced by the rail transportation has been a very serious problem in the United States. For the year 1973, a total of $232,576,501 was claimed for L & D (1-1). This amount, being 0.8% less than that reported in 1972, was still very sizable compared with the earnings of the rail transportation of freight.

Increasingly, attempts have been made to identify the causes and to solve the principal freight damage problems. In the recent years, extra emphasis has been placed on the aspect of careful freight handling, by the manufacturers and the railroad shippers. Proper loading methods for different freight in a railroad car are recommended by the Freight Loading and Container Section of the Association of American Railroads (1-2). Defective or unfit equipment in railroad cars will render the freight more susceptable to damages. Shippers are urged to replace such equipment when necessary. Moreover, the dynamics of the car and that of the freight/package system contribute significantly to much of the direct or indirect causes of freight damage.

The dynamics of the car are primarily of two kinds--longitudinal shocks and over-the-road type of vibrations. Longitudinal shocks on the cars due to coupling and humping actions in classification yards give rise to high acceleration level impulse type of excitations to the freight within the car. Most of today's package design criteria are aimed at protecting freight elements from such shocks and usually drop tests are used to study and subsequently to choose certain cushioning materials as for packaging a particular kind of freight. Over-the-road vibrations, although usually do not give rise to high amplitudes of excitation, may have a prolonged effect on the freight elements. Moreover, package materials designed to protect the freight primarily from shocks may turn out to be providing bad dynamic environments to the products within, (1-3) while the train is running over the tracks. If a resonant frequency of the freight and cushioning material combination is encountered, the cushioning material may in effect magnify the input vibration levels to the freight and cause damage instead of protecting it. Vibrations, on the other hand, may not necessarily be the direct cause of freight damages. Prolonged vibrations can shake loose the freight elements from their initial loading positions. This loosening effect, which is called rattle, allows free motion of the freight element. This, coupled with the longitudinal shocks during run-ins and run-outs (depending on the type of terrain that the train is running through) can cause tumbling and falling of freight elements and subsequently damage them. It can be seen then that a severe vibration environment may contribute very much to freight damage both directly and indirectly. The indirect contribution is made when vibrations establish a damage-prone situation. Thus it becomes clear that for prevention of freight damage it is necessary to fully understand the vibration environment in a typical rail car and then isolate the freight from such vibrations as much as economically possible.

In the past years, tremendous emphasis has been given to the handling aspects by shippers and the rail industry. It is evidently very important now to study the dynamics of the car together with that of the freight/package systems. This study can enable better future design of packaging materials for the dynamic requirements of freight in a railroad car. Furthermore, a study as such may yield better design criteria for the railroad truck suspension, damping systems and track requirements for a smoother ride for the freight. At this stage of research, it is our primary interest to study the freight environment in a typical railroad car.

One effective way to study the freight environment in a rail car which allows the option of modifying and redesigning fairly easily the various parameters such as suspension and damping systems, etc., is by the mathematical model study approach. Several mathematical models have been developed in this field and used in simulation studies of various aspects of the freight car dynamics (1-4, 1-5, 1-6, 1-7, 1-8, 1-9, 1-10, 1-11, 1-12). Each has made contributions toward solutions of dynamic problems in the field. Industry models and detailed information on them is not freely available because of proprietory rights. Other models, while good for individual tasks for which they were developed were not deemed suitable for determination of the three dimensional coupled car-freight element dynamics problem being addressed here.

A comprehensive study of freight dynamics necessitates a simulation of truck, car and freight components, all in one mathematical model, so that the interactions between these various components can be properly represented. The various mathematical models set up by the Stucki Company (1-10), Patel/Martin (1-11) and Tse/Martin (1-12) address the dynamics of rail car as a complete unit, but each is directed towards a specific goal. For example, the Stucki Company analysis attempts to establish the damping requirements to control vertical and roll motions of rail cars, and does not incorporate effects of these motions on freight elements. The model developed by Patel/Martin is relatively simple and explores a basic method of approach to computer simulations of rail cars. The Tse/Martin model was only recently developed, and was an independent parallel study conducted at the Association of American Railroads, under the Track Train Dynamics Program. It describes the rail car and truck using a 20-degree of freedom model and simulates the car body as a flexible (2 mass) system. The model was specifically developed for use in parametric studies on the dynamic characteristics of rail cars. The IIT simulation developed in the present study includes the coupling of freight elements, car body, truck motions and track characteristics in one mathematical model.

The mathematical-computer model developed here at IIT is the first known solution incorporating the dynamics of the freight car truck, the bolsters, the car body and the freight element into one comprehensive analysis. It has been developed from a very basic and simple model, through several iterations of the design cycle, up to the current configuration, which has been shown from a dynamics point of view, to be a good representation of a real freight car system.

The report given here is to describe the work conducted in developing this mathematical model, and to describe the details of the model.

2. SELECTION OF A CAR AND DETERMINATION OF ITS CHARACTERISTICS

The dynamics of the car and that of the freight/package system may be studied by mathematically modeling the system and performing simulations on the model. For maximum value of such a study to the American Railroad Industry, the model should be based on a car and its truck that is widely used in North America.

The majority of commodities shipped by rail today are carried inside closed cars (2-1). The dynamic environment in these cars then can be considered as representative of the service environment encountered by freight packages. In a recent "over the road" test program (for determining the acceleration levels and forces in various parts of a box car under service conditions) performed by A.A.R. (2-2), a 70-ton box car with a typical truck was selected as a representative vehicle, to run the test over 5000 miles on different railroads. Based on this information, a 70-ton box car with a typical truck is chosen here for the basis of the mathematical model. It is highly representative of the North American railcars. In addition, future correlation studies with the 5000 Mile Road Test data may be conducted, and the 'ground work' for this has now been completed.

After selection of a car and truck, work was conducted to identify the significant components of the car/truck system and to determine their characteristics. As originally proposed, the approach adopted here has been to model the car as simple as possible for the initial analysis, and only when this model is properly functioning, and its simulation possibilities fully explored, then the additional complexities involved in improving simulation have been added in a stepwise manner. Initially then, the car had to be regarded as a five mass system, with linear springs and dampers. Nonlinear effects (spring bottoming, clearances, etc.) have subsequently been added. At the present time, the development of the mathematical model components is as follows.

2.1 The Car Body

As shown by the Freight Car System tests of Nasa-Martin Marietta Corp., a rigid car is a good representation of a current typical U.S. box car, that is, the flexibility of the box car body may be considered negligible. In such a case, the mathematical representation may be made as a rigid body, with its mass concentrated at the center of gravity. It is, therefore, appropriate in the present model to assume a single lumped mass for the car body. This requires that the following parameters be determined: mass and mass moments of inertia about the three principal axes, location of the center of gravity of the car body, width of the car and other geometric parameters determined from referenced literature and/or railroad publications (for details refer to Appendix C).

2.2 The Center Plates

These are locations for support of the car body weight on the truck bolsters. They are the circular plates of diameters 13-14 inches about which the car body swivels. Lubrication pads are introduced between the center plates to reduce friction. Relative vertical motions between the car body and truck bolsters may cause center plate separation. Severe car roll motion may result in only partial contacts of the center plates. For all practical purposes, then, center plates may be modeled as two stiff vertical springs.

2.3 The Side Bearings

These are the stoppers located on the upper surface of the truck bolsters. Some of them may be spring loaded and some are just rollers. Their function is to inhibit the car from rolling about its longitudinal axis indefinitely. If the weight of the car body plus that of the freight is fairly evenly distributed, all side bearing clearances statically are of the order of 1/4 inch. When the car rolls severely, this clearance may vanish and the car load is shared by the center plate and the side bearings. This nonlinear effect is modeled into the computer simulation.

2.4 The Center Plate Extension Pads (C-PEP Pads)

On more recent designs of bolsters by some railcar manufacturers, a new feature called the C-PEP Pads is incorporated at a location between the center plate and the side bearings.

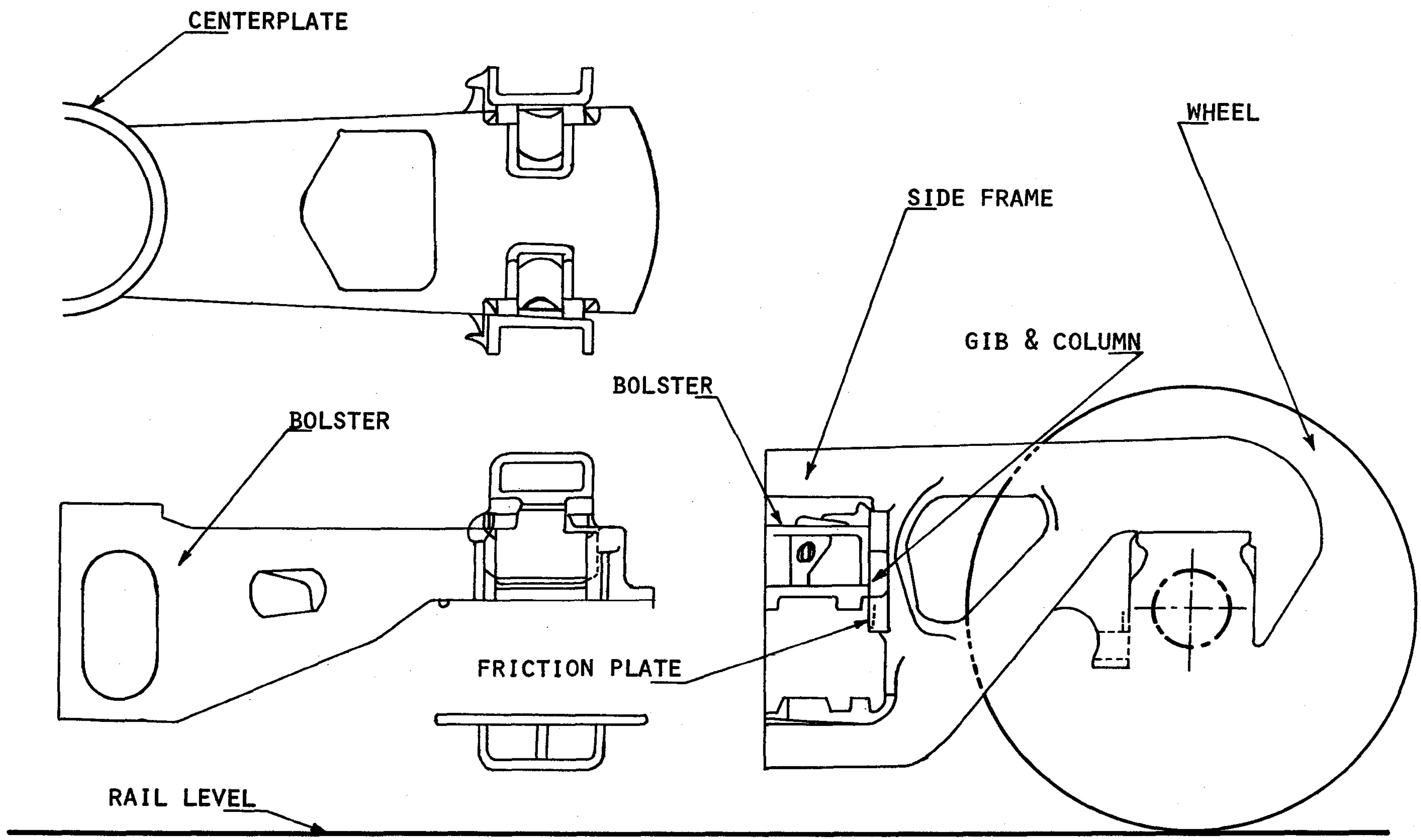


FIG. 2-1. A TYPICAL FREIGHT CAR TRUCK

These pads are made of a certain elastomeric material that provides horizontal rotational control in addition to complementing the vertical damping.

In order not to further complicate the mathematical model at this time, this effect is reserved as an option for further modifications on the model. At present, bolsters without this feature constitute the more conventional designs which are still widely used in the rail industry of today.

2.5 The Axle, Wheel Set and Sideframes

These are all rigid masses and have been assumed to be equivalent to one lumped mass, located at the center of gravity of the truck. Geometric dimensions of each have been determined.

2.6 Suspension Springs

The typical truck with 3 11/16 inch travel linear springs are studied from mechanical drawings and their stiffnesses determined. (For details see Appendix C.)

2.7 The Friction Shoe and Spring-Steel Wear Plate

Together these form a typical damping system, and the friction shoe can be spring loaded to give variable force depending on the loading and the direction of the damping stroke. From some railroad company experimental studies, it was found that the frictional force developed on the wear plate for the upward stroke was different from that for the downward stroke. The energy dissipated by the upward stroke is 65% of the complete cycle. This information was made use of in later deriving the equivalent viscous damping coefficient (see Appendix C).

It became apparent that the modeling of truck damping systems is possible as both Coulomb and/or viscous, and that perhaps an equivalent viscous damping approach would be fruitful, allowing this model to be compared with both existing (Coulomb) damping systems, and at the same time, used for future design and evaluation of hydraulic dampers. Consequently, a new analytical method for incorporating freight car truck damping effects into the mathematical model has been developed, based on an equivalent viscous damping concept. The computer model now has the capability of either Coulomb, viscous or equivalent viscous dampings.

2.8 Rail Profile and Subgrade Structures

Noting that these are not rigid structures, the elasticity of the subgrade has to be considered (2-3), and is modeled into this computer program as a vertical and lateral road bed stiffness.

2.9 Couplers

The car is subjected to longitudinal coupler inputs depending on the type of truck, track condition, terrain, coupler design, etc.

Note: Details of numerical values on spring stiffness, equivalent viscous damping coefficient, and descriptive data for a 70-ton box car are reported in Appendix C.

3. THE MATHEMATICAL MODEL

3.1 General Description

In the mathematical modeling of the freight car, a 27-degree of freedom nonlinear model has been developed. This simulates a 70-ton box car as 6 lumped masses connected by spring damper groups, and each mass has several degrees of freedom. The 6 masses respectively represent the car body, the front and rear bolsters, the front and rear side frame/axle sets (trucks), and an element of freight.

The degrees of freedom modeled for the various masses are shown in the table below.

Table 1. Degrees of Freedom

Freight car element	Degree of Freedom					
	Translational			Rotational About:		
	Vert.	Lat.	Long.	Vert. (yaw)	Lat. (pitch)	Long. (roll)
Car body	X	X	X	X	X	X
Front bolster	X	-	-	X	X	X
Rear bolster	X	-	-	X	X	X
Front truck	X	X	-	X	X	X
Rear truck	X	X	-	X	X	X
Freight element	X	X	X	-	-	-

3.2 Car Body and Truck Bolsters Interface

3.2.1 Center Plates. These are modeled here as four vertical springs, each with a very high stiffness. This allows for relative vertical motions between the car and the truck bolsters. At this time, relative lateral motions between the car body and the two truck bolsters are assumed constrained to be zero.

3.2.2 Sidebearings. The sidebearing reaction is modeled as a spring with a clearance between the car body underframe and the spring. It does not have any stiffness until there is no clearance.

3.3 Truck Bolsters and Sideframes Interface

3.3.1 Suspension Springs. A total of four vertical springs is modeled per truck, with two springs on each side. In order to

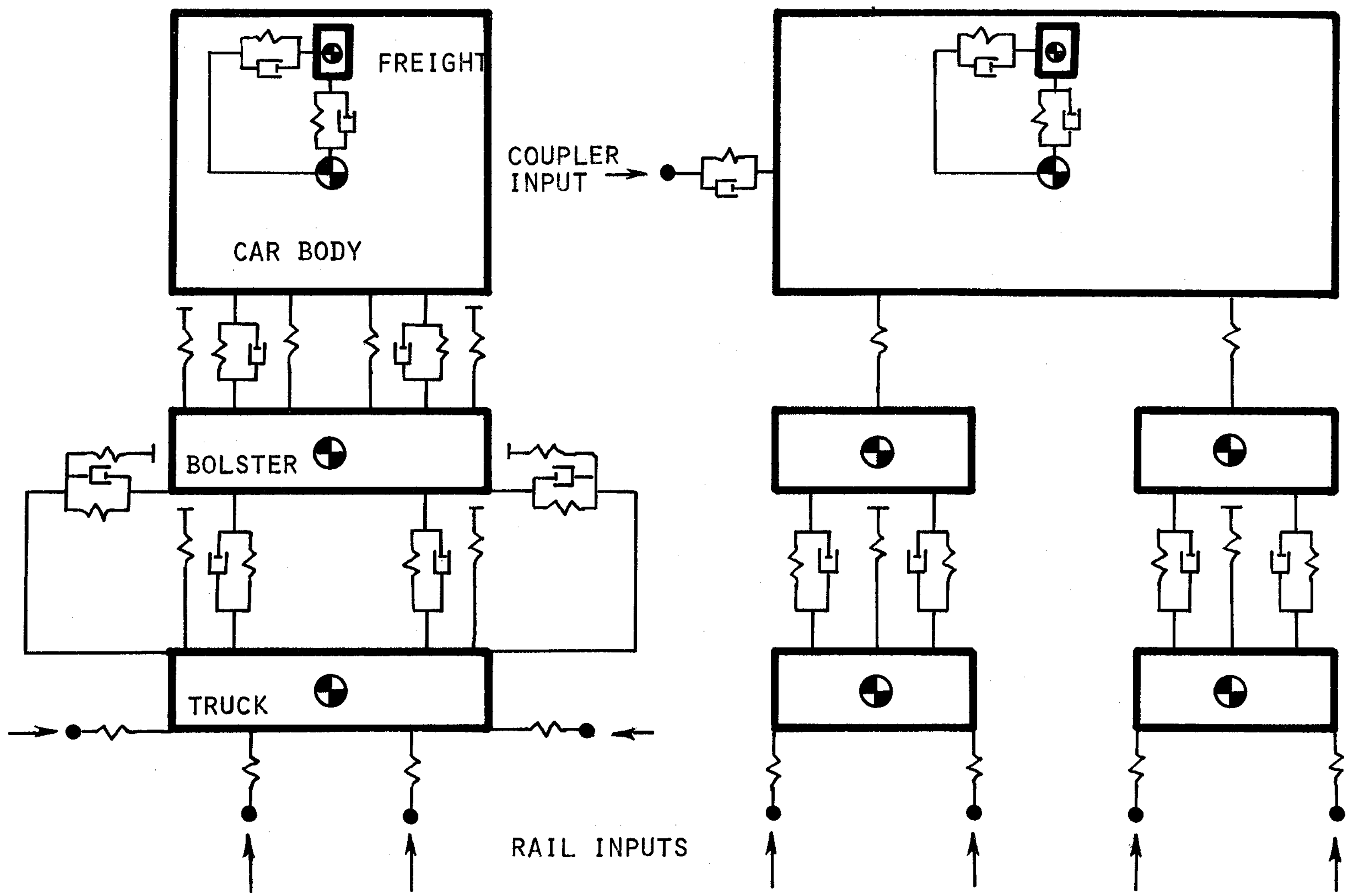
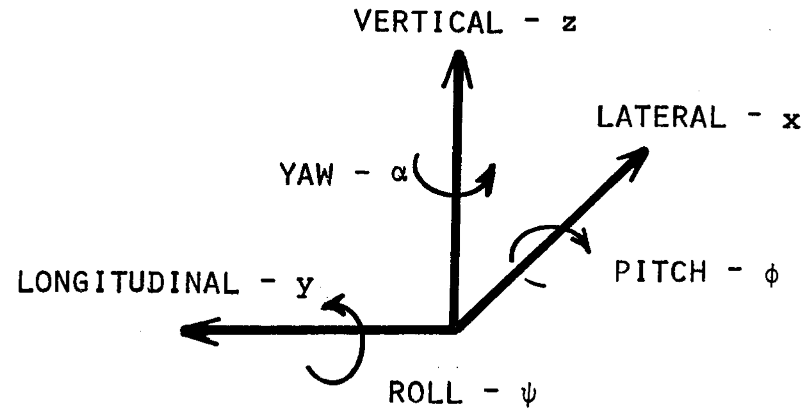


FIG. 3-1A. THE IIT MATHEMATICAL MODEL OF A FREIGHT CAR WITH A FREIGHT ELEMENT



TRANSLATION	ROTATION
4 LATERALS	5 PITCHING
2 LONGITUDINALS	5 ROLLING
6 VERTICALS	5 YAWING

TOTAL DEGREE OF FREEDOM: 27

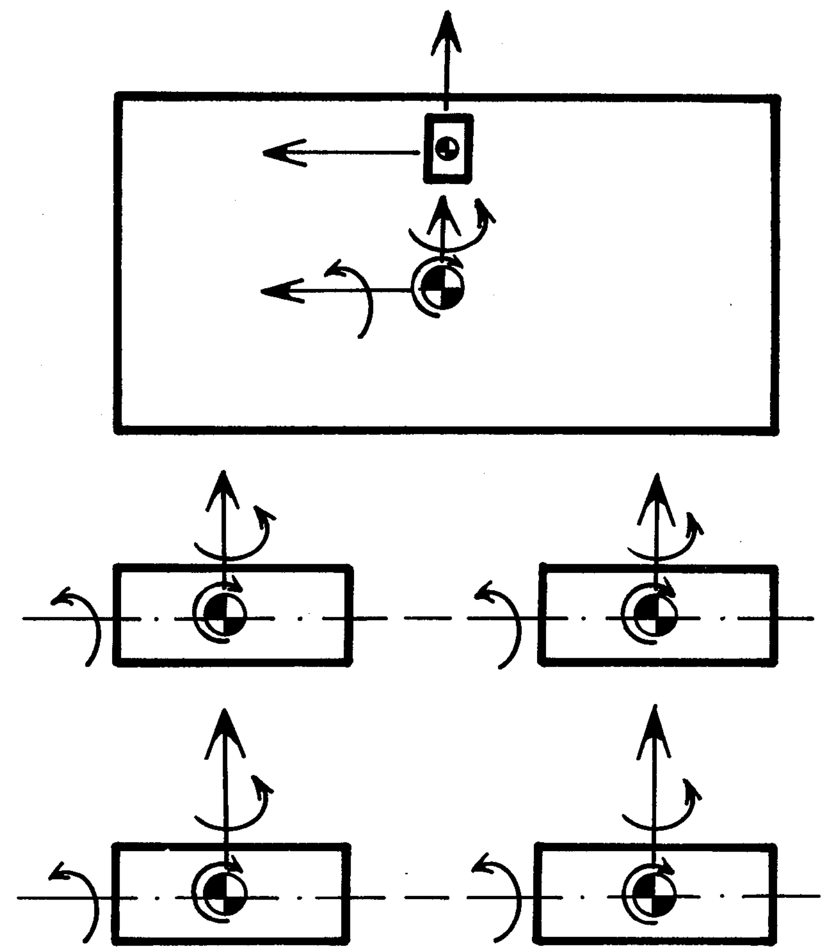


FIG. 3-1B. DEGREES OF FREEDOM OF THE IIT MATHEMATICAL MODEL incorporate the bending effect of the bolsters, an analysis was made to estimate the bending stiffness of the bolster (see Appendix C). Consequently, the values of the stiffness used for suspension springs are the effective spring stiffness of the suspension group in series with the bending stiffness of the truck bolster. Two additional nonlinear springs are used to model the effect of spring bottoming, which happens at severe bounce situations.

Although the actual suspension springs are in the vertical plane, when the truck bolster moves laterally relative to the sideframe, certain lateral elastic constraint is introduced into the system. These spring actions are accounted for in this mathematical model by four lateral springs per truck.

3.3.2 Friction Plates. These are the energy dissipation elements used in the truck for damping down the amplitudes of the vibratory motions of the truck. In this mathematical model the friction plates are represented as either an equivalent viscous damping (see Appendix C) or Coulomb's friction damping. Incorporating viscous type damping models into the computer simulation at this time is considered advantageous since it both simplifies the mathematics and in addition leaves the way open for future investigations of modified damper designs. The Computer Program developed here (see section on Computer Program) allows the option of running the computer simulation with either dry or the equivalent viscous damping. Consequently, it has not lost the reality of dry friction type damping, used on the majority of today's trucks.

3.3.3 Gib Clearance. This is the lateral clearance between the bolster and the sideframe. A nonlinear spring with a very high stiffness is used for modeling the gib effect.

3.4 Rail Profile and Subgrade

Depending on the quality of the subgrade under the rails, and the dynamics of the railcar, up to several inches of rail depressions have been observed when a train passes over it. This effect is modeled as eight track springs in each of the vertical and lateral directions, located at the contact points between the wheels and the rails. The North American Continent rail tracks are usually half-staggered (i.e., the rail joints on one side of the track is at mid-length of the rail on the opposite track) and depressions at the rail joints can conservatively be estimated to be of the order of 3/4 inch. Previous studies, made by other railroad researchers (2-3) showed that the profiles of some of the revenue tracks actually look like "rectified sine waves". From this, the rail profile used in this simulation is that of a rectified sine wave, with an amplitude of 3/4 inch vertically, and 3/8 inch laterally, in phase (Figure C-3c).

3.5 Coupler Forces

When trains are moving over hilly areas, run-ins and run-outs can create a very significant shock and vibratory environment inside the car. Longitudinal coupler inputs can be modeled as if they act at the car body center of gravity by introducing an equivalent force and couple combination at the center of gravity. The spring-damper model used here for this purpose is excited by the equivalent force only, in order to verify that the model functions properly. Rotational responses due to couples at the center of gravity are checked out when the model is excited in the roll or the bounce modes, and so does not need to be input for this purpose at this point. Other severe coupler shock forces are also developed in service, due to the freight yard humping action. This depends, among other things, on the speeds at which the cars are coupled together (3-1). At this stage of the research, only the vibratory coupler inputs are being considered.

3.6 Car and Freight Element Interface

A linear spring-damper system is again assumed. The dynamic damping characteristics have been obtained experimentally as part of this total project here at Illinois Institute of Technology. This part of the program will be reported in a separate Project Technical Report.

It can be seen from the discussion given above, that the mathematical model developed here is nonlinear, as in the actual rail car. Gib clearances, sidebearing clearances and the spring bottoming effects are some of the nonlinearities in the system.

These are simulated by the system of equations developed, which are then solved on the digital computer, yielding displacements, velocities and accelerations at any point(s) in the freight car. Dynamic loadings at the center plates, truck bolster lateral reactions and wheel loads, wheel lifts, suspension spring deflections plus all the forces and reactions in the system can be studied in the present model.

4. EQUATIONS OF MOTION AND METHOD OF SOLUTION

4.1 Equations of Motion

After setting up the mathematical model, equations of motion have to be derived and solved for accelerations, velocities and displacements (both rotational and translational).

For a simple spring-mass-damper system in which the body is under excitation from an external force p, the equation of motion is

$$m\ddot{x} + c\dot{x} + kx = p, \tag{4-1}$$

where m - mass of the body

c - viscous damping coefficient

k - spring stiffness.

If m, c, k, p, $\dot{x}$ and x initial are prescribed, the acceleration $\ddot{x}$ can be computed.

For more complicated spring-mass-damper systems such as the one representing the 70-ton box car with the typical truck a systematic way of writing the equations of motion step by step without much complication is the Lagrange's Equation of motion which can have the form

$$\frac{d}{dt}\left(\frac{\partial E}{\partial \dot{q}_i}\right) - \frac{\partial E}{\partial q_i} + \frac{\partial V}{\partial q_i} + \frac{\partial D}{\partial \dot{q}_i} = Q_i, \tag{4-2}$$

where E - kinetic energy of the system

V - potential energy of the system

D - Rayleigh's dissipation function of the system

Q_i - generalized forces, and

q_i - generalized coordinates i = 1, 2, ..., 27 (x_1, y_1, z_1, ψ_1, ϕ_1, α_1, z_2, ψ_2, ϕ_2, α_2, z_3, ψ_3, ϕ_3, α_3, x_4, z_4, ψ_4, ϕ_4, α_4, x_5, z_5, ψ_5, ϕ_5, α_5, x_6, y_6, z_6)

With 27 generalized coordinates, there are 27 equations of motion similar to the one in eq. (4-1). Details of the derivation of these equations are listed in Appendix A Equations of Motion.

Some of the equations derived are coupled with one another and they are grouped into 5 different matrices. All the equations, both coupled and uncoupled are programmed (in Fortran language) and then solved for 27 accelerations simultaneously. Integrating these accelerations twice leads to the corresponding velocities and displacements.

4.2 Solution by Computer Iterative Method

For dynamic simulation on the computer model, an iterative method has been developed, which uses the UNIVAC 1108 computer (on campus). Rail surface variations as well as coupler inputs are sources of excitation. Simulation is started at time equals zero, with the system at static equilibrium. Excitation is applied to the model and the resulting accelerations computed. The coupled accelerations are solved by a subroutine LSIMEO (which is currently available in the Math Pak at the Information Processing Center of Illinois Institute of Technology, and the listing of which is given in Appendix B Computer Program Listings). All accelerations are then numerically integrated by the Runge-Kutta Integration technique to obtain velocities and displacements at the center of gravity of the masses in the model. These new velocities and displacements, together with the excitation, are the values based upon which the acceleratin of the second time step are computed. This process is repeated for each time step and new values are computed on old ones. An iteration method of this nature enables us to study the dynamic responses of the system up to any desired period of simulation.

5. COMPUTER PROGRAM

A computer program has been developed for solving the equations and simulating the dynamic responses of the box car/freight system. This program consists of a Main program and nine subroutines. The function and philosophy of each is discussed below.

5.1 MAIN Program

This program functions as the coordinator for the subroutines. It calls the subroutines ACCELN and RUNG to obtain values of acceleration, velocities and displacements at centers of gravity of the masses in the model. Geometric parameters such as truck center distance, length of the various components of the car and truck, spring stiffnesses, and moments of inertia, etc., are entered in this portion of the program.

5.2 Subroutine DELGAP

At each time step, this subroutine computes all four of the sidebearing clearances between the body bolsters and the truck bolsters. If a sidebearing is touching the body bolster, there is an additional reaction at the point of contact other than those at the center plate locations. This reaction is accommodated by adding, at this time step, an additional spring in parallel with the center plate vertical springs. This additional spring will remain in effect for as long as there is no sidebearing clearance.

5.3 Subroutine DELGIB

Between the truck bolster and the column of the sideframe, there is a small clearance (of the order of 3/4 inch) laterally. This is the gib clearance. When the box car is running on the rails, lateral motions exist such that the truck bolster may be hitting the column. This zero gib clearance effect is modeled here by adding a gib spring in parallel with the lateral springs whenever such condition exists. This is done by first checking the gib clearance. A coefficient denoted by the symbol δ is utilized to facilitate the modeling of the spring nonlinearity. A value of 1 or 0 is correspondingly assigned to δ to bring in the gib spring action or remove it. A total of 4 gib springs is used to model the gib effect on the two truck bolsters.

5.4 Subroutine SPRING

During severe conditions of the bouncing mode, especially with heavily loaded cars, spring bottoming phenomenon is not uncommon. In order that the computer model can simulate and predict the suspension spring bottoming effect, a SPRING subroutine is developed for this purpose. At every time step, the vertical motions of the truck bolsters relative to those of the sideframe are checked against the allowable travel length of the spring as specified by designers and manufacturers. If it is found that any suspension spring group reaches its solid length, a comparatively high stiffness spring is introduced in the vertical model to stop the truck bolster from compressing on the suspension springs very much further. This high stiffness spring will be removed by the subroutine if spring bottoming does not exist.

5.5 Subroutine ACCELN

The function of this subroutine is to compute the acceleration values at each iteration. First, it calls the subroutines DELGAP, DELGIB and SPRING to update the total number of springs suitable for use at each iteration time step. For example, if after calling the three subroutines it is found that two of the four sidebearing clearances reduce to zero and two suspension groups bottom out but the truck bolsters are not hitting the column of the sideframe, then two additional sidebearing springs and two bottoming springs but no gib springs will be included in the current iteration to calculate the new acceleration values.

Sixteen of the acceleration variables are coupled (e.g. those of the rotational coordinates) and they are grouped into five matrices. This grouping into smaller matrices, rather than retaining all the coupled equations in one big matrix, significantly helps to speed up the solving process by LSIMEO. The rest of the acceleration variables are computed independently.

ACCELN also calls other subroutines CPLATE, CAL and T5000, the functions of which will be discussed in their own sections later in this report.

5.6 Subroutine RUNG

After accelerations are computed by ACCLN, their values are transferred back to the main program. Then MAIN calls subroutine RUNG to integrate numerically the accelerations twice yielding the corresponding velocities and displacements.

Subroutine RUNG is a standard program available in most computer-aided numerical analysis books. The one used here is a fourth order Runge-Kutta method which predicts the value of y_{i+1} from the value of y_i at time t_i based on the formula:

$$y_{i+1} = y_i + \frac{h}{6}(K_1 + 2K_2 + 2K_3 + K_4)$$

where K_1, K_2, K_3 and K_4 are weighted averages as follows:

$$K_1 = f(y_i, t_i)$$
$$K_2 = f(y_i + \frac{h}{2} K_1, t_i + \frac{h}{2})$$
$$K_3 = f(y_i + \frac{h}{2} K_2, t_i + \frac{h}{2})$$
$$K_4 = f(y_i + h K_3, t_i + h)$$

and h is the step size. Note: The fourth order Runge-Kutta technique gives as accurate value of y_{i+1} as a fourth order Taylor series. The choice of the step size h has to depend on the response frequency of the system studied. The following exercise serves as an example on estimating the step size.

Estimation on the time step for using Runge-Kutta technique based on the natural frequency of the bolster:

Total lateral spring stiffness = 4(4425 + 666000)lb/in.
= 2,681,700 lb/in.

Mass of the bolster $= \frac{1150}{g}$

Natural frequency $f_n = \sqrt{\frac{K}{M}}$

$$= \sqrt{\frac{2,681,700 \times 386.4}{1150}} \text{ Hz}$$

= 950 Hz

Periodic time $= \frac{1}{f_n} = \frac{1}{950}$ sec. = .00105 sec.

Rule of thumb: 8 iterations is minimum per response cycle for using Runge-Kutta

$$\text{Time step} = \frac{.00105 \text{ sec.}}{8}$$

$$= 0.00013 \text{ sec.}$$

This analysis can be considered as a rough guide to estimate the order of the step size. However, the optimum value still needs to be obtained through experience and trial and error.

5.7 Subroutine CPLATE

This subroutine computes the vertical reaction of the front and rear center plates.

5.8 Subroutine CAL

Each time this subroutine is called it computes the front bolster lateral reaction as well as all the wheel loads. This subroutine gives valuable information as to the loading on the column of the sideframe and the phenomenon of wheel-lifts, the severe cases of which may cause derailments.

5.9 Subroutine T5000

This subroutine computes various accelerations at selected locations within the dynamic system, based on the accelerations of the rigid body masses and the geometry of the rigid bodies. In the analysis performed so far, the vertical and lateral accelerations on the roof and the floor of the box car at the front and rear ends of the car have been computed. These specific locations have been chosen to yield data for later comparison with field data obtained by the 5000 MILE BOX CAR VIBRATION TEST.

5.10 Subroutine SGNFUN

A special feature introduced in this computer program is the option of simulating the suspension damping system either as an equivalent viscous damping or as Coulomb's damping. If the latter is preferred, one statement is added in the subroutine ACCLN to call on SGNFUN.

Subroutine SGNFUN computes, at every time step, the relative velocities, both vertical and lateral, between the bolsters and sideframe columns. It then assigns a positive or a negative sign to the constant damping force such that the damping force always opposes the motion.

With such an option, two simulations can be run on the computer using the Coulomb's friction or equivalent viscous damping without any major modifications on the program at all. This is of great value in both development of new suspension damping systems or in evaluation studies of existing systems. Hence, this method of modeling the damping system of the typical truck as an equivalent viscous damping can readily be justified.

5.11 Nonlinear Modeling

It can be seen that in this computer simulation a great number of nonlinear springs can now be introduced by DELGAP, DELGIB and SPRING subroutines. These effects have been added to the model in a stepwise manner, and this has added a considerable amount of complexity to the computer program. It is felt, however, that the nonlinear model so developed is now a close simulation of a railcar dynamic system, and the complexity is justified. The correlation of this model with other data which has been verified by tests is discussed later in this report, and this nonlinear modeling of the railcar is shown to be validated.

5.12 Computer Program for Frequency Analysis

The main program and the nine subroutines discussed so far give as output data on acceleration, velocity and displacement at any point(s) in the car and the truck, in addition to the dynamic loadings on the car structure. These output data are given as a function of time. However, road test data often express acceleration levels (g) in the frequency domain. The frequency analysis on excitation levels becomes more meaningful to designers of freight cushioning and packaging materials, and also useful to the rail car and truck designers. With this in mind, the program FREQ was developed.

From our computer simulation results based on the rectified sine wave vertical rail inputs as well as the Stucki Company's data, we observed that the accelerations of the car body and those of the freight are quite periodic. In order to study the frequency contributions of these acceleration responses, one convenient method is to apply the Fourier analysis. Any periodic motion (including those which are complex) can be represented by a series of sines and cosines which are harmonically related. For example, if x(t) is a periodic function of the period τ, it can be represented by the Fourier series

$$x(t) = \frac{a_0}{2} + a_1 \cos \omega_1 t + a_2 \cos 2\omega_1 t + \ldots$$

$$+ b_1 \sin \omega_1 t + b_2 \sin 2\omega_1 t + \ldots$$

where $\omega_1 = 2\pi/\tau$ is the fundamental frequency and the coefficients a_0, a_1, a_2, ..., a_n and b_1, b_2, ..., b_n completely define the harmonic contribution of the period wave. With some manipulation of algebra, the coefficients can be obtained by hand calculation. In order to minimize the computational time, the digital computer is again used.

A program has been developed by the Information Processing Center at Illinois Institute of Technology to specifically compute the coefficients for the Fourier series. This we incorporate as a subroutine in our Frequency Analysis program, FREQ, and have it compute the amplitudes of contribution at each of the harmonics. Then the Fourier spectrum of the wave form can be plotted. Since some of the road test data from the 5000 Mile Box Car Vibration Test will eventually be presented as PSD (Power Spectral Density) functions, the computer program developed here in this project can output data suitable for PSD analysis. This will be useful in later correlation studies.

At present, FREQ is an independent computer program from the MAIN and its nine subroutines. Future work is planned to incorporate FREQ as an additional subroutine to MAIN so that we can run rail car freight dynamic simulations on our model with output acceleration levels in the time and/or frequency domain(s).

Note: All program listings are in Appendix B Computer Program Listings.

6. COMPUTER MODEL VALIDATION

The computer programs have been thoroughly debugged to give the correct logic and mathematical solutions. However, in order to show that the computer model properly simulates a real rail car dynamic system, it is necessary at this stage to validate the computer model. This ensures that any future dynamic analysis based on this model for design purposes will be meaningful.

6.1 Methods of Validation

Validation can be done in two ways. First, other computer model outputs can be checked against this model. One such computer model in the rail industry now is the model that Stucki Company developed (1-10) to study the damping requirements to control vertical and roll motions of freight cars. This A. Stucki model has been validated against test data. The model is proprietary to the Stucki Company and is not freely available. However, some output data based on this model has been published (1-10) and can be of value to other workers for validation purposes. Recently, the Association of American Railroads, as part of the Track Train Dynamics Programs, have developed a computer model (1-11) in which the box car body is modeled as two lumped masses joined by torsional springs to incorporate flexibility. These models, among numerous others, can be used to help establish validity for the model developed here, even though many significant differences in these various models exist.

As a second method of validation, the computer model simulation outputs can be correlated with field test data generated by other researchers. The most recent and extensive piece of road test data on shock and vibration service conditions was completed by the Research and Test Department of the Association of American Railroads (2-2). In this test, a 70-ton box car with a typical truck was placed in actual revenue trains on a number of railroads. Data so collected will reflect a wide range in speed, track characteristics and operating terrain. Since the type of car and track for this test is the same as modeled here, it will be very beneficial to try to correlate with such test data when it is available. Currently, data from this test program is being prepared by AAR for spectral and RMS analysis, and it is intended to further validate the IIT mathematical model with this data when it has been reduced to the appropriate form.

6.2 Comparison of IIT Model with Other Models

Before comparing the IIT computer simulation outputs with others, it is desirable to consider the following table, which briefly summarizes the similarities and differences between this model and some others.

Table 2. Comparison of IIT, AAR and Stucki Models

	AAR	STUCKI	IIT
Degrees of freedom	20	27	27
Rigid car body	2 masses	1 mass	1 mass
Damping model	Friction	Friction and Viscous	Friction or Equivalent Viscous
Freight Element Model	-	-	Lat, Vert, Long
Track input type	Vert, Lat	Vert	Vert, Lat
Options: Center plate extension pads	-	-	Yes
Longitudinal input	-	-	At car body

For all simulations on the IIT computer model, a rectified sine wave has been used as vertical input displacements on the wheels with maximum rail surface variation equal to 3/4 inch. At this time, the lateral rail input is assumed to exist on one track only with maximum variation equal to half of that of the vertical. Details of track input equations are found in Appendix C.

6.3 Validation of IIT Model

To study the dynamic responses of the box car/freight system for the rocking mode, the conventionally half-staggered rail joints are adopted in the simulation. Different car speeds were simulated to identify the critical speed, which is 17.5 mph for our system, compared with 15 mph for the Stucki model system.

The following output data were plotted and compared with A. Stucki Company data on a 100-ton hopper car for purposes of comparison of the wave shapes of the dynamic responses for the two systems modeled.

(a) Car body roll angle (Fig. 6-1)

(b) Front center plate vertical reaction (Fig. 6-2a)

(c) Wheel loads (Fig. 6-2b)

(d) Front bolster lateral reaction (Fig. 6-3)

The following table lists the comparisons of results of computer simulations for the rocking mode.

Table 3. Comparison of Results with Stucki's - Rocking Mode

	STUCKI (100-ton hopper car)	IIT (70-ton box car)
1. Max. roll angle P-P	11.5° (15 mph)	11.4° (17.5 mph)
2. Time occurred (from beginning of simulation)	9.4 sec	8.6 sec
3. Max. front center plate loading	160 K lb	105 K lb
4. First occupance of center plate separation	5.2 sec	5.0 sec
5. Max. front bolster lateral reaction	40 K	45 K*
6. Max. wheel loads -2 wheels (left front)	145 K lb	138 K lb
7. First occurrence of wheel lift (left front)	6.0 sec	5.4 sec
8. Duration of wheel lift (left front)	0.3-0.5 sec	0.2-0.4 sec

*Lateral bolster loadings for IIT model result from both vertical and lateral rail excitations. This value is slightly higher than that of the Stucki's which is based on vertical input only.

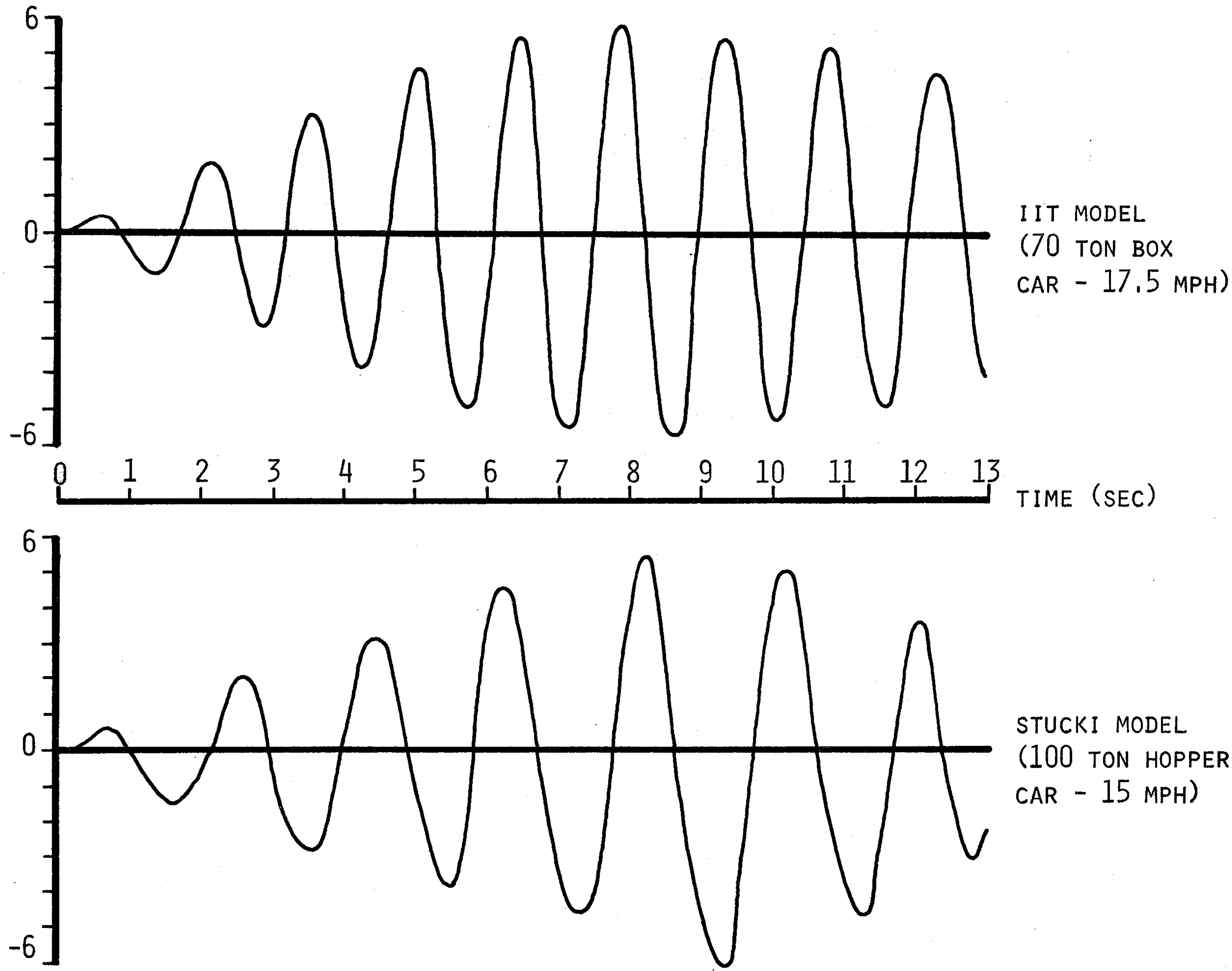


FIG. 6-1. CAR BODY ROLL ANGLE (DEGREES)

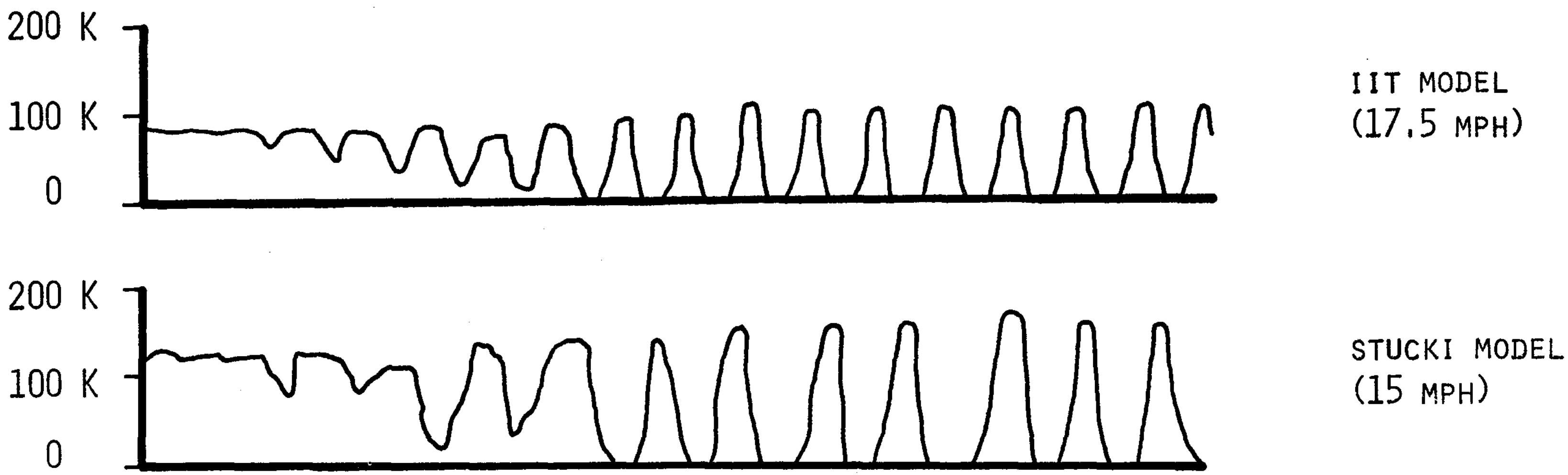


FIG. 6-2A. FRONT CENTER PLATE VERTICAL REACTION (POUNDS)

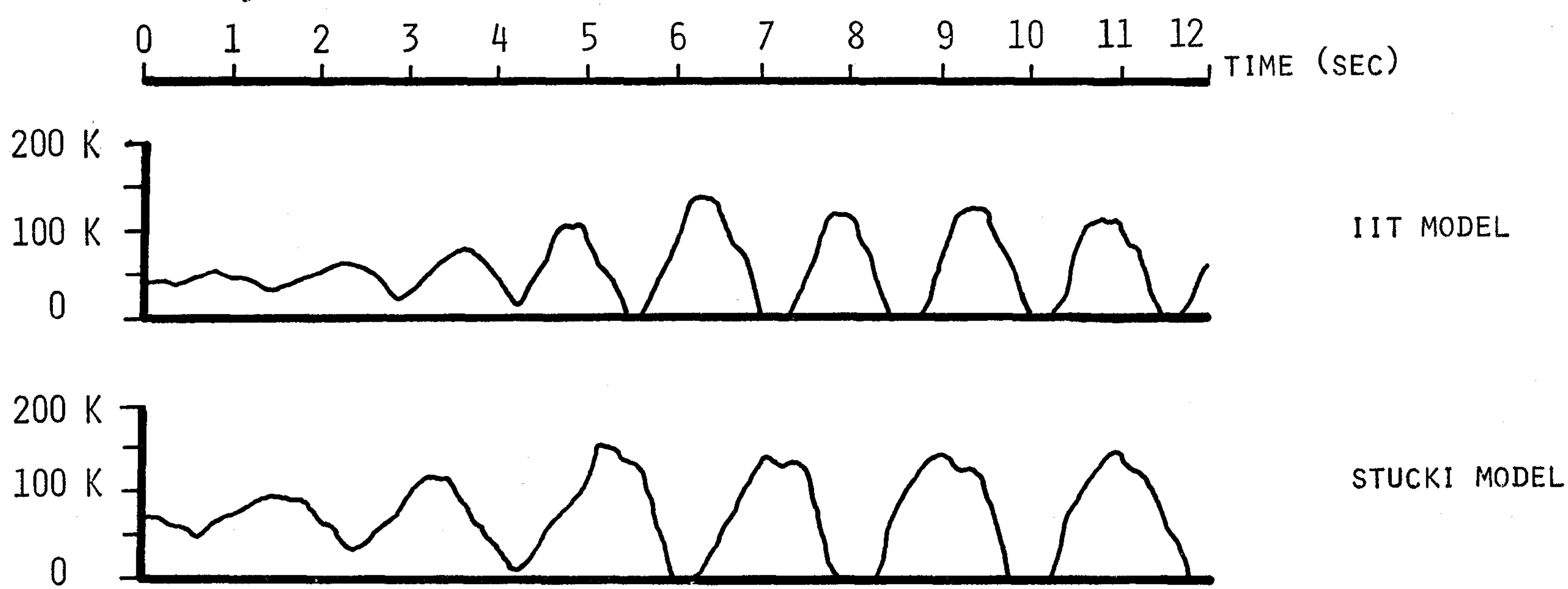


FIG. 6-2B. WHEEL LOADS (POUNDS), LEFT FRONT, TOTAL - 2 WHEELS

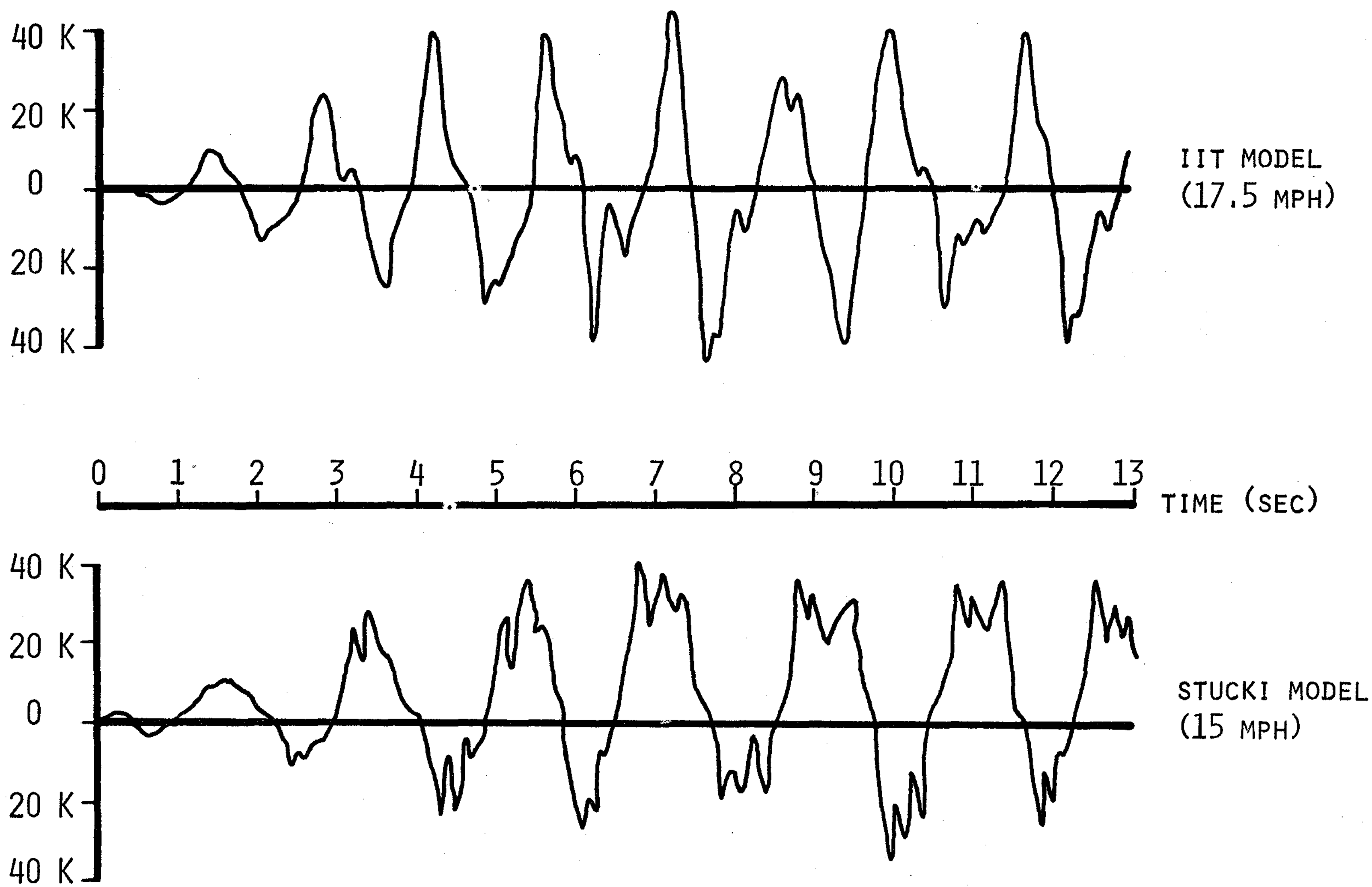


FIG. 6-3. FRONT BOLSTER LATERAL REACTION (POUNDS)

To further correlate with the Stucki data, the IIT model is simulated in a bounce mode in which the car first runs over 2 bumps of amplitude 1 1/2 inch at 60 mph on both tracks and then runs on smooth tracks.

Figures 6-4 and 6-5 show the corresponding loadings on the front and rear center plates respectively as compared to the Stucki data. The general response patterns of the two models are very similar although the maximum center plate loadings on the Stucki model are higher due to a heavier car.

The suspension spring group compressions on such a simulation are represented in Figures 6-6 and 6-7. The corresponding data from the Stucki model was also presented for purpose of comparison. Both models show springs solid, which is a phenomenon actually observed in cases of severe bumps on the rail.

Both center plate loadings and suspension spring compressions on the IIT model are quite similar in amplitudes for the front and rear. This can be explained by the fact that the truck center distance used in the IIT model is 39.5 ft, almost the same as the length of rail joints (39 ft). However, in the case of the Stucki model, a distance of 45 ft was used and this accounts for the general dissimilarities between responses on its front and rear plots.

Table 4 summarizes the comparisons of results between the IIT model and the Stucki model on the bounce mode.

The comparisons show that the two sets of computer output data concerning roll angle, various loadings at the car and truck and spring compressions are extremely close. The Stucki data, however, has been validated against field test data performed on the L & N Railroad Co. rocking test track at Frankfort, Kentucky, on June 24, 1969 (1-10). These comparisons, therefore, show that the current IIT computer model is valid, and based on this, further refinement and development of the mathematical model can continue with confidence. Unfortunately, no test data for freight element response (which may be considered as corresponding to the Stucki data) is currently available to allow total correlation of the motion of a freight element in a box car.

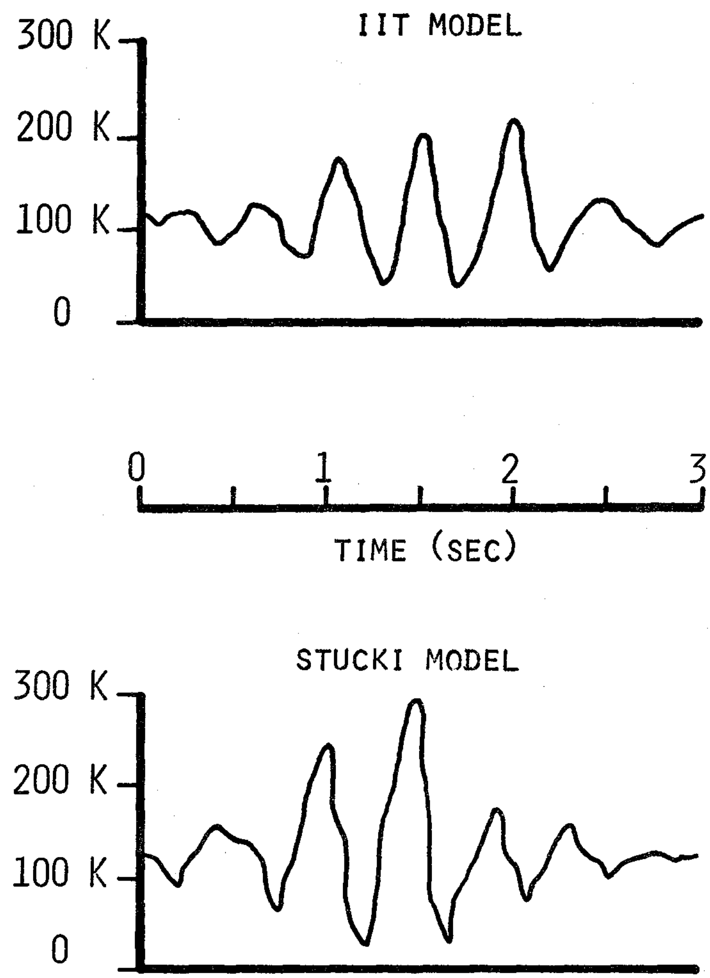


FIG. 6-4. FRONT CENTER PLATE VERTICAL REACTION (LB), BOUNCE MODE - 60 MPH

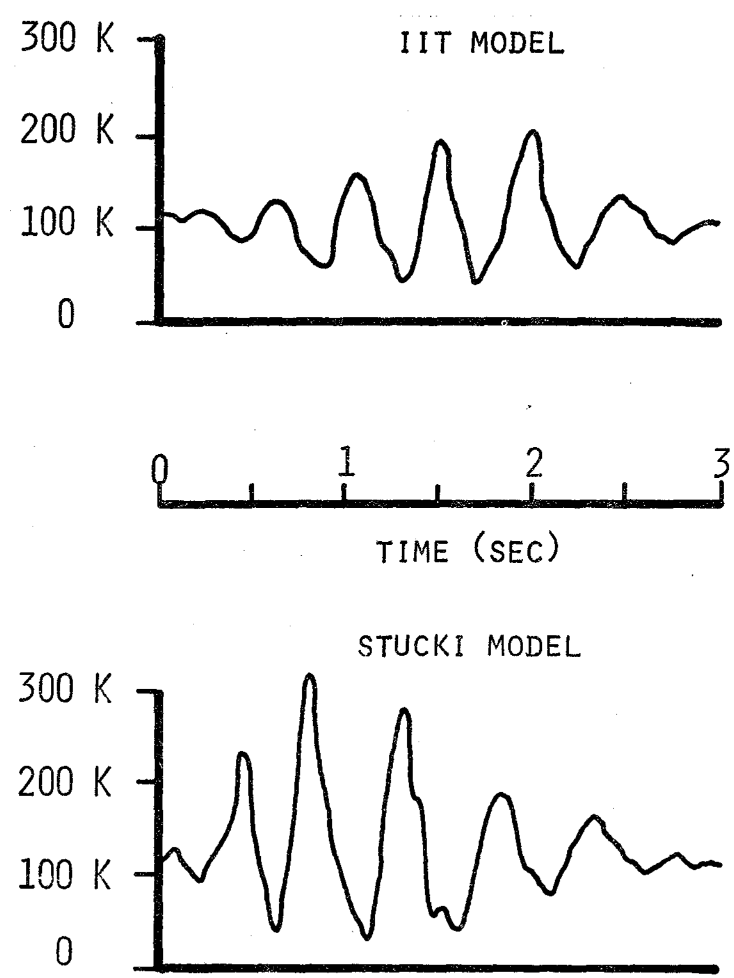


FIG. 6-5. REAR CENTER PLATE VERTICAL REACTION (LB), BOUNCE MODE - 60 MPH

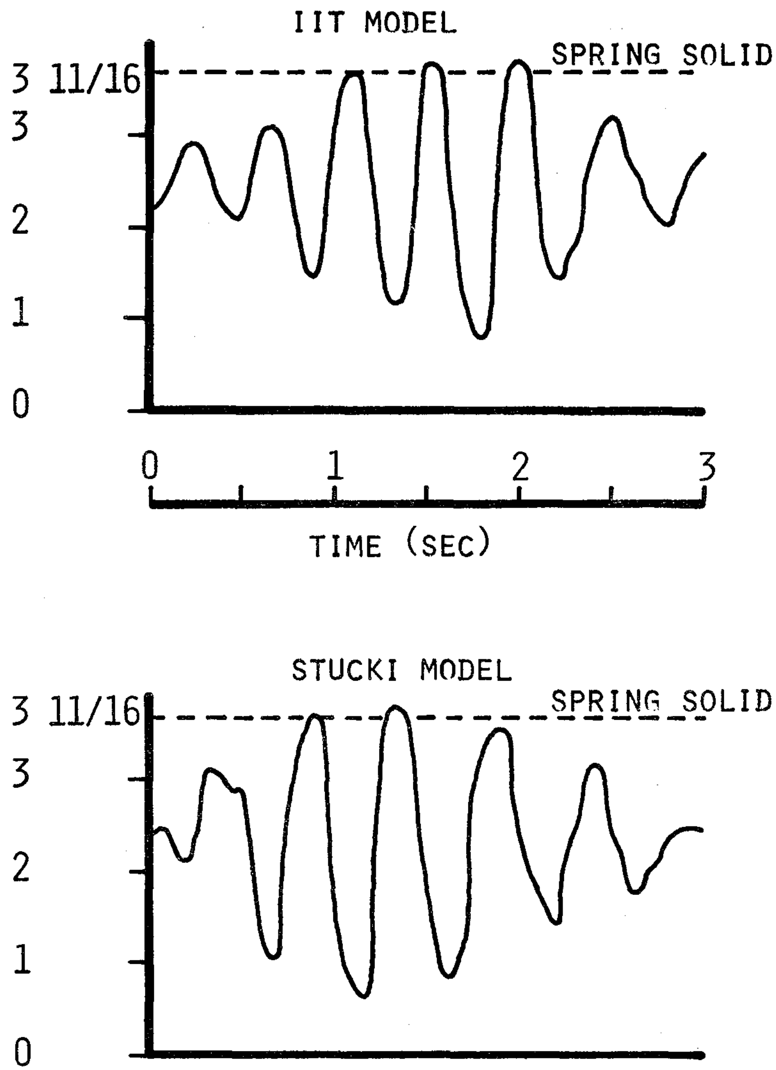


FIG. 6-6. SPRING GROUP COMPRESSION (IN.) - RIGHT FRONT, BOUNCE MODE - 60 MPH

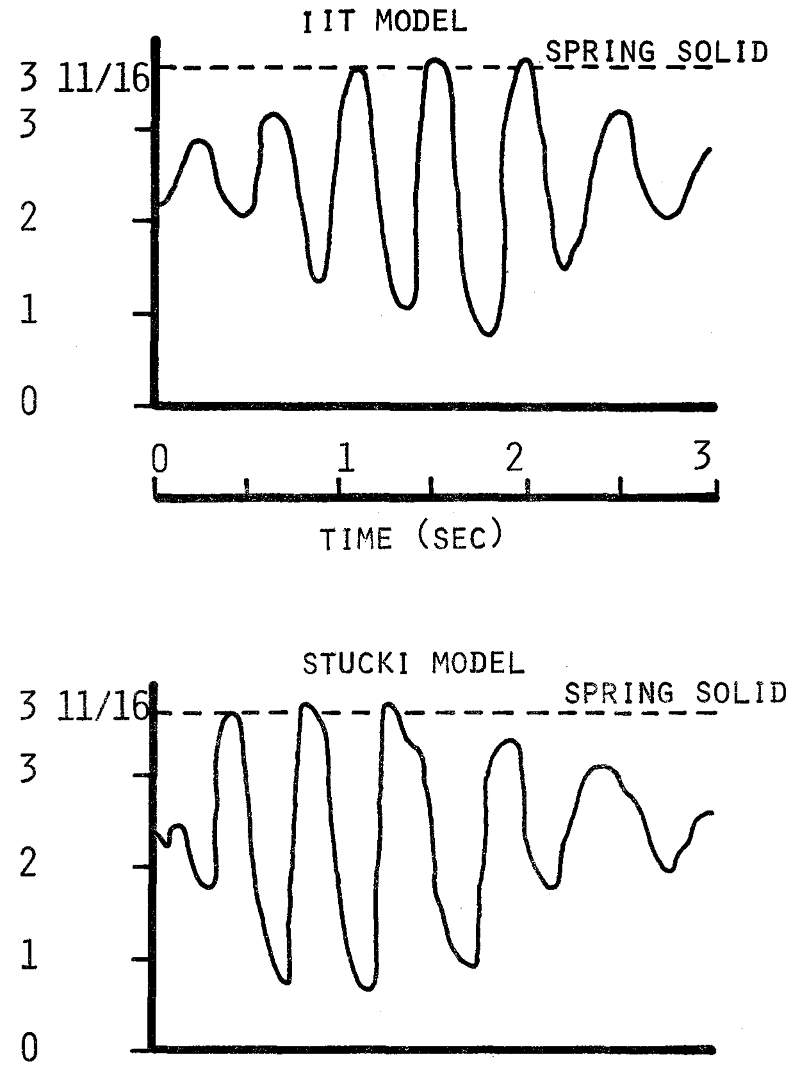


FIG. 6-7. SPRING GROUP COMPRESSION (IN.) - RIGHT REAR, BOUNCE MODE - 60 MPH

This effect has to be deduced from the accurate modeling of the freight dynamic environment in the box car, and then from laboratory simulation of this environment for individual freight elements. The net response of the Freight Element in the box car is the composite motion derived from these two effects.

Table 4. Comparison of Results with Stucki's - Bounce Mode

	STUCKI (100-ton hopper car)	IIT (70-ton box car)
I. Center plate loadings		
a. Maximum - front	290 K lb	215 K lb
b. Maximum - rear	320 K lb	205 K lb
c. No. of response cycles - front	6	6
- rear	6	6
II. Suspension spring compressions		
a. Spring bottoming occurrence - right front	2	2
- right rear	2	2
b. Interval between spring bottomings - right front	0.45 sec	0.5 sec
- right rear	0.45 sec	0.5 sec

7. DYNAMICS OF A SINGLE FREIGHT ELEMENT

7.1 Freight Dynamics Study by Computer Model Simulation

Computer models developed by Track Train Dynamics groups throughout the nation and other railroad researchers are primarily designed for studying the dynamics of the car and the truck. As far as these aspects are concerned, we have already been able to generate information from our present computer model. However, our main attention is given to simulate the type of dynamic environment that revenue trains generate in freight cars. Knowing this freight environment as part of a mathematical model, valuable information on the dynamics of the freight can readily be predicted.

Other researchers have made some attempts to study the responses of a freight element under the impact-shock type of excitation (7-1, 1-3). However, the study of the freight dynamics in a 70-ton box car with a typical truck on the road type of environment by computer simulation is believed to be first of its kind.

A freight element located at different places of the box car is likely to be experiencing different acceleration levels. Hence, attempts were made here to consider a fragile freight element (in this case a 150-lb refrigerator) packaged in some cushion material at different locations of the box car and to study the responses of the freight. Some severe over-the-road conditions like the rocking mode and bounce mode were used to excite the model moving at resonant speeds. Study as such will give guidelines to the more severe environment freight responses that the usual revenue train may experience.

7.2 Case Studies

The following cases were simulated for illustration purposes on the freight dynamics:

Case (1) - Rocking mode (17.5 mph), freight element near roof of the box car. The 150-lb freight element is placed at a point 4 ft above the center of gravity of the car body. The lateral accelerations on the freight were plotted as a function of time (Fig. 7-1a). A maximum peak to peak acceleration level

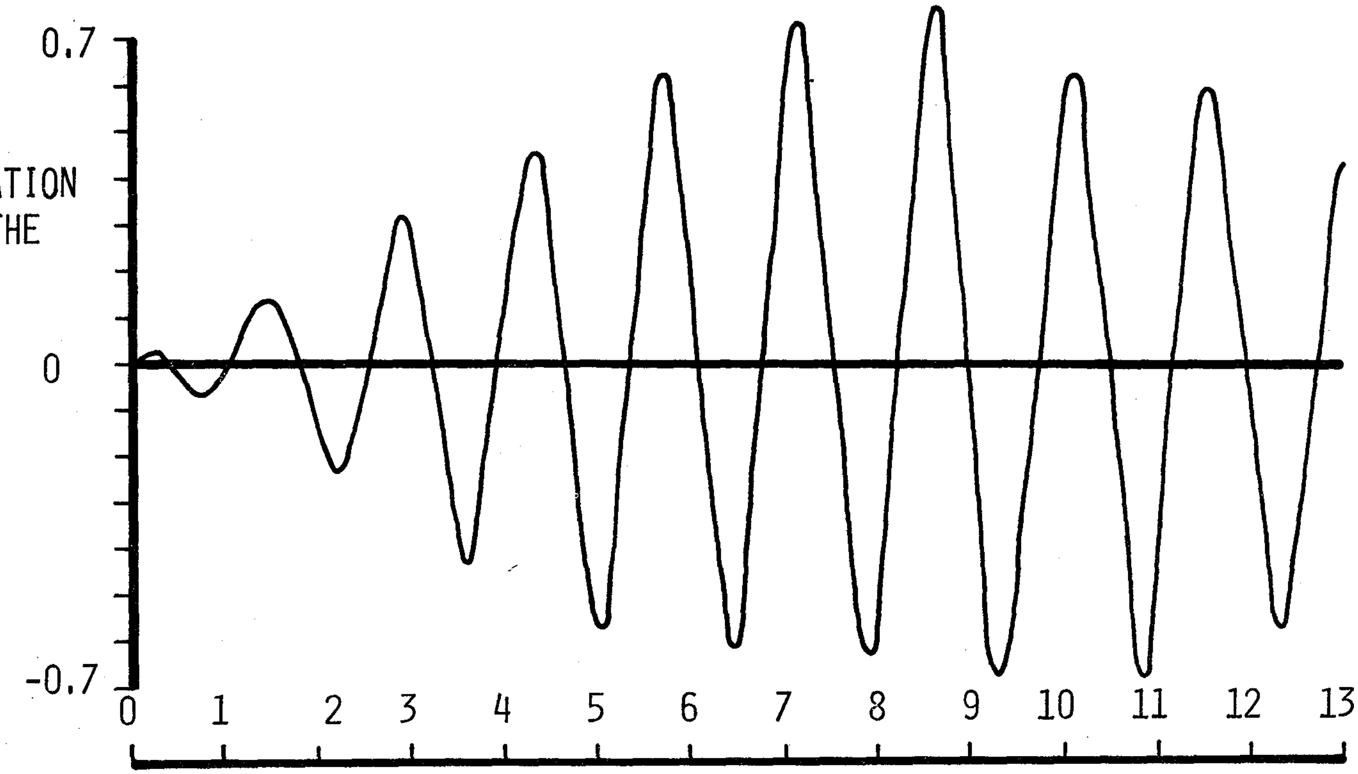


FIG. 7-1A.
FREIGHT ELEMENT
LATERAL ACCELERATION
(g) AT ROOF OF THE
CAR, ROCKING
MODE - 17.5 MPH

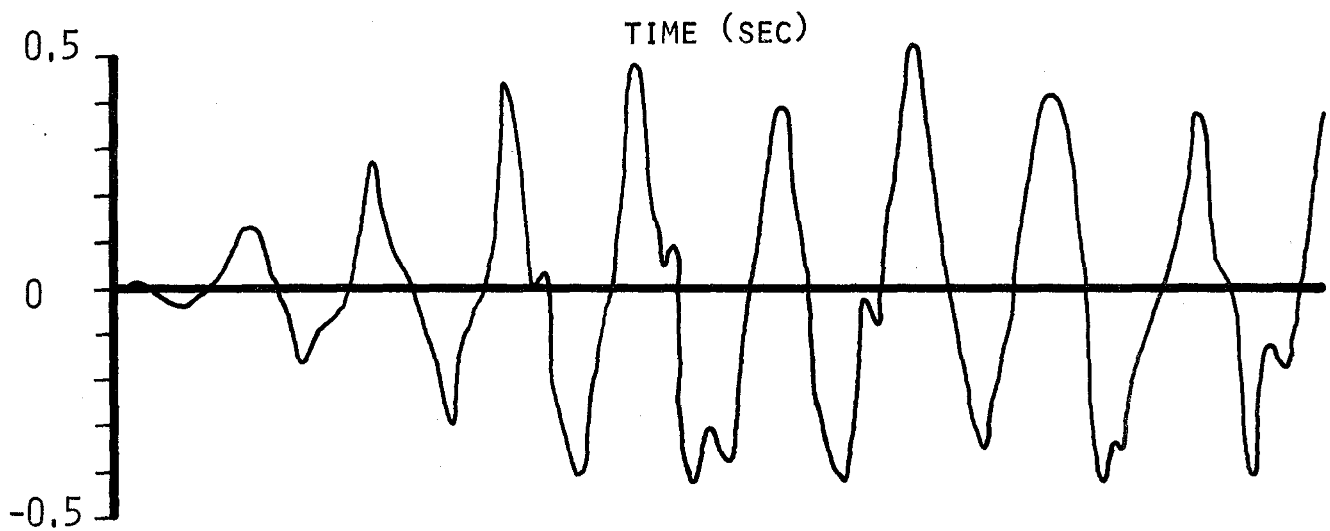


FIG. 7-1B.
CAR BODY LATERAL
ACCELERATION (g)
AT ITS CENTER OF
GRAVITY, ROCKING
MODE - 17.5 MPH

of 1.5 g is encountered after 8.5 sec of simulation. These accelerations of the freight were compared with the lateral acceleration at the car body center of gravity (Fig. 7-1b) which has a maximum peak to peak value of 1 g. This demonstrates that freight element at locations other than the center of gravity of the car body will sometimes experience more severe levels of accelerations during the rocking mode which is inherited with the conventionally half-staggered rail joint system in North America.

Based on these accelerations, a frequency analysis was made using the program FREQ to identify the most significant freight element vibration frequency, the corresponding Power Spectral Density (PSD) was computed and results plotted in Figures 7-2a and 7-2b. The frequency associated with the highest level of accelerations (.314 g) is around 0.64 Hz at which the PSD value is also a maximum.

Case (2) - <u>Rocking mode (50 mph), freight element near roof of the box car</u>. This is defined as rocking mode simply because of the 1/2 stagger used in the rail joints. The freight element lateral and vertical accelerations versus time are plotted in Figures 7-3a and 7-3b. The lateral peak to peak accelerations on the freight is 0.2 g at a frequency of 2 Hz. The vertical peak to peak accelerations is 0.1 g at 1.25 Hz (Fig. 7-3b).

Case (3) - <u>Bounce mode (50 mph), freight element at front end of the box car (near floor)</u>. The freight element is placed at the front end of the car body. Due to the severe pitching motions of the car in the bounce mode, at which the rail joints on opposite tracks are in phase, the accelerations that the freight element is experiencing is higher at this location than that at the center of gravity of the car. Computer simulations were made and the freight vertical accelerations plotted against time in Figure 7-4a. The frequency analysis (Fig. 7-5a) shows that the frequency associated with the highest level of accelerations on the freight (.424 g) is 2 Hz. The PSD plot (Fig. 7-5b) also indicates that most of the vibratory energy is around this particular frequency.

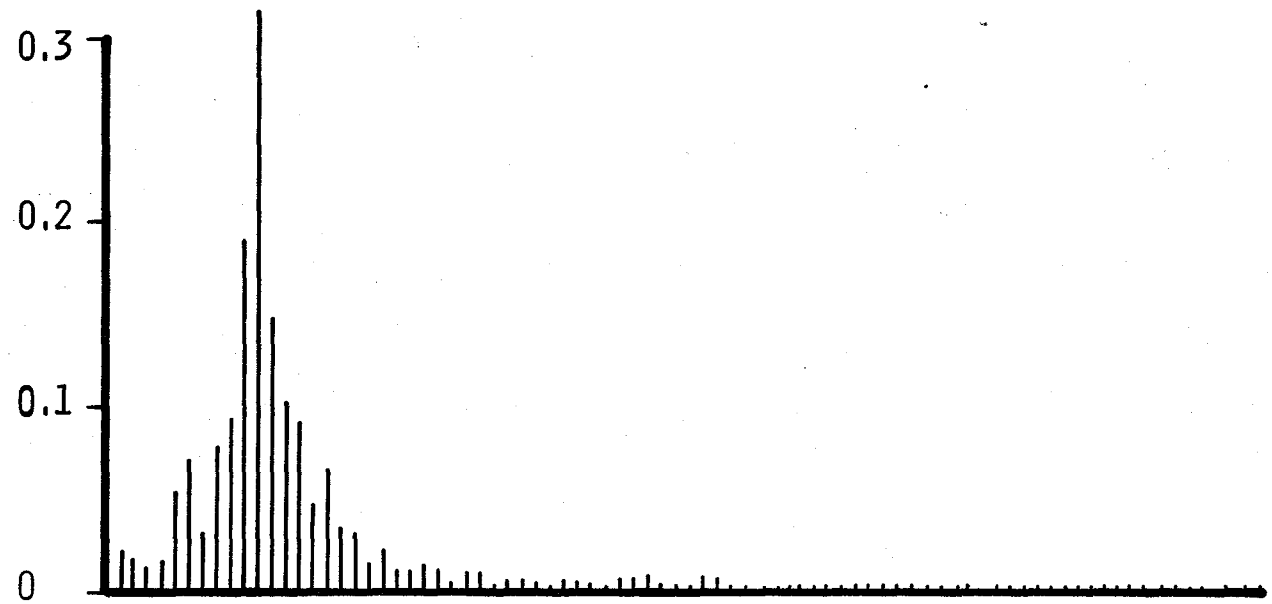


FIG. 7-2A. FREIGHT ELEMENT ACCELERATION (g) SPECTRUM, LATERAL, ROCKING MODE - 17.5 MPH

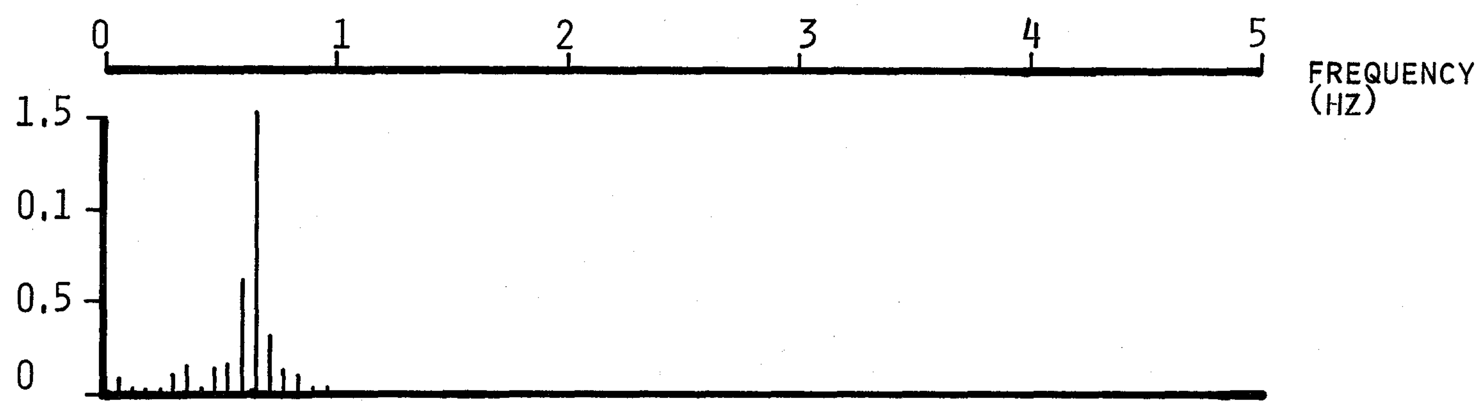


FIG. 7-2B. PSD ANALYSIS (g^2/HZ), FREIGHT ELEMENT LATERAL, ROCKING MODE - 17.5 MPH

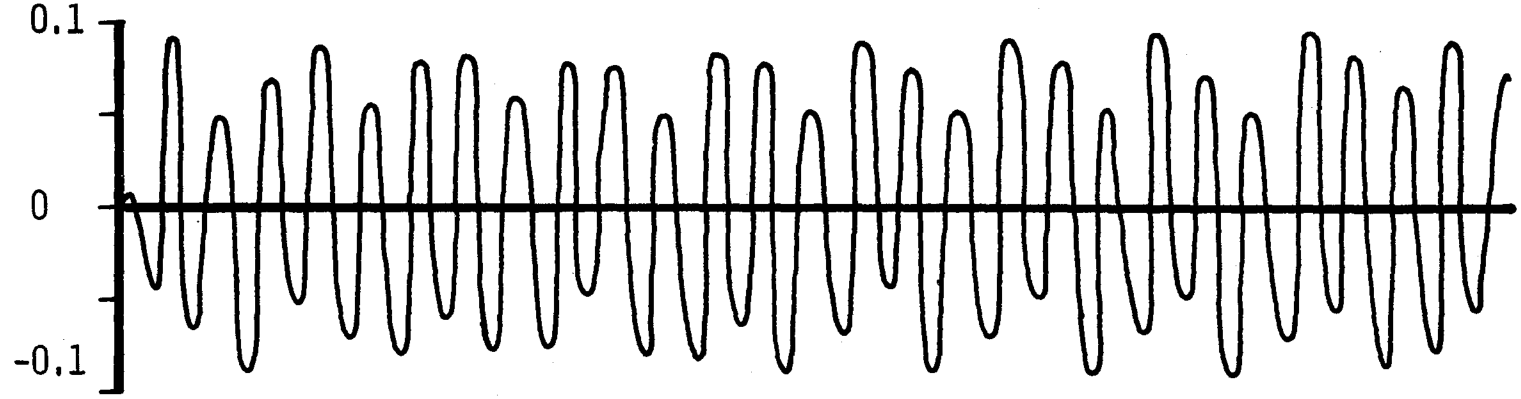


FIG. 7-3A. LATERAL ACCELERATION (g) OF FREIGHT ELEMENT AT ROOF OF THE CAR, ROCKING MODE - 50 MPH

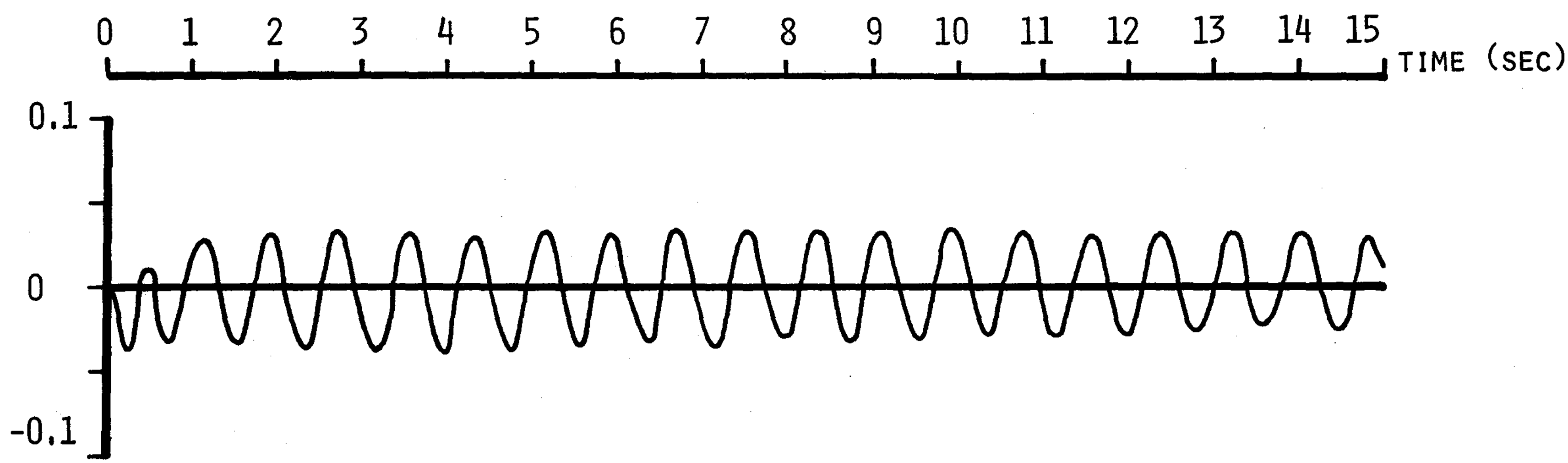


FIG. 7-3B. VERTICAL ACCELERATION (g) OF FREIGHT ELEMENT AT CENTER OF GRAVITY OF CAR BODY, ROCKING MODE - 50 MPH

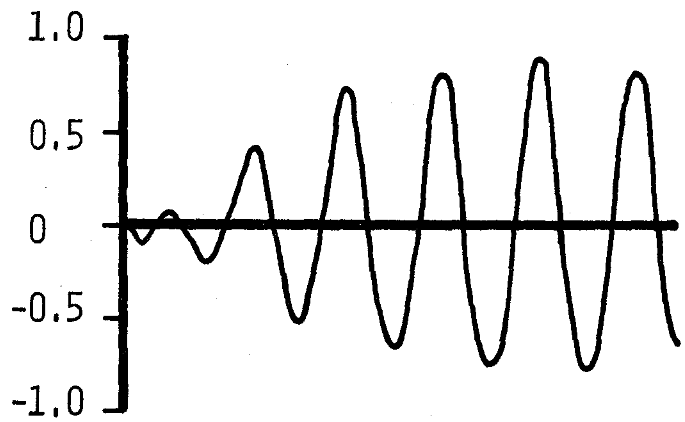


FIG. 7-4A. VERTICAL ACCELERATION (g) OF FREIGHT ELEMENT AT FRONT END OF CAR, BOUNCE MODE - 50 MPH

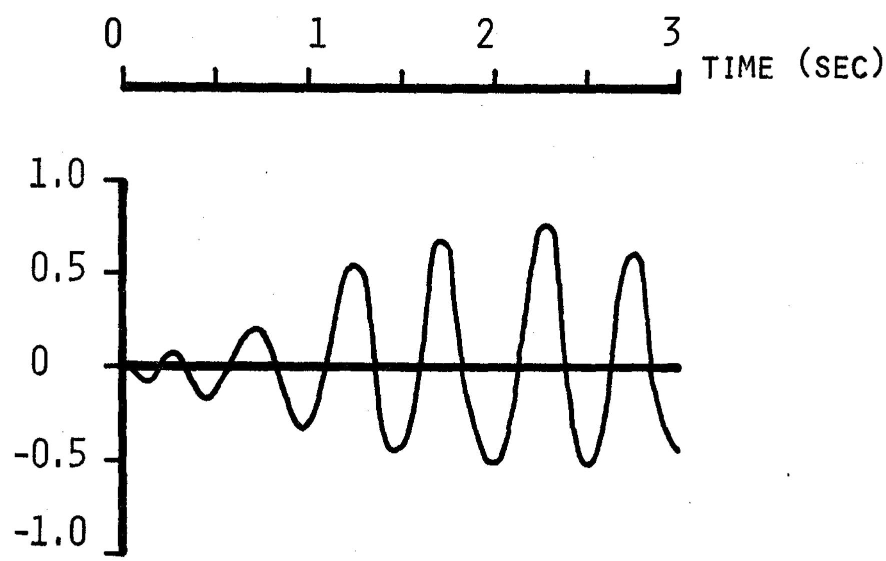


FIG. 7-4B. VERTICAL ACCELERATION (g) OF FREIGHT ELEMENT AT CENTER OF GRAVITY OF CAR BODY, BOUNCE MODE - 50 MPH

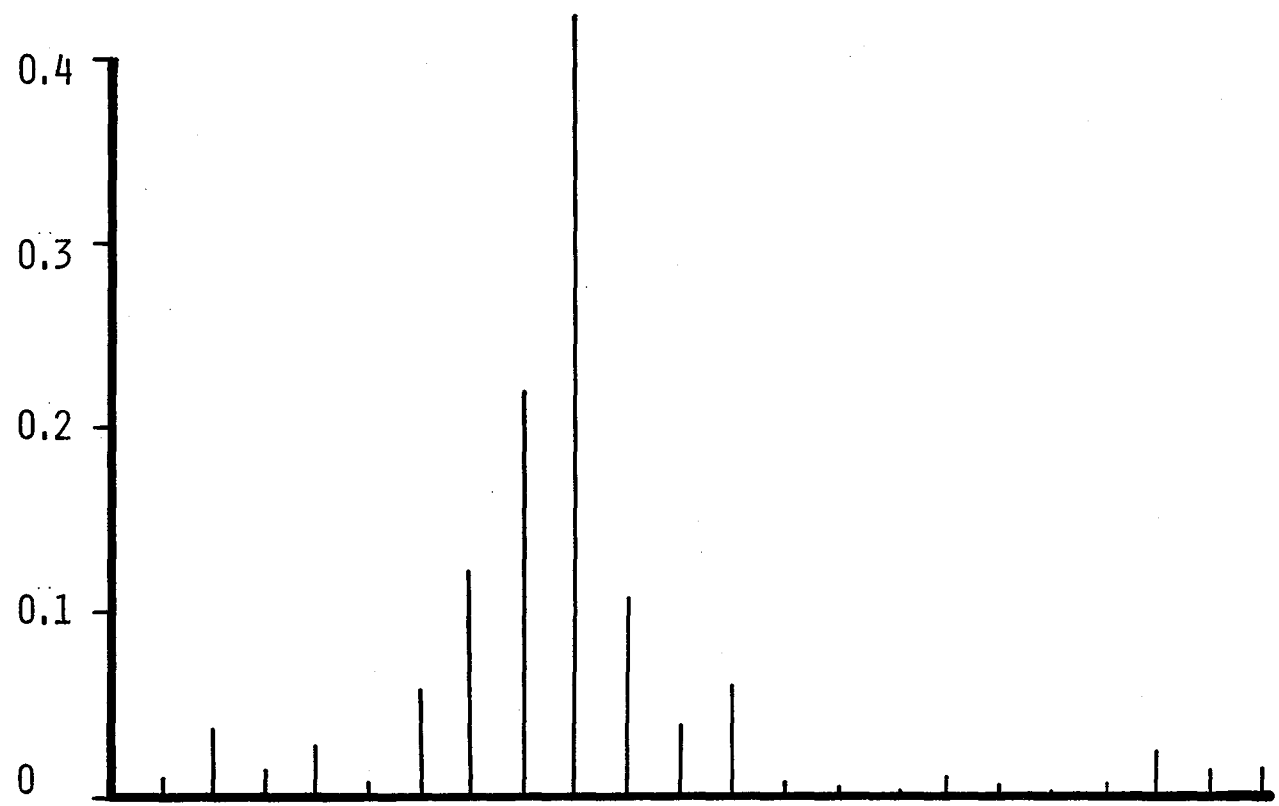


FIG. 7-5A. FREIGHT ELEMENT ACCELERATION (g) SPECTRUM, VERTICAL BOUNCE MODE - 50 MPH

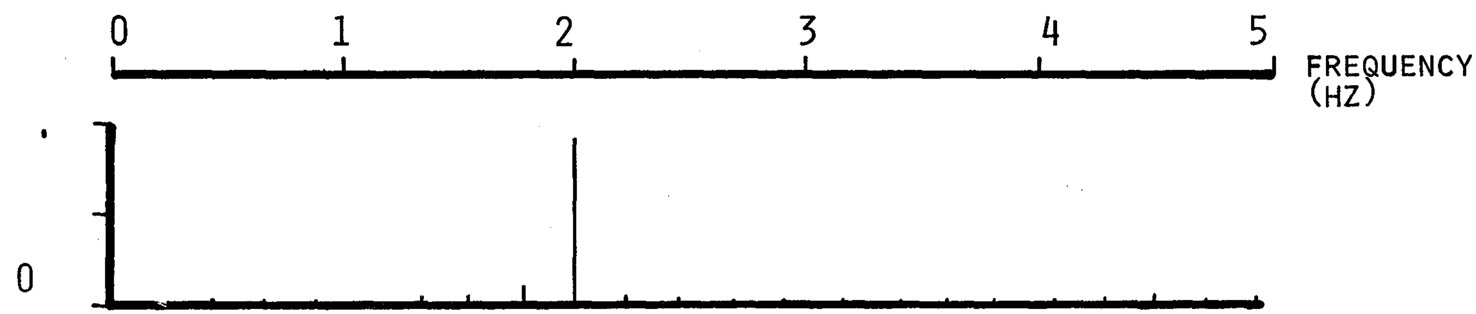


FIG. 7-5B. PSD ANALYSIS (g^2/HZ), FREIGHT ELEMENT VERTICAL, BOUNCE MODE - 50 MPH

Case (4) - Bounce mode (50 mph), freight at center of gravity of the car body. Figure 7-4b shows the freight element vertical accelerations at center of gravity of the car at 50 mph in the bounce mode. A maximum peak to peak accelerations of 1.3 g is predicted as compared to 1.75 g when the freight is at the front end of the car.

The above examples of freight dynamics simulation illustrates some of the bounds on the freight responses when moving over the railroads. Detail study on freight dynamics will be part of the next phase of work.

8. CONCLUSIONS AND RESULTS

1. A mathematical model of the dynamics of a railroad box car carrying a freight element has been developed.

2. It has been shown by comparison with test data, that this model, which represents the car body, freight element, truck bolsters, wheelsets and sideframes, suspension systems and track elasticity and profile as a twenty-seven degree of freedom system, can satisfactorily simulate the dynamics of the typical U.S. box car and freight traveling on tangent track at various operating speeds.

3. Using the computer simulation developed here, it is possible to determine the accelerations, velocities displacements and forces experienced by a freight element, or by the various components of a box car/truck, due to excitation from the rail and/or coupler forces.

4. The output values may be expressed in either the time domain or in the frequency domain, at the option of the user.

5. This study has shown that Coulomb friction damping in a railcar suspension system can be satisfactorily modeled as equivalent viscous damping.

6. This new method facilitates development of new truck designs by making it easier for the designer to investigate the effects of a wide range of configurations and specifications of both friction and hydraulic dampers.

7. When compared to data published by the A. Stucki Company, the simulation developed here shows excellent results for prediction of the box car motions in the Rocking Mode and the Vertical Bounce Mode.

8. The dynamic response of a typical freight element subjected to the vibration environment in a box car under operating conditions can now be predicted by this model.

9. Further pursuit of this freight response study will lead to a thorough understanding of the dynamic response of the freight element. This will indicate possible design modifications in packaging systems, freight car and truck characteristics, etc., which will minimize freight damage due to vibrations.

APPENDIX A

EQUATIONS OF MOTION

The motion of a rigid body in space can be described by rotations and translations. Rotations can be referenced from any inertial coordinates or body axes. However, the angular velocity components about the body axes (which rotates with the body) cannot be integrated to obtain angular displacements about these axes (Chapter 4 Rigid Body Dynamics, Methods of Analytical Dynamics, Leonard Meirovitch). Therefore, it will be unsatisfactory to describe the orientation of a rigid body in space by the body angular velocity components.

One set of independent coordinates which can carry out the transformation from one Cartesian system of axes to another is the Euler's angles. The reason for using Euler's angles is that the three components of the body angular velocity can be expressed in terms of Euler's angles and their time derivatives. Thus, in this analysis, the Euler's angles method is adapted for description of rotations.

The translational coordinates used are inertial coordinates fixed in space, with the coordinate origins at the center of gravity of the various masses when the springs are not extended or compressed, i.e. masses sitting at the free length of the springs. All displacements, velocities and accelerations in translation are referenced from these coordinate origins.

The translation coordinates, together with the Euler's angles constitute the generalized coordinates of the system. In the present model analysis, there are 27 such generalized coordinates.

Once the generalized coordinates are set up, equations of motion can be written for the system by using the method of Lagrange's equation. The Lagrange's equation is an equation of motion in each of the generalized coordinates. It sums up the forces acting on a mass due to the kinetic energy, both rotational and translational, the potential energy associated with spring and gravity, the dissipation energy from damping system and the generalized forces. Details of derivation of each of the energies are discussed later in this section of the appendix.

Figure A-1a shows how Euler's angles provide a description of the body orientation in space. The transformation of body axes x, y, z to one set of inertial Cartesian coordinates X, Y, Z is carried out by three successive rotations:

(1) rotate about x axis through an angle ϕ brings x, y, z into x, y', z'

(2) rotate about y' axis through an angle ψ brings x, y', z' into x', y', Z

(3) rotate about Z axis through an angle α brings x', y', z into X, Y, Z.

Since x, y' and Z are the axes of rotation, the body angular velocity components $\dot{\phi}$, $\dot{\psi}$ and $\dot{\alpha}$ are directed along these axes respectively.

Denoting the inertial angular components about X, Y, Z axes as ω_1, ω_2 and ω_3 respectively, it can readily be seen, from Figure A-1b, that the following relationships are true by resolving the body angular velocities $\dot{\phi}$, $\dot{\psi}$ and $\dot{\alpha}$ along:

(1) X axis, $\omega_1 = \dot{\phi} \cos \psi \cos \alpha + \dot{\psi} \sin \alpha$,

(2) Y axis, $\omega_2 = \dot{\psi} \cos \alpha - \dot{\phi} \cos \psi \sin \alpha$, and

(3) Z axis, $\omega_3 = \dot{\phi} \sin \psi + \dot{\alpha}$

Similarly, writing the above relationships ω_{ij} for the five masses, where $i = 1, \ldots, 5$ and $j = 1, 2, 3$:

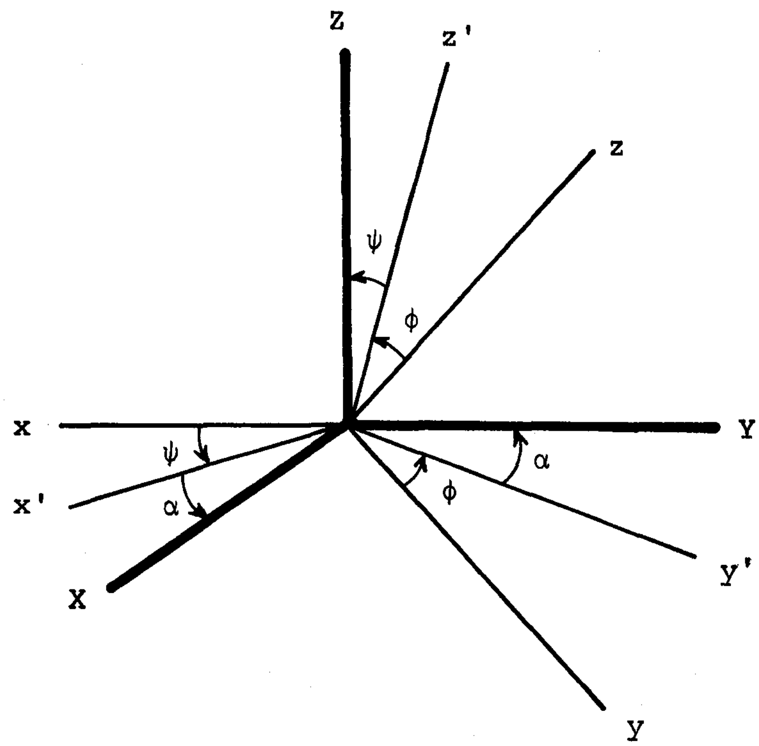


FIG. A-1A. EULER'S ANGLES

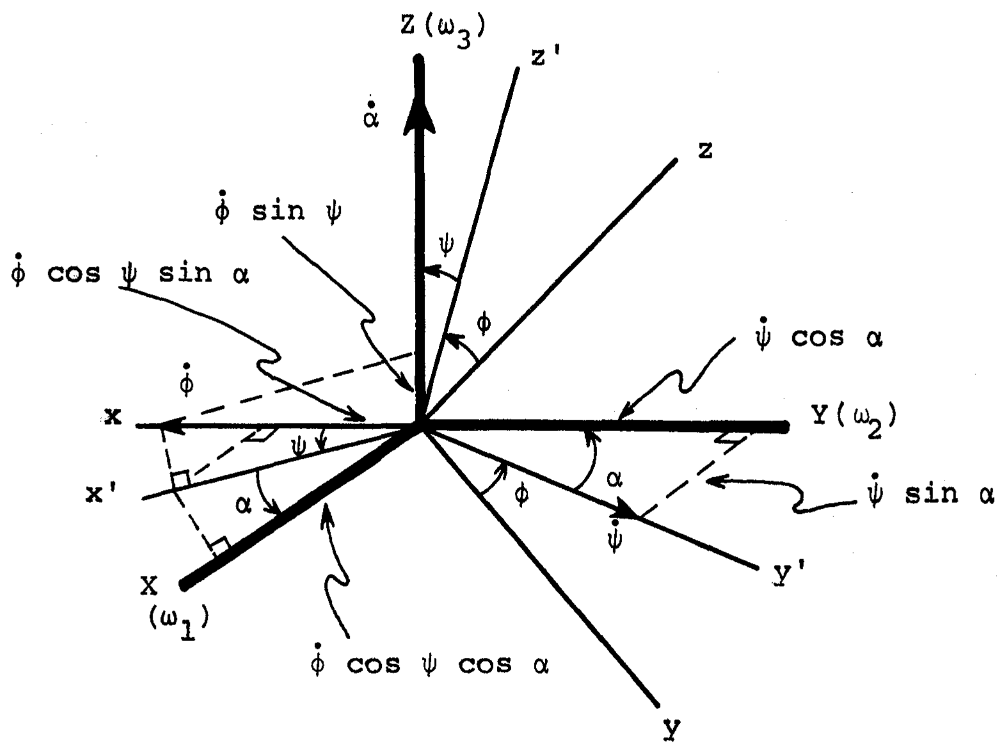


FIG. A-1B. TRANSFORMATION OF BODY ANGULAR VELOCITIES TO INERTIAL ANGULAR COMPONENTS

a) For the car body:

$$\omega_{11} = \dot{\phi}_1 \cos\psi_1 \cos\alpha_1 + \dot{\psi}_1 \sin\alpha_1$$

$$\omega_{12} = \dot{\psi}_1 \cos\alpha_1 - \dot{\phi}_1 \cos\psi_1 \sin\alpha_1$$

$$\omega_{13} = \dot{\phi}_1 \sin\psi_1 + \dot{\alpha}_1$$

b) For the front bolster:

$$\omega_{21} = \dot{\phi}_2 \cos\psi_2 \cos\alpha_2 + \dot{\psi}_2 \sin\alpha_2$$

$$\omega_{22} = \dot{\psi}_2 \cos\alpha_2 - \dot{\phi}_2 \cos\psi_2 \sin\alpha_2$$

$$\omega_{23} = \dot{\phi}_2 \sin\psi_2 + \dot{\alpha}_2$$

c) For the rear bolster:

$$\omega_{31} = \dot{\phi}_3 \cos\psi_3 \cos\alpha_3 + \dot{\psi}_3 \sin\alpha_3$$

$$\omega_{32} = \dot{\psi}_3 \cos\alpha_3 - \dot{\phi}_3 \cos\psi_3 \sin\alpha_3$$

$$\omega_{33} = \dot{\phi}_3 \sin\psi_3 + \dot{\alpha}_3$$

d) For the front axle-wheel set-sideframe assembly:

$$\omega_{41} = \dot{\phi}_4 \cos\psi_4 \cos\alpha_4 + \dot{\psi}_4 \sin\alpha_4$$

$$\omega_{42} = \dot{\psi}_4 \cos\alpha_4 - \dot{\phi}_4 \cos\psi_4 \sin\alpha_4$$

$$\omega_{43} = \dot{\phi}_4 \sin\psi_4 + \dot{\alpha}_4$$

e) For the rear axle-wheel set-sideframe assembly:

$$\omega_{51} = \dot{\phi}_5 \cos\psi_5 \cos\alpha_5 + \dot{\psi}_5 \sin\alpha_5$$

$$\omega_{52} = \dot{\psi}_5 \cos\alpha_5 - \dot{\phi}_5 \cos\psi_5 \sin\alpha_5$$

$$\omega_{53} = \dot{\phi}_5 \sin\psi_5 + \dot{\alpha}_5$$

where ϕ--angle of pitch, ψ--angle of roll and α--angle of yaw. Note: The freight element at this stage is assumed to have only translational degrees of freedom and hence no rotational angles associated with it.

Kinetic Energy of the System

The total kinetic energy is the sum of the rotational kinetic energy and translational kinetic energy.

<u>Kinetic Energy of the Car Body</u>. Assuming all rotations about the principal axes allowed for the car body

$$K.E._{rotational} = \frac{1}{2}(I_{x1}\omega_{11}^2 + I_{y1}\omega_{12}^2 + I_{z1}\omega_{13}^2)$$

where I_{x1}, I_{y1} and I_{z1} are the principal moments of inertia about x_1, y_1 and z_1 axes of the car body.

Assuming the car body can translate in the three principal directions

$$K.E._{translational} = \frac{1}{2} m_1 (\dot{x}_1^2 + \dot{y}_1^2 + \dot{z}_1^2)$$

where m_1 = mass of the car body.

Substituting the Euler's angles for ω_{11}, ω_{12} and ω_{13}, we have, for kinetic energy of the car body

$$\begin{aligned} E_1 = {} & \frac{1}{2} I_{x1}(\dot{\phi}_1 \cos \psi_1 \cos \alpha_1 + \dot{\psi}_1 \sin \alpha_1)^2 \\ & + \frac{1}{2} I_{y1}(\dot{\psi}_1 \cos \alpha_1 - \dot{\phi}_1 \cos \psi_1 \sin \alpha_1)^2 \\ & + \frac{1}{2} I_{z1}(\dot{\phi}_1 \sin \psi_1 + \dot{\alpha}_1)^2 + \frac{1}{2} m_1 (\dot{x}_1^2 + \dot{y}_1^2 + \dot{z}_1^2) \end{aligned}$$

<u>Kinetic Energy for the Front Bolster</u>. Similarly,

$$E_2 = \frac{1}{2}(I_{x2}\omega_{21}^2 + I_{y2}\omega_{22}^2 + I_{z2}\omega_{23}^2) + \frac{1}{2} m_2 (\dot{x}_2^2 + \dot{y}_2^2 + \dot{z}_2^2)$$

Since we assume the lateral displacements of the bolsters constrained to that of the car body, we have, see Figure A-2,

$$x_2 = x_1 - r_1 \sin \psi_1 - d_1 \sin \alpha_1$$

$$\dot{x}_2 = \dot{x}_1 - r_1 \cos \psi_1 \dot{\psi}_1 - d_1 \cos \alpha_1 \dot{\alpha}_1$$

Substituting ω_{21}, ω_{22}, ω_{23} and $\dot{x}_2$, we have

$$\begin{aligned} E_2 = {} & \frac{1}{2} I_{x2}(\dot{\phi}_2 \cos \psi_2 \cos \alpha_2 + \dot{\psi}_2 \sin \alpha_2)^2 \\ & + \frac{1}{2} I_{y2}(\dot{\psi}_2 \cos \alpha_2 - \dot{\phi}_2 \cos \psi_2 \sin \alpha_2)^2 \\ & + \frac{1}{2} I_{z2}(\dot{\phi}_2 \sin \psi_2 + \dot{\alpha}_2)^2 + \frac{1}{2} m_2 (\dot{x}_1^2 - 2\dot{x}_1 r_1 \cos \psi_1 \dot{\psi}_1 \\ & + r_1^2 \cos^2 \psi_1 \dot{\psi}_1^2 - 2d_1 \dot{x}_1 \cos \alpha_1 \dot{\alpha}_1 + 2r_1 d_1 \cos \psi_1 \cos \alpha_1 \dot{\psi}_1 \dot{\alpha}_1 \\ & + d_1^2 \cos \alpha_1^2 \dot{\alpha}_1^2) + \frac{1}{2} m_2 \dot{y}_2^2 + \frac{1}{2} m_2 \dot{z}_2^2 \end{aligned}$$

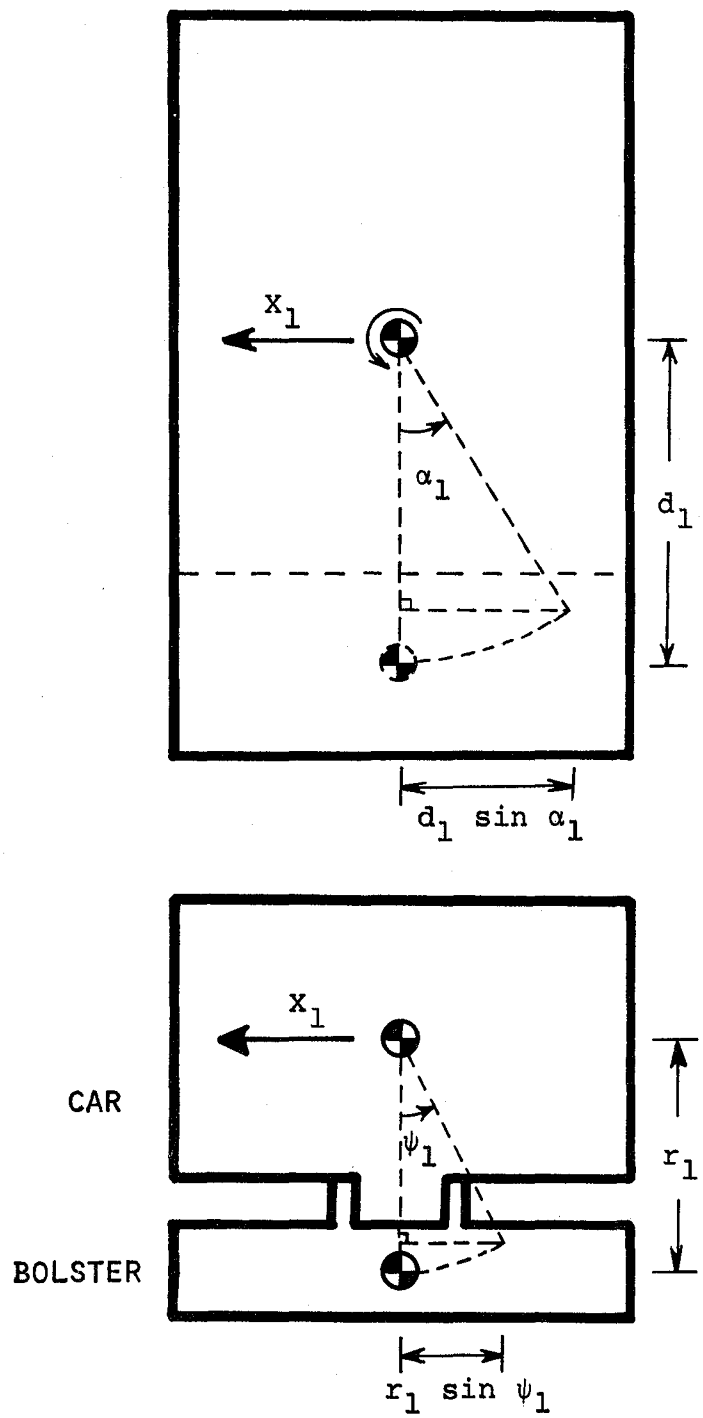


FIG. A-2. BOLSTER LATERAL CONSTRAINT

Kinetic Energy of the Rear Bolster.

$$E_3 = \frac{1}{2} I_{x3}(\dot{\phi}_3 \cos \psi_3 \cos \alpha_3 + \dot{\psi}_3 \sin \alpha_3)^2$$
$$+ \frac{1}{2} I_{y3}(\dot{\psi}_3 \cos \alpha_3 - \dot{\phi}_3 \cos \psi_3 \sin \alpha_3)^2$$
$$+ \frac{1}{2} I_{z3}(\dot{\phi}_3 \sin \psi_3 + \dot{\alpha}_3)^2 + \frac{1}{2} m_3(\dot{x}_1^2 - 2r_1\dot{x}_1 \cos \psi_1\dot{\psi}_1$$
$$+ r_1^2 \cos^2 \psi_1\dot{\psi}_1^2 + 2d_1\dot{x}_1 \cos \alpha_1\dot{\alpha}_1 - 2r_1d_1 \cos \psi_1\dot{\psi}_1 \cos \alpha_1\dot{\alpha}_1$$
$$+ d_1^2 \cos^2 \alpha_1\dot{\alpha}_1^2) + \frac{1}{2} m_3\dot{y}_3^2 + \frac{1}{2} m_3\dot{z}_3^2$$

Kinetic Energy of the Front Axle-Wheel Set-Sideframe Assembly.

$$E_4 = \frac{1}{2} I_{x4}(\dot{\phi}_4 \cos \psi_4 \cos \alpha_4 + \dot{\psi}_4 \sin \alpha_4)^2$$
$$+ \frac{1}{2} I_{y4}(\dot{\psi}_4 \cos \alpha_4 - \dot{\phi}_4 \cos \psi_4 \sin \alpha_4)^2$$
$$+ \frac{1}{2} I_{z4}(\dot{\phi}_4 \sin \psi_4 + \dot{\alpha}_4)^2 + \frac{1}{2} m_4(\dot{x}_4^2 + \dot{y}_4^2 + \dot{z}_4^2)$$

Kinetic Energy of the Rear Axle-Wheel Set-Sideframe Assembly.

$$E_5 = \frac{1}{2} I_{x5}(\dot{\phi}_5 \cos \psi_5 \cos \alpha_5 + \dot{\psi}_5 \sin \alpha_5)^2$$
$$+ \frac{1}{2} I_{y5}(\dot{\psi}_5 \cos \alpha_5 - \dot{\phi}_5 \cos \psi_5 \sin \alpha_5)^2$$
$$+ \frac{1}{2} I_{z5}(\dot{\phi}_5 \sin \psi_5 + \dot{\alpha}_5)^2 + \frac{1}{2} m_5(\dot{x}_5^2 + \dot{y}_5^2 + \dot{z}_5^2)$$

Kinetic Energy of the Freight Element.

$$E_6 = \frac{1}{2} m_6(\dot{x}_6^2 + \dot{y}_6^2 + \dot{z}_6^2)$$

Therefore, the total kinetic energy of the system is

$$E = E_1 + E_2 + E_3 + E_4 + E_5 + E_6$$

Potential Energy of the System

The total potential energy is the sum of the spring potential energy and the gravitational potential energy of the entire system.

In our present mathematical model there are six masses connected by springs and dampers. The corresponding potential energy associated with each group of springs is derived as follows.

Spring Potential Energy Between the Car Body and the Front Truck Bolster. When a spring is displaced distance x units, the spring potential energy is simply $1/2\ Kx^2$, where K is the stiffness of the spring.

Referring to Figure A-3, the spring potential energy associated with the vertical springs with stiffness K_1 and K_2, respectively are

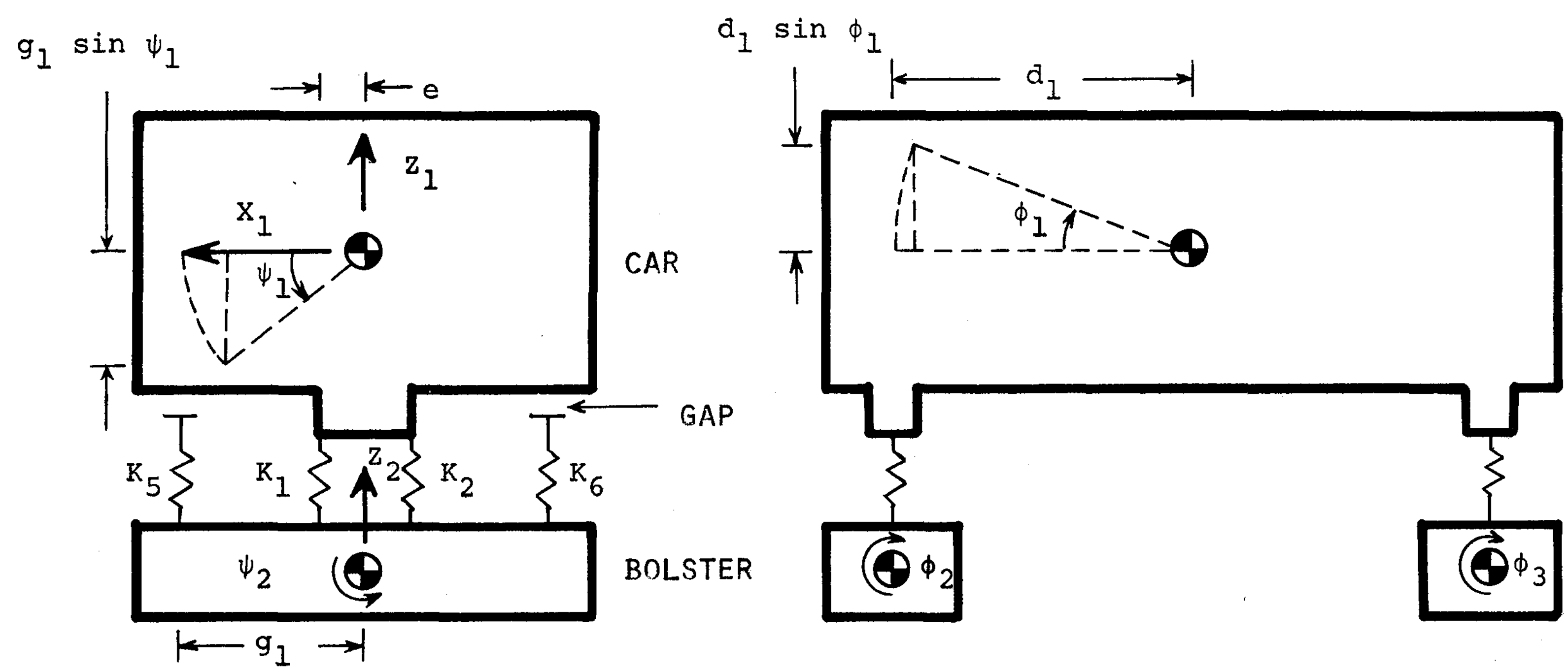


FIG. A-3. VERTICAL SPRINGS AT CAR BODY AND TRUCK BOLSTER INTERFACE

$$V_1 = \frac{1}{2} K_1 (z_1 - z_2 - e(\sin\psi_1 - \sin\psi_2) + d_1 \sin\phi_1)^2$$

$$V_2 = \frac{1}{2} K_2 (z_1 - z_2 + e(\sin\psi_1 - \sin\psi_2) + d_1 \sin\phi_1)^2$$

For the two springs modelling the side bearings, i.e. K_5 and K_6, we have

$$V_5 = \frac{1}{2} K_5 \delta_1 \ (z_1 - z_2 - g_1(\sin\psi_1 - \sin\psi_2) + d_1 \sin\phi_1 + GAP)^2$$

where $\delta_1 = 1$ when

$$(z_1 - z_2 - g_1(\sin\psi_1 - \sin\psi_2) + d_1 \sin\phi_1 + GAP) < 0$$

and $\delta_1 = 0$ when

$$(z_1 - z_2 - g_1(\sin\psi_1 - \sin\psi_2) + d_1 \sin\phi_1 + GAP) \geq 0$$

$$V_6 = \frac{1}{2} K_6 \delta_2 \ (z_1 - z_2 + g_1(\sin\psi_1 - \sin\psi_1) + d_1 \sin\phi_1 + GAP)^2$$

Similarly, $\delta_2 = 1$ when

$$(z_1 - z_2 + g_1(\sin\psi_1 - \sin\psi_2) + d_1 \sin\phi_1 + GAP) < 0$$

and $\delta_2 = 0$ when

$$(z_1 - z_2 + g_1(\sin\psi_1 - \sin\psi_2) + d_1 \sin\phi_1 + GAP) \geq 0$$

The function of δ_1 and δ_2 is to check the side bearing clearance between the car body and truck bolster and if no clearance exists, the corresponding spring is added in parallel to K_1 and K_2.

Spring Potential Energy Between the Car Body and the Rear Truck Bolster. Similarly we can write

$$V_7 = \frac{1}{2} K_7 (z_1 - z_3 - e(\sin\psi_1 - \sin\psi_3) - d_1 \sin\phi_1)^2$$

$$V_8 = \frac{1}{2} K_8 (z_1 - z_3 + e(\sin\psi_1 - \sin\psi_3) - d_1 \sin\phi_1)^2$$

$$V_{11} = \frac{1}{2} K_{11} \delta_3 (z_1 - z_3 - g_1(\sin\psi_1 - \sin\psi_3) - d_1 \sin\phi_1 + GAP)^2$$

$$V_{12} = \frac{1}{2} K_{12} \delta_4 (z_1 - z_3 + g_1(\sin\psi_1 - \sin\psi_3) - d_1 \sin\phi_1 + GAP)^2$$

Spring Potential Energy for the Front Truck Suspension Springs. Refer to Figure A-4, we have

$$V_{13} = \frac{1}{2} K_{13} (z_2 - z_4 - h_2(\sin\psi_2 - \sin\psi_4) + d_2 \sin\phi_2 - d_4 \sin\phi_4)^2$$

$$V_{14} = \frac{1}{2} K_{14} (z_2 - z_4 + h_2(\sin\psi_2 - \sin\psi_4) + d_2 \sin\phi_2 - d_4 \sin\phi_4)^2$$

$$V_{15} = \frac{1}{2} K_{15}(z_2 - z_4 - h_2(\sin \psi_2 - \sin \psi_4)$$
$$- (d_2 \sin \phi_2 - d_4 \sin \phi_4))^2$$

$$V_{16} = \frac{1}{2} K_{16}(z_2 - z_4 + h_2(\sin \psi_2 - \sin \psi_4)$$
$$- (d_2 \sin \phi_2 - d_4 \sin \phi_4))^2$$

$$VBOM_9 = \frac{1}{2} KBOM\delta_9(z_2 - z_4 - h_2(\sin \psi_2 - \sin \psi_4) + TL)^2$$

$$VBOM_{10} = \frac{1}{2} KBOM\delta_{10}(z_2 - z_4 + h_2(\sin \psi_2 - \sin \psi_4) + TL)^2$$

where δ_9, δ_{10}= 1 or 0 depending on whether suspension springs bottom out or not.

<u>Spring Potential Energy for the Rear Truck Suspension Springs</u>.

$$V_{17} = \frac{1}{2} K_{17}(z_3 - z_5 - h_3(\sin \psi_3 - \sin \psi_5)$$
$$+ d_3 \sin \phi_3 - d_5 \sin \phi_5)^2$$

$$V_{18} = \frac{1}{2} K_{18}(z_3 - z_5 + h_3(\sin \psi_3 - \sin \psi_5)$$
$$+ d_3 \sin \phi_3 - d_5 \sin \phi_5)^2$$

$$V_{19} = \frac{1}{2} K_{19}(z_3 - z_5 - h_3(\sin \psi_3 - \sin \psi_5)$$
$$- (d_3 \sin \phi_3 - d_5 \sin \phi_5))^2$$

$$V_{20} = \frac{1}{2} K_{20}(z_3 - z_5 + h_3(\sin \psi_3 - \sin \psi_5)$$
$$- (d_3 \sin \phi_3 - d_5 \sin \phi_5))^2$$

$$VBOM_{11} = \frac{1}{2} KBOM\delta_{11}(z_3 - z_5 - h_3(\sin \psi_3 - \sin \psi_5) + TL)^2$$

$$VBOM_{12} = \frac{1}{2} KBOM\delta_{12}(z_3 - z_5 + h_3(\sin \psi_3 - \sin \psi_5) + TL)^2$$

where δ_{11}, δ_{12} = 1 or 0 depending on whether rear suspension groups bottom out or not.

<u>Spring Potential Energy for Lateral Springs Between Front Bolster and Sideframe</u>. Refer to Figure A-5,

$$V_{13}L = \frac{1}{2} K_{13}L(x_1 - r_1 \sin \psi_1 - d_1 \sin \alpha_1 - x_4 - (d_2 \sin \alpha_2$$
$$- d_4 \sin \alpha_4) - (r_2 \sin \psi_2 + r_4 \sin \psi_4))^2$$

$$V_{14}L = \frac{1}{2} K_{14}L(x_1 - r_1 \sin \psi_1 - d_1 \sin \alpha_1 - x_4 - (d_2 \sin \alpha_2$$
$$- d_4 \sin \alpha_4) - (r_2 \sin \psi_2 + r_4 \sin \psi_4))^2$$

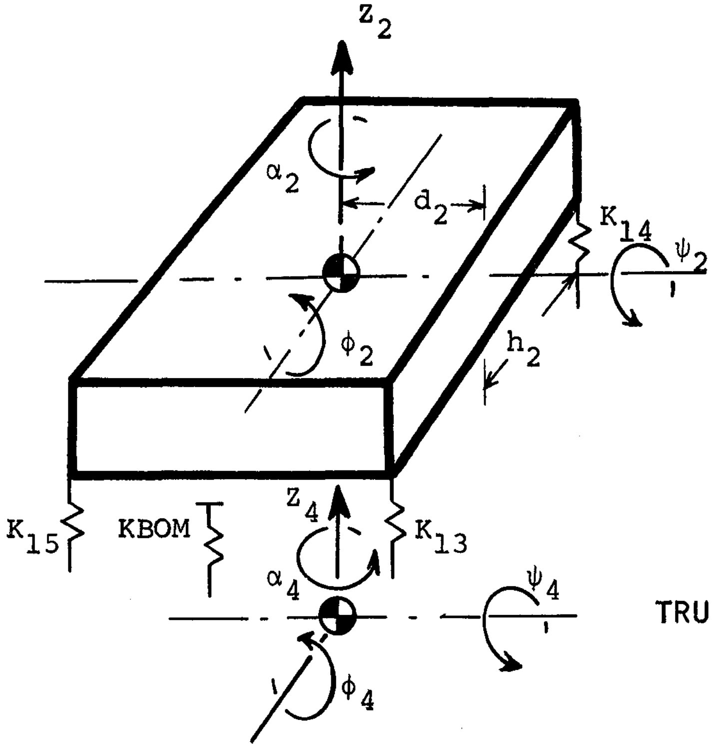


FIG. A-4. VERTICAL SPRINGS AT BOLSTER AND TRUCK INTERFACE

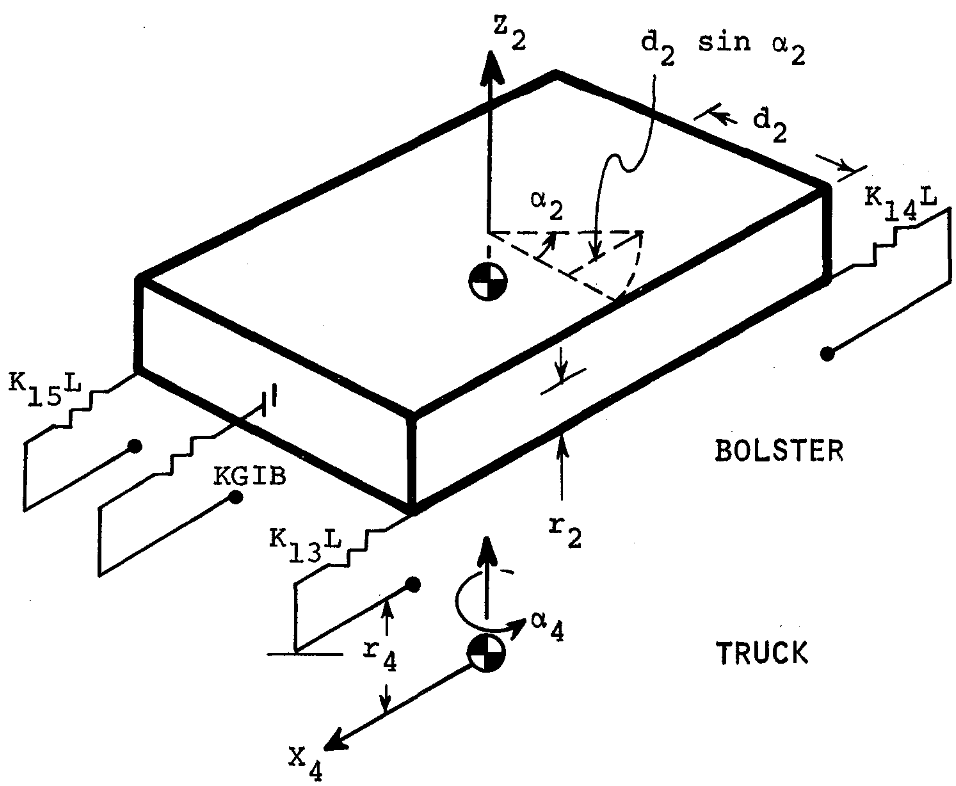


FIG. A-5. LATERAL SPRINGS BETWEEN FRONT BOLSTER AND TRUCK

$$V_{15}L = \frac{1}{2} K_{15}L(x_1 - r_1 \sin\psi_1 - d_1 \sin\alpha_1 - x_4 + d_2 \sin\alpha_2 - d_4 \sin\alpha_4 - (r_2 \sin\psi_2 + r_4 \sin\psi_4))^2$$

$$V_{16}L = \frac{1}{2} K_{16}L(x_1 - r_1 \sin\psi_1 - d_1 \sin\alpha_1 - x_4 + d_2 \sin\alpha_2 - d_4 \sin\alpha_4 - (r_2 \sin\psi_2 + r_4 \sin\psi_4))^2$$

$$VGIB_5 = \frac{1}{2} KGIB\delta_5(x_1 - r_1 \sin\psi_1 - d_1 \sin\alpha_1 - x_4 - GIB)^2$$

$$VGIB_6 = \frac{1}{2} KGIB\delta_6(x_1 - r_1 \sin\psi_1 - d_1 \sin\alpha_1 - x_4 + GIB)^2$$

where δ_5 and δ_6 are 1 or 0 depending on the GIB clearance between the bolster and the column of the sideframe.

<u>Potential Energy for the Lateral Springs Between Rear Bolster and Sideframe</u>.

$$V_{17}L = \frac{1}{2} K_{17}L(x_1 - r_1 \sin\psi_1 + d_1 \sin\alpha_1 - x_5 - (d_3 \sin\alpha_3 - d_5 \sin\alpha_5) - (r_3 \sin\psi_3 + r_5 \sin\psi_5))^2$$

$$V_{18}L = \frac{1}{2} K_{18}L(x_1 - r_1 \sin\psi_1 + d_1 \sin\alpha_1 - x_5 - (d_3 \sin\alpha_3 - d_5 \sin\alpha_5) - (r_3 \sin\psi_3 + r_5 \sin\psi_5))^2$$

$$V_{19}L = \frac{1}{2} K_{19}L(x_1 - r_1 \sin\psi_1 + d_1 \sin\alpha_1 - x_5 + (d_3 \sin\alpha_3 - d_5 \sin\alpha_5) - (r_3 \sin\psi_3 + r_5 \sin\psi_5))^2$$

$$V_{20}L = \frac{1}{2} K_{20}L(x_1 - r_1 \sin\psi_1 + d_1 \sin\alpha_1 - x_5 + d_3 \sin\alpha_3 - d_5 \sin\alpha_5 - (r_3 \sin\psi_3 + r_5 \sin\psi_5))^2$$

$$VGIB_7 = \frac{1}{2} KGIB\delta_7(x_1 - r_1 \sin\psi_1 + d_1 \sin\alpha_1 - x_5 - GIB)^2$$

$$VGIB_8 = \frac{1}{2} KGIB\delta_8(x_1 - r_1 \sin\psi_1 + d_1 \sin\alpha_1 - x_5 + GIB)^2$$

<u>Potential Energy for the Track Springs</u>. Refer to Figure A-6 (for front truck),

<u>Vertical Springs</u>.

$$V_{21} = \frac{1}{2} K_{21}(z_4 - H \sin\psi_4 + D \sin\phi_4 - V_1)^2$$

$$V_{22} = \frac{1}{2} K_{22}(z_4 + H \sin\psi_4 + D \sin\phi_4 - V_2)^2$$

$$V_{23} = \frac{1}{2} K_{23}(z_4 - H \sin\psi_4 - D \sin\phi_4 - V_3)^2$$

$$V_{24} = \frac{1}{2} K_{24}(z_4 + H \sin\psi_4 - D \sin\phi_4 - V_4)^2$$

<u>Lateral Springs</u>.

$$V_{21}L = \frac{1}{2} K_{21}L(x_4 - D \sin\alpha_4 - R \sin\psi_4 + R_9)^2$$

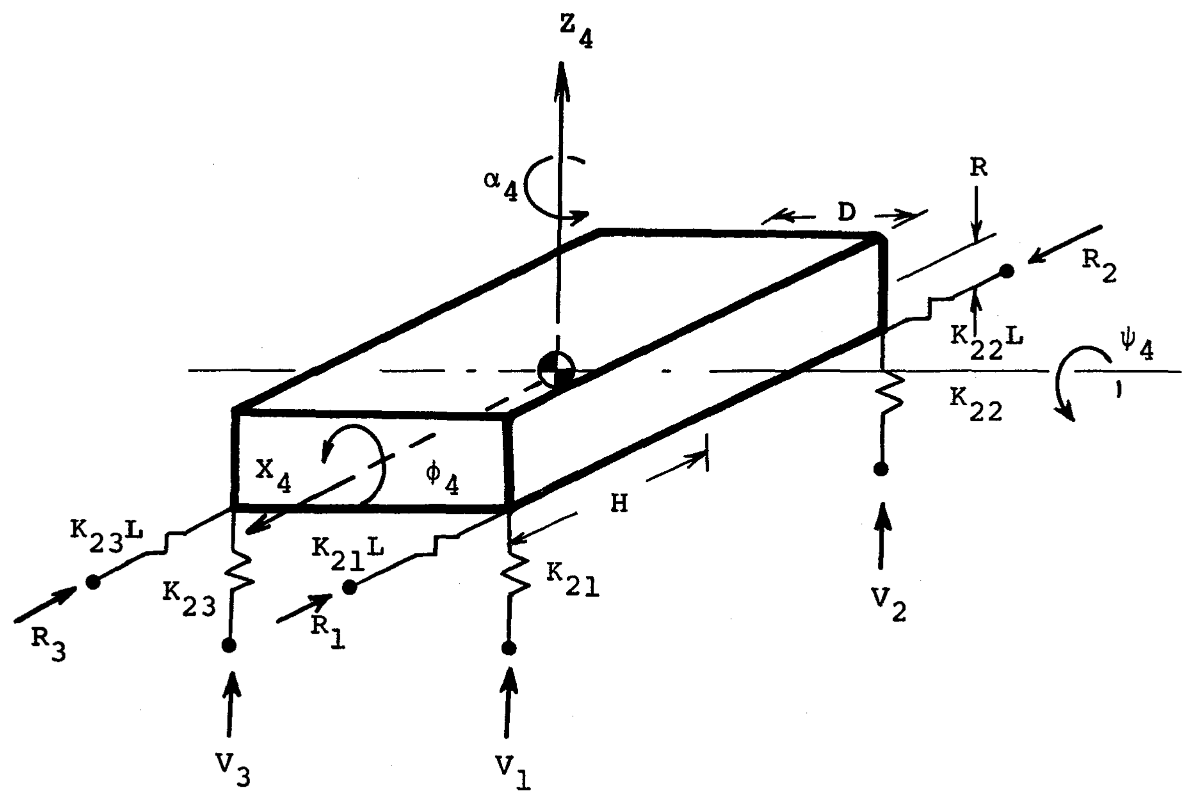


FIG. A-6. TRACK SPRINGS AT FRONT TRUCK

$$V_{22}L = \frac{1}{2} K_{22}L(x_4 - D \sin \alpha_4 - R \sin \psi_4 - R_{10})^2$$

$$V_{23}L = \frac{1}{2} K_{23}L(x_4 + D \sin \alpha_4 - R \sin \psi_4 + R_{11})^2$$

$$V_{24}L = \frac{1}{2} K_{24}L(x_4 + D \sin \alpha_4 - R \sin \psi_4 - R_{12})^2$$

Similarly, for rear truck,

$$V_{25} = \frac{1}{2} K_{25}(z_5 - H \sin \psi_5 + D \sin \phi_5 - V_5)^2$$

$$V_{26} = \frac{1}{2} K_{26}(z_5 + H \sin \psi_5 + D \sin \phi_5 - V_6)^2$$

$$V_{27} = \frac{1}{2} K_{27}(z_5 - H \sin \psi_5 - D \sin \phi_5 - V_7)^2$$

$$V_{28} = \frac{1}{2} K_{28}(z_5 + H \sin \psi_5 - D \sin \phi_5 - V_8)^2$$

$$V_{25}L = \frac{1}{2} K_{25}L(x_5 - D \sin \alpha_5 - R \sin \psi_5 + R_{13})^2$$

$$V_{26}L = \frac{1}{2} K_{26}L(x_5 - D \sin \alpha_5 - R \sin \psi_5 - R_{14})^2$$

$$V_{27}L = \frac{1}{2} K_{27}L(x_5 + D \sin \alpha_5 - R \sin \psi_5 + R_{15})^2$$

$$V_{28}L = \frac{1}{2} K_{28}L(x_5 + D \sin \alpha_5 - R \sin \psi_5 - R_{16})^2$$

Potential Energy for the Torsional and Pitching Springs.

$$VT_{24} = \frac{1}{2} KT_{24}(\alpha_2 - \alpha_4)^2$$

$$VT_{35} = \frac{1}{2} KT_{35}(\alpha_3 - \alpha_5)^2$$

$$VP_{24} = \frac{1}{2} KP_{24}(\phi_2 - \phi_4)^2$$

$$VP_{35} = \frac{1}{2} KP_{35}(\phi_3 - \phi_5)^2$$

Potential Energy for Springs Between Freight Element and the C.G. of Car Body.

$$V_F = \frac{1}{2} KF(x_6 - x_1)^2 + \frac{1}{2}KF(y_6 - y_1)^2 + \frac{1}{2} KF(z_6 - z_1)^2$$

Gravitational Potential Energy of the System.

$$V_G = m_1gz_1 + m_2gz_2 + m_3gz_3 + m_4gz_4 + m_5gz_5 + m_6gz_6$$

Total Potential Energy of the System.

$$V = V_1 + V_2 + V_5 + V_6 + V_7 + V_8 + V_{11} + V_{12} + V_{13} + V_{14}$$
$$+ V_{15} + V_{16} + VBOM_9 + VBOM_{10} + V_{17} + V_{18} + V_{19} + V_{20}$$
$$+ VBOM_{11} + VBOM_{12} + V_{13}L + V_{14}L + V_{15}L + V_{16}L + VGIB_5$$
$$+ VGIB_6 + V_{17}L + V_{18}L + V_{19}L + V_{20}L + VGIB_7 + VGIB_8 + V_{21}$$
$$+ V_{22} + V_{23} + V_{24} + V_{21}L + V_{22}L + V_{23}L + V_{24}L + V_{25}$$
$$V_{26} + V_{27} + V_{25}L + V_{26}L + V_{27}L + V_{28}L + VT_{24} + VT_{35}$$
$$+ VP_{24} + VP_{35} + VF + VG$$

Dissipation Energy

Energy is dissipated through the various dampers of the system. If treated as viscous dampers, the damping force is proportional to the velocity of the damping action, i.e. of the form $c\dot{x}$ and the energy dissipated is of the form $1/2\ c\dot{x}^2$. The dissipation energy of the system is

$$D = \frac{1}{2} C_{13}(\dot{z}_2 - \dot{z}_4 - h_2(\cos\psi_2\dot{\psi}_2 - \cos\psi_4\dot{\psi}_4) + d_2\cos\phi_2\dot{\phi}_2$$
$$- d_4\cos\phi_4\dot{\phi}_4)^2 + \frac{1}{2} C_{14}(\dot{z}_2 - \dot{z}_4 + h_2(\cos\psi_2\dot{\psi}_2 - \cos\psi_4\dot{\psi}_4)$$
$$+ d_2\cos\phi_2\dot{\phi}_2 - d_4\cos\phi_4\dot{\phi}_4)^2 + \frac{1}{2} C_{15}(\dot{z}_2 - \dot{z}_4 - h_2(\cos\psi_2\dot{\psi}_2$$
$$- \cos\psi_4\dot{\psi}_4) - (d_2\cos\phi_2\dot{\phi}_2 - d_4\cos\phi_4\dot{\phi}_4))^2 + \frac{1}{2} C_{16}(\dot{z}_2 - \dot{z}_4$$
$$+ h_2(\cos\psi_2\dot{\psi}_2 - \cos\psi_4\dot{\psi}_4) - (d_2\cos\phi_2\dot{\phi}_2 - d_4\cos\phi_4\dot{\phi}_4))^2$$
$$+ \frac{1}{2} C_{17}(\dot{z}_3 - \dot{z}_5 - h_3(\cos\psi_3\dot{\psi}_3 - \cos\psi_5\dot{\psi}_5) + d_3\cos\phi_3\dot{\phi}_3$$
$$- d_5\cos\phi_5\dot{\phi}_5)^2 + \frac{1}{2} C_{18}(\dot{z}_3 - \dot{z}_5 + h_3(\cos\psi_3\dot{\psi}_3 - \cos\psi_5\dot{\psi}_5)$$
$$+ d_3\cos\phi_3\dot{\phi}_3 - d_5\cos\phi_5\dot{\phi}_5)^2 + \frac{1}{2} C_{19}(\dot{z}_3 - \dot{z}_5 - h_3(\cos\psi_3\dot{\psi}_3$$
$$- \cos\psi_5\dot{\psi}_5) - (d_3\cos\phi_3\dot{\phi}_3 - d_5\cos\phi_5\dot{\phi}_5))^2 + \frac{1}{2} C_{20}(\dot{z}_3 - \dot{z}_5$$
$$+ h_3(\cos\psi_3\dot{\psi}_3 - \cos\psi_5\dot{\psi}_5) - (d_3\cos\phi_3\dot{\phi}_3 - d_5\cos\phi_5\dot{\phi}_5))^2$$
$$+ \frac{1}{2}(C_{13}L + C_{14}L)(\dot{x}_1 - r_1\cos\psi_1\dot{\psi}_1 - d_1\cos\alpha_1\dot{\alpha}_1 - \dot{x}_4$$
$$- (d_2\cos\alpha_2\dot{\alpha}_2 - d_4\cos\alpha_4\dot{\alpha}_4) - (r_2\cos\psi_2\dot{\psi}_2 + r_4\cos\psi_4\dot{\psi}_4))^2$$
$$+ \frac{1}{2}(C_{15}L + C_{16}L)(\dot{x}_1 - r_1\cos\psi_1\dot{\psi}_1 - d_1\cos\alpha_1\dot{\alpha}_1 - \dot{x}_4$$
$$+ (d_2\cos\alpha_2\dot{\alpha}_2 - d_4\cos\alpha_4\dot{\alpha}_4) - (r_2\cos\psi_2\dot{\psi}_2 + r_4\cos\psi_4\dot{\psi}_4))^2$$
$$+ \frac{1}{2}(C_{17}L + C_{18}L)(\dot{x}_1 - r_1\cos\psi_1\dot{\psi}_1 + d_1\cos\alpha_1\dot{\alpha}_1 - \dot{x}_5$$

$$- (d_3 \cos \alpha_3\dot{\alpha}_3 - d_5 \cos \alpha_5\dot{\alpha}_5) - (r_3 \cos \psi_3\dot{\psi}_3 + r_5 \cos \psi_5\dot{\psi}_5))^2$$

$$+ \frac{1}{2}(C_{19}L + C_{20}L)(\dot{x}_1 - r_1 \cos \psi_1\dot{\psi}_1 + d_1 \cos \alpha_1\dot{\alpha}_1 - \dot{x}_5$$

$$+ (d_3 \cos \alpha_3\dot{\alpha}_3 - d_5 \cos \alpha_5\dot{\alpha}_5) - (r_3 \cos \psi_3\dot{\psi}_3 + r_5 \cos \psi_5\dot{\psi}_5))^2$$

$$+ \frac{1}{2} CF(\dot{x}_6 - \dot{x}_1)^2 + \frac{1}{2} CF(\dot{y}_6 - \dot{y}_1)^2 + \frac{1}{2} CF(\dot{z}_6 - \dot{z}_1)^2$$

Lagrange's equation can have the form

$$\frac{d}{dt}\left(\frac{\partial E}{\partial \dot{q}_i}\right) - \frac{\partial E}{\partial q_i} + \frac{\partial V}{\partial q_i} + \frac{\partial D}{\partial \dot{q}_i} = Q_i$$

After some algebraic manipulation on each of the generalized coordinates and making the following substitutions, we can then write 27 equations of motion.

$SAA = z_1 - z_2$

$SAB = \sin \psi_1 - \sin \psi_2$

$SAC = z_1 - z_3$

$SAD = \sin \psi_1 - \sin \psi_3$

$SAF = x_1 - r_1 \sin \psi_1 - d_1 \sin \alpha_1 - x_4$

$SAI = d_1 \sin \phi_1$

$SAM = z_2 - z_4$

$SAN = \dot{z}_2 - \dot{z}_4$

$SAO = \sin \psi_2 - \sin \psi_4$

$SAP = \cos \psi_2\dot{\psi}_2 - \cos \psi_4\dot{\psi}_4$

$SAQ = d_2 \sin \phi_2 - d_4 \sin \phi_4$

$SAR = d_2 \cos \phi_2\dot{\phi}_2 - d_4 \cos \phi_4\dot{\phi}_4$

$SAU = x_1 - r_1 \sin \psi_1 + d_1 \sin \alpha_1$

$SBA = z_3 - z_5$

$SBB = \sin \psi_3 - \sin \psi_5$

$SBC = d_3 \sin \phi_3 - d_5 \sin \phi_5$

$SBD = \dot{z}_3 - \dot{z}_5$

$SBE = \cos \psi_3\dot{\psi}_3 - \cos \psi_5\dot{\psi}_5$

$$SBF = d_3 \cos \phi_3\dot{\phi}_3 - d_5 \cos \phi_5\dot{\phi}_5$$

$$SBM = h_4 \sin \psi_4$$

$$SBN = D \sin \phi_4$$

$$SBO = D \sin \alpha_4$$

$$SBP = R \sin \alpha_4$$

$$SBQ = h_5 \sin \psi_5$$

$$SBR = D \sin \phi_5$$

$$SBS = D \sin \alpha_5$$

$$SBT = R \sin \psi_5$$

$$SCA = \dot{x}_1 - r_1 \cos \psi_1\dot{\psi}_1 - d_1 \cos \alpha_1\dot{\alpha}_1 - \dot{x}_4 - (d_2 \cos \alpha_2\dot{\alpha}_2 - d_4 \cos \alpha_4\dot{\alpha}_4) - (r_2 \cos \psi_2\dot{\psi}_2 + r_4 \cos \psi_4\dot{\psi}_4)$$

$$SCB = \dot{x}_1 - r_1 \cos \psi_1\dot{\psi}_1 - d_1 \cos \alpha_1\dot{\alpha}_1 - \dot{x}_4 + (d_2 \cos \alpha_2\dot{\alpha}_2 - d_4 \cos \alpha_4\dot{\alpha}_4) - (r_2 \cos \psi_2\dot{\psi}_2 + r_4 \cos \psi_4\dot{\psi}_4)$$

$$SCC = \dot{x}_1 - r_1 \cos \psi_1\dot{\psi}_1 + d_1 \cos \alpha_1\dot{\alpha}_1 - \dot{x}_5 - (d_3 \cos \alpha_3\dot{\alpha}_3 - d_5 \cos \alpha_5\dot{\alpha}_5) - (r_3 \cos \psi_3\dot{\psi}_3 + r_5 \cos \psi_5\dot{\psi}_5)$$

$$SCD = \dot{x}_1 - r_1 \cos \psi_1\dot{\psi}_1 + d_1 \cos \alpha_1\dot{\alpha}_1 - \dot{x}_5 + (d_3 \cos \alpha_3\dot{\alpha}_3 - d_5 \cos \alpha_5\dot{\alpha}_5) - (r_3 \cos \psi_3\dot{\psi}_3 + r_5 \cos \psi_5\dot{\psi}_5)$$

$$SCE = x_1 - r_1 \sin \psi_1 - d_1 \sin \alpha_1 - x_4 - (d_2 \sin \alpha_2 - d_4 \sin \alpha_4) - (r_2 \sin \psi_2 + r_4 \sin \psi_4)$$

$$SCF = x_1 - r_1 \sin \psi_1 - d_1 \sin \alpha_1 - x_4 + (d_2 \sin \alpha_2 - d_4 \sin \alpha_4) - (r_2 \sin \psi_2 + r_4 \sin \psi_4)$$

$$SCG = x_1 - r_1 \sin \psi_1 + d_1 \sin \alpha_1 - x_5 - (d_3 \sin \alpha_3 - d_5 \sin \alpha_5) - (r_3 \sin \psi_3 + r_5 \sin \psi_5)$$

$$SCH = x_1 - r_1 \sin \psi_1 + d_1 \sin \alpha_1 - x_5 + (d_3 \sin \alpha_3 - d_5 \sin \alpha_5) - (r_3 \sin \psi_3 + r_5 \sin \psi_5)$$

These are the 27 equations of motion in the generalized coordinate of the system:

For $q_1 = x_1$

$$(m_1 + 2m_2)\ddot{x}_1 + 2m_2r_1 \sin\psi_1\dot{\psi}_1^2 - 2m_2r_1\cos\psi_1\ddot{\psi}_1 + (K13L + K14L)SCE$$
$$+ (K_{15}L + K_{16}L)SCF + (K_{17}L + K_{18}L)SCH + (K_{19}L + K_{20}L)SCH + (C_{13}L$$
$$+ C_{14}L)SCA + (C_{15}L + C_{16}L)SCB + (C_{17}L + C_{18}L)SCC + (C_{19}L + C_{20}L)SCD$$
$$- CF(\dot{x}_6 - \dot{x}_1) + KGIB\delta_5(SAF - GIB) + KGIB\delta_6(SAF + GIB) +$$
$$KGIB\delta_7(SAU - GIB) + KGIB\delta_8(SAU + GIB) - KF(\dot{x}_6 - \dot{x}_1) = 0$$

For $q_2 = y_1$

$$m_1\ddot{y}_1 + KC(y_1 - y_S) + CC(\dot{y}_1 - \dot{y}_S) = 0$$

For $q_3 = z_1$

$$m_1\ddot{z}_1 + K_1(SAA - SAB + SAI) + K_2(SAA + SAB + SAI) + K_5\delta_1(SAA$$
$$+ g_1SAB + SAI + GAP) + K_6\delta_2(SAA + g_1SAB + SAI + GAP) + K_7(SAC$$
$$- eSAD - SAI) + K_8(SAC + eSAD - SAI) + K_{11}\delta_3(SAC - g_1SAD - SAI$$
$$+ GAP) + K_{12}\delta_4(SAC + g_1SAD - SAI + GAP) + m_1g - KF(z_6 - z_1)$$
$$- CF(\dot{z}_6 - \dot{z}_1) = 0$$

For $q_4 = \psi_1$

$$(-2m_2r_1)\ddot{x}_1 + (I_{x_1}\sin^2\alpha_1 + I_{y_1} + 2m_2r_1^2)\ddot{\psi}_1 + (I_{x_1} - I_{y_1})\sin\alpha_1\ddot{\phi}_1$$
$$+ I_{x_1}\cos\alpha_1\dot{\alpha}_1(\dot{\phi}_1\cos\psi_1\cos\alpha_1 + \dot{\psi}_1\sin\alpha_1)$$
$$+ I_{x_1}\sin\alpha_1(-\dot{\phi}_1\cos\alpha_1\sin\psi_1\dot{\psi}_1 - \dot{\phi}_1\cos\psi_1\sin\alpha_1\dot{\alpha}_1$$
$$+ \dot{\psi}_1\cos\alpha_1\dot{\alpha}_1) - I_{y_1}\sin\alpha_1\dot{\alpha}_1(\dot{\psi}_1\cos\alpha_1 - \dot{\phi}_1\cos\psi_1\sin\alpha_1)$$
$$+ I_{y_1}\cos\alpha_1(-\dot{\psi}_1\sin\alpha_1\dot{\alpha}_1 + \dot{\phi}_1\sin\alpha_1\sin\psi_1\dot{\psi}_1$$
$$- \dot{\phi}_1\cos\psi_1\cos\alpha_1\dot{\alpha}_1) + I_{x_1}\dot{\phi}_1\cos\alpha_1\sin\psi_1(\dot{\phi}_1\cos\psi_1\cos\alpha_1$$
$$+ \dot{\psi}_1\sin\alpha_1) - I_{y_1}\dot{\phi}_1\sin\alpha_1\sin\psi_1(\dot{\psi}_1\cos\alpha_1 - \dot{\phi}_1\cos\psi_1\sin\alpha_1)$$
$$- I_{z_1}\dot{\phi}_1\cos\psi_1(\dot{\phi}_1\sin\psi_1 + \dot{\alpha}_1) - K_1e\cos\psi_1(SAA - eSAB + SAI) +$$
$$K_2e\cos\psi_1(SAA + eSAB + SAI) - K_5\delta_1g_1\cos\psi_1(SAA - g_1SAB + SAI$$
$$+ GAP) + K_6\delta_2g_1\cos\psi_1(SAA + g_1SAB + SAI + GAP) - K_7e\cos\psi_1(SAC$$
$$- eSAD - SAI) + K_8e\cos\psi_1(SAC + eSAD - SAI) - K_{11}\delta_3g_1\cos\psi_1(SAC$$
$$- g_1SAD - SAI + GAP) + K_{12}\delta_4g_1\cos\psi_1(SAC + g_1SAD - SAI + GAP)$$

$- (K_{13}L + K_{14}L)r_1 \cos \psi_1 SCE - (K_{15}L + K_{16}L)r_1 \cos \psi_1 SCF - (K_{17}L$
$+ K_{18}L)r_1 \cos \psi_1 SCG - (K_{19}L + K_{20}L)r_1 \cos \psi_1 SCH$
$- KGIB\delta_5 r_1 \cos \psi_1 (SAF - GIB) - KGIB\delta_6 r_1 \cos \psi_1 (SAF + GIB)$
$- KGIB\delta_7 r_1 \cos \psi_1 (SAU - GIB) - KGIB\delta_8 r_1 \cos \psi_1 (SAU + GIB)$
$- (C_{13}L + C_{14}L)r_1 \cos \psi_1 SCA - (C_{15}L + C_{16}L)r_1 \cos \psi_1 SCB$
$- (C_{17}L + C_{18}L)r_1 \cos \psi_1 SCC - (C_{19}L + C_{20}L)r_1 \cos \psi_1 SCD = 0$

<u>For $q_5 = \phi_1$</u>

$(I_{x1} - I_{y1})\sin \alpha_1 \ddot{\psi}_1 + (I_{x1} + I_{y1} \sin^2 \alpha_1 + I_{z1} \sin^2 \psi_1)\ddot{\phi}_1$
$+ I_{z1} \sin \psi_1 \ddot{\alpha}_1 - I_{x_1} \cos \alpha_1 \sin \psi_1 \dot{\psi}_1 (\dot{\phi}_1 \cos \psi_1 \cos \alpha_1 + \dot{\psi}_1 \sin \alpha_1)$
$- I_{x_1} \cos \psi_1 \sin \alpha_1 \dot{\alpha}_1 (\dot{\phi}_1 \cos \psi_1 \cos \alpha_1 + \dot{\psi}_1 \sin \alpha_1) +$
$+ I_{x_1} \cos \psi_1 \cos \alpha_1 (- \dot{\phi}_1 \cos \alpha_1 \sin \psi_1 \dot{\psi}_1 - \dot{\phi}_1 \cos \psi_1 \sin \alpha_1 \dot{\alpha}_1$
$+ \dot{\psi}_1 \cos \alpha_1 \dot{\alpha}_1) + I_{y_1} \sin \alpha_1 \sin \psi_1 \dot{\psi}_1 (\dot{\psi}_1 \cos \alpha_1 - \dot{\phi}_1 \cos \psi_1 \sin \alpha_1)$
$- I_{y1} \cos \psi_1 \cos \alpha_1 \dot{\alpha}_1 (\dot{\psi}_1 \cos \alpha_1 - \dot{\phi}_1 \cos \psi_1 \sin \alpha_1)$
$- I_{y_1} \cos \psi_1 \sin \alpha_1 (- \dot{\psi}_1 \sin \alpha_1 \dot{\alpha}_1 + \dot{\phi}_1 \sin \alpha_1 \sin \psi_1 \dot{\psi}_1$
$- \dot{\phi}_1 \cos \psi_1 \cos \alpha_1 \dot{\alpha}_1) + I_{z1} \cos \psi_1 \dot{\psi}_1 (\dot{\phi}_1 \sin \psi_1 + \dot{\alpha}_1)$
$+ I_{z1} \sin \psi_1 \dot{\phi}_1 \cos \psi_1 \dot{\psi}_1 + K_1 d_1 \cos \phi_1 (SAA - eSAB + SAI)$
$+ K_2 d_1 \cos \phi_1 (SAA + eSAB + SAI) + K_5 \delta_1 d_1 \cos \phi_1 (SAA - g_1 SAB + SAI$
$+ GAP) + K_6 \delta_2 d_1 \cos \phi_1 (SAA + g_1 SAB + SAI + GAP) - K_7 d_1 \cos \phi_1 (SAC$
$- eSAD - d_1 \sin \phi_1) - K_8 d_1 \cos \phi_1 (SAC + eSAD - SAI)$
$- K_{11} \delta_3 d_1 \cos \phi_1 (SAC - g_1 SAD - SAI + GAP) - K_{12} \delta_4 d_1 \cos \phi_1 (SAC$
$+ g_1 SAD - SAI + GAP) = 0$

<u>For $q_6 = \alpha_1$</u>

$I_{z1} \sin \psi_1 \ddot{\phi}_1 + I_{z1} + 2m_2 d_1^2 \cos^2 \alpha_1 + I_{z1} \cos \psi_1 \dot{\phi}_1 \dot{\psi}_1$
$- 4m_2 d_1^2 \cos \alpha_1 \sin \alpha_1 \dot{\alpha}_1^2 - I_{x_1}(- \dot{\phi}_1 \cos \psi_1 \sin \alpha_1 + \dot{\psi}_1 \cos \alpha_1)$
$(\dot{\phi}_1 \cos \psi_1 \cos \alpha_1 + \dot{\psi}_1 \sin \alpha_1) - I_{y_1}(- \dot{\psi}_1 \sin \alpha_1 - \dot{\phi}_1 \cos \psi_1 \cos \alpha_1)$
$(\dot{\psi}_1 \cos \alpha_1 - \dot{\phi}_1 \cos \psi_1 \sin \alpha_1) + 2m_2 d_1^2 \cos \alpha_1 \sin \alpha_1 \dot{\alpha}_1^2 - (K_{13}L$
$+ K_{14}L)d_1 \cos \alpha_1 SCE - (K_{15}L + K_{16}L)d_1 \cos \alpha_1 SCF + (K_{17}L +$
$+ K_{18}L)d_1 \cos \alpha_1 SCG + (K_{19}L + K_{20}L)d_1 \cos \alpha_1 SCH - (C_{13}L$

$+ C_{14}L)d_1 \cos \alpha_1 SCA - (C_{15}L + C_{16}L)d_1 \cos \alpha_1 SCB + (C_{17}L$

$+ C_{18}L)d_1 \cos \alpha_1 SCC + (C_{19}L + C_{20}L)d_1 \cos \alpha_1 SCD$

$- KGIB\delta_5 d_1 \cos \alpha_1 (SAF - GIB) - KGIB\delta_6 d_1 \cos \alpha_1 (SAF + GIB)$

$+ KGIB\delta_7 d_1 \cos \alpha_1 (SAU - GIB) + KGIB\delta_8 d_1 \cos \alpha_1 (SAU + GIB) = 0$

<u>For $q_7 = z_2$</u>

$m_2\ddot{z}_2 - K_1(SAA - eSAB + SAI) - K_2(SAA + eSAB + SAI) - K_5\delta_1(SAA$

$- g_1SAB + SAI + GAP) - K_6\delta_2(SAA + g_1SAB + SAI + GAP) + K_{13}(SAM$

$- h_2SAO + SAQ) + K_{14}(SAM + h_2SAO + SAQ) + K_{15}(SAM - h_2SAO - SAQ)$

$+ K_{16}(SAM + h_2SAO - SAQ) + m_2g + C_{13}(SAN - h_2SAP + SAR) + C_{14}(SAN$

$+ h_2SAP + SAR) + C_{15}(SAN - h_2SAP - SAR) + C_{16}(SAN + h_2SAP - SAR)$

$+ KBOM\delta_9(SAM - h_2SAO + TL) + KBOM\delta_{10}(SAM + h_2SAO + TL) = 0$

<u>For $q_8 = \psi_2$</u>

$(I_{x2} \sin^2 \alpha_2 + I_{y2})\ddot{\psi}_2 + (I_{x2} - I_{y2})\sin \alpha_2\ddot{\phi}_2 + I_{x2} \cos \alpha_2\dot{\alpha}_2$

$(\dot{\phi}_2 \cos \psi_2 \cos \alpha_2 + \dot{\psi}_2 \sin \alpha_2) + I_{x2} \sin \alpha_2(- \dot{\phi}_2 \cos \alpha_2 \sin \psi_2\dot{\psi}_2$

$- \dot{\phi}_2 \cos \psi_2 \sin \alpha_2\dot{\alpha}_2 + \dot{\psi}_2 \cos \alpha_2\dot{\alpha}_2) - I_{y2} \sin \alpha_2\dot{\alpha}_2(\dot{\psi}_2 \cos \alpha_2$

$- \dot{\phi}_2 \cos \psi_2 \sin \alpha_2) + I_{y2} \cos \alpha_2(- \dot{\psi}_2 \sin \alpha_2\dot{\alpha}_2 + \dot{\phi}_2 \sin \alpha_2 \sin \psi_2\dot{\psi}_2$

$- \dot{\phi}_2 \cos \psi_2 \cos \alpha_2\dot{\alpha}_2) + I_{x2} \sin \psi_2\dot{\phi}_2 \cos \alpha_2(\dot{\phi}_2 \cos \psi_2 \cos \alpha_2$

$+ \dot{\psi}_2 \sin \alpha_2) - I_{y2}\dot{\phi}_2 \sin \psi_2 \sin \alpha_2(\dot{\psi}_2 \cos \alpha_2 - \dot{\phi}_2 \cos \psi_2 \sin \alpha_2)$

$- I_{z2}\dot{\phi}_2 \cos \psi_2(\dot{\phi}_2 \sin \psi_2 + \dot{\alpha}_2) + K_1e(SAA - eSAB + SAI)$

$- K_2e(SAA + eSAB + SAI) + K_5\delta_1g_1 \cos \psi_2(SAA - g_1SAB + SAI + GAP)$

$- K_6\delta_2g_1 \cos \psi_2(SAA + g_1SAB + SAI + GAP) - K_{13}h_2 \cos \psi_2(SAM$

$- h_2SAO + SAQ) + K_{14}h_2 \cos \psi_2(SAM + h_2SAO + SAQ) - K_{15}h_2 \cos \psi_2(SAM$

$- h_2SAO - SAQ) + K_{16}h_2 \cos \psi_2(SAM + h_2SAO - SAQ) - (K_{13}L$

$+ K_{14}L)r_2 \cos \psi_2SCE - (K_{15}L + K_{16}L)r_2 \cos \psi_2SCF$

$- KBOM\delta_9h_2 \cos \psi_2(SAM - h_2SAO + TL) + KBOM\delta_{10}h_2 \cos \psi_2(SAM$

$+ h_2SAO + TL) - C_{13}h_2 \cos \psi_2(SAN - h_2SAP + SAR) + C_{14}h_2 \cos \psi_2(SAN$

$+ h_2 SAP + SAR) - C_{15}h_2 \cos\psi_2 (SAN - h_2 SAP - SAR)$

$+ C_{16}h_2 \cos\psi_2 (SAN + h_2 SAP - SAR) - (C_{13}L + C_{14}L)r_2 \cos\psi_2 SCA$

$- (C_{15}L + C_{16}L)r_2 \cos\psi_2 SCB = 0$

<u>For $q_9 = \phi_2$</u>

$(I_{x2} - I_{y2})\sin\alpha_2\ddot{\psi}_2 + (I_{x2} + I_{y2}\sin^2\alpha_2 + I_{z2}\sin^2\psi_2)\ddot{\phi}_2$

$+ I_{z2}\sin\psi_2\ddot{\alpha}_2 - I_{x2}\cos\alpha_2\sin\psi_2\dot{\psi}_2(\dot{\phi}_2\cos\psi_2\cos\alpha_2 + \dot{\psi}_2\sin\alpha_2)$

$- I_{x2}\cos\psi_2\sin\alpha_2\dot{\alpha}_2(\dot{\phi}_2\cos\psi_2\cos\alpha_2 + \dot{\psi}_2\sin\alpha_2)$

$+ I_{x2}\cos\psi_2\cos\alpha_2(-\dot{\phi}_2\cos\alpha_2\sin\psi_2\dot{\psi}_2 - \dot{\phi}_2\cos\psi_2\sin\alpha_2\dot{\alpha}_2$

$+ \dot{\psi}_2\cos\alpha_2\dot{\alpha}_2) + I_{y2}\sin\alpha_2\sin\psi_2\dot{\psi}_2(\dot{\psi}_2\cos\alpha_2 - \dot{\phi}_2\cos\psi_2\sin\alpha_2)$

$- I_{y2}\cos\psi_2\cos\alpha_2\dot{\alpha}_2(\dot{\psi}_2\cos\alpha_2 - \dot{\phi}_2\cos\psi_2\sin\alpha_2)$

$- I_{y2}\cos\psi_2\sin\alpha_2(-\dot{\psi}_2\sin\alpha_2\dot{\alpha}_2 + \dot{\phi}_2\sin\alpha_2\sin\psi_2\dot{\psi}_2$

$- \dot{\phi}_2\cos\psi_2\cos\alpha_2\dot{\alpha}_2) + I_{z2}\cos\psi_2\dot{\psi}_2(\dot{\phi}_2\sin\psi_2 + \dot{\alpha}_2)$

$+ I_{z2}\sin\psi_2\dot{\phi}_2\cos\psi_2\dot{\psi}_2 + K_{13}d_2\cos\phi_2(SAM - h_2 SAO + SAQ)$

$+ K_{14}d_2\cos\phi_2(SAM + h_2 SAO + SAQ) - K_{15}d_2\cos\phi_2(SAM - h_2 SAO - SAQ)$

$- K_{16}d_2\cos\phi_2(SAM + h_2 SAO - SAQ) + KP_{24}(\phi_2 - \phi_4)$

$+ C_{13}d_2\cos\phi_2(SAN - h_2 SAP + SAR) + C_{14}d_2\cos\phi_2(SAN + h\ SAP + SAR)$

$- C_{15}d_2\cos\phi_2(SAN - h_2 SAP - SAR) - C_{16}d_2\cos\phi_2(SAN + h_2 SAP$

$- SAR) = 0$

<u>For $q_{10} = \alpha_2$</u>

$I_{z2}(\ddot{\phi}_2\sin\psi_2 + \dot{\phi}_2\cos\psi_2\dot{\psi}_2 + \ddot{\alpha}_2) - I_{x2}(-\dot{\phi}_2\cos\psi_2\sin\alpha_2$

$+ \dot{\psi}_2\cos\alpha_2)(\dot{\phi}_2\cos\psi_2\cos\alpha_2 + \dot{\psi}_2\sin\alpha_2) - I_{y2}(-\dot{\psi}_2\sin\alpha_2$

$- \dot{\phi}_2\cos\psi_2\cos\alpha_2)(\dot{\psi}_2\cos\alpha_2 - \dot{\phi}_2\cos\psi_2\sin\alpha_2) - (K_{13}L$

$+ K_{14}L)d_2\cos\alpha_2 SCE + (K_{15}L + K_{16}L)d_2\cos\alpha_2 SCF + KT_{24}(\alpha_2 - \alpha_4)$

$- (C_{13}L + C_{14}L)d_2\cos\alpha_2 SCA + (C_{15}L + C_{16}L)d_2\cos\alpha_2 SCB = 0$

For $q_{11} = z_3$

$$m_3\ddot{z}_3 - K_7(SAC - eSAD - SAI) - K_8(SAC + eSAD - SAI) - K_{11}\delta_3(SAC - g_1SAD - SAI + GAP) - K_{12}\delta_4(SAC + g_1SAD + SAI + GAP) + K_{17}(SBA - h_3SBB + SBC) + K_{18}(SBA + h_3SBB + SBC) + K_{19}(SBA - h_3SBB - SBC) + K_{20}(SBA + h_3SBB - SBC) + m_3g + KBOM\delta_{11}(SBA - h_3SBB + TL) + KBOM\delta_{12}(SBA + h_3SBB + TL) + C_{17}(SBD - h_3SBE + SBF) + C_{18}(SBD + h_3SBE + SBF) + C_{19}(SBD - h_3SBE - SBF) + C_{20}(SBD + h_3SBE - SBF) = 0$$

For $q_{12} = \psi_3$

$$(I_{x_3} \sin^2 \alpha_3 + I_{y_3})\ddot{\psi}_3 + (I_{x_3} \sin \alpha_3 - I_{y_3} \sin \alpha_3)\ddot{\phi}_3 + I_{x_3} \cos \alpha_3\dot{\alpha}_3(\dot{\phi}_3 \cos \psi_3 \cos \alpha_3 + \dot{\psi}_3 \sin \alpha_3) + I_{x_3} \sin \alpha_3(- \dot{\phi}_3 \cos \alpha_3 \sin \psi_3\dot{\psi}_3 - \dot{\phi}_3 \cos \psi_3 \sin \alpha_3\dot{\alpha}_3 + \dot{\psi}_3 \cos \alpha_3\dot{\alpha}_3) - I_{y_3} \sin \alpha_3\dot{\alpha}_3(\dot{\psi}_3 \cos \alpha_3 - \dot{\phi}_3 \cos \psi_3 \sin \alpha_3) + I_{y_3} \cos \alpha_3(- \dot{\psi}_3 \sin \alpha_3\dot{\alpha}_3 + \dot{\phi}_3 \sin \alpha_3 \sin \psi_3\dot{\psi}_3 - \dot{\phi}_3 \cos \psi_3 \cos \alpha_3\dot{\alpha}_3) - I_{x_3}(- \dot{\phi}_3 \cos \alpha_3 \sin \psi_3)(\dot{\phi}_3 \cos \psi_3 \cos \alpha_3 + \dot{\psi}_3 \sin \alpha_3) - I_{y_3}\dot{\phi}_3 \sin \alpha_3 \sin \psi_3(\psi_3 \cos \alpha_3 - \dot{\phi}_3 \cos \psi_3 \sin \alpha_3) - I_{z_3}\dot{\phi}_3 \cos \psi_3(\dot{\phi}_3 \sin \psi_3 + \dot{\alpha}_3) + K_7e \cos \psi_3(SAC - eSAD - SAI) - K_8e \cos \psi_3(SAC + eSAD - SAI) + K_{11}\delta_3g_1 \cos \psi_3(SAC - g_1SAD - SAI + GAP) - K_{12}\delta_4g_1 \cos \psi_3(SAC + g_1SAD - SAI + GAP) - K_{17}h_3 \cos \psi_3(SBA - h_3SBB + SBC) + K_{18}h_3 \cos \psi_3(SBA + h_3SBB + SBC) - K_{19}h_3 \cos \psi_3(SBA - h_3SBB - SBC) + K_{20}h_3 \cos \psi_3(SBA + h_3SBB - SBC) - (K_{17}L + K_{18}L)r_3 \cos \psi_3SCG - (K_{19}L + K_{20}L)r_3 \cos \psi_3SCH - KBOM\delta_{11}h_3 \cos \psi_3(SBA - h_3SBB + TL) + KBOM\delta_{12}h_3 \cos \psi_3(SBA + h_3SBB + TL) - C_{17}h_3 \cos \psi_3(SBD - h_3SBE + SBF) + C_{18}h_3 \cos \psi_3(SBD + h_3SBE + SBF) - C_{19}h_3 \cos \psi_3(SBD - h_3SBE - SBF) + C_{20}h_3 \cos \psi_3(SBD + h_3SBE - SBF) - (C_{17}L + C_{18}L)r_3 \cos \psi_3SCC - (C_{19}L + C_{20}L)r_3 \cos \psi_3SCD = 0$$

For $q_{13} = \phi_3$

$$(I_{x_3} - I_{y_3})\sin\alpha_3\ddot{\psi}_3 + (I_{x_3} + I_{y_3}\sin^2\alpha_3 + I_{z_3}\sin^2\psi_3)\ddot{\phi}_3$$
$$+ I_{z_3}\sin\psi_3\ddot{\alpha}_3 - I_{x_3}\cos\alpha_3\sin\psi_3\dot{\psi}_3(\dot{\phi}_3\cos\psi_3\cos\alpha_3 + \dot{\psi}_3\sin\alpha_3)$$
$$- I_{x_3}\cos\psi_3\sin\alpha_3\dot{\alpha}_3(\dot{\phi}_3\cos\psi_3\cos\alpha_3 + \dot{\psi}_3\sin\alpha_3)$$
$$+ I_{x_3}\cos\psi_3\cos\alpha_3(-\dot{\phi}_3\cos\alpha_3\sin\psi_3\dot{\psi}_3 - \dot{\phi}_3\cos\psi_3\sin\alpha_3\dot{\alpha}_3$$
$$+ \dot{\psi}_3\cos\alpha_3\dot{\alpha}_3) + I_{y_3}\sin\alpha_3\sin\psi_3\dot{\psi}_3(\dot{\psi}_3\cos\alpha_3 - \dot{\phi}_3\cos\psi_3\sin\alpha_3)$$
$$- I_{y_3}\cos\psi_3\cos\alpha_3\dot{\alpha}_3(\dot{\psi}_3\cos\alpha_3 - \dot{\phi}_3\cos\psi_3\sin\alpha_3)$$
$$- I_{y_3}\cos\psi_3\sin\alpha_3(-\dot{\psi}_3\sin\alpha_3\dot{\alpha}_3 + \dot{\phi}_3\sin\alpha_3\sin\psi_3\dot{\psi}_3$$
$$- \dot{\phi}_3\cos\psi_3\cos\alpha_3\dot{\alpha}_3) + I_{z_3}\cos\psi_3\dot{\psi}_3(\dot{\phi}_3\sin\psi_3 + \dot{\alpha}_3)$$
$$+ I_{z_3}\sin\psi_3\dot{\phi}_3\cos\psi_3\dot{\psi}_3 + K_{17}d_3\cos\phi_3(SBA - h_3SBB + SBC)$$
$$+ K_{18}d_3\cos\phi_3(SBA + h_3SBB + SBC) - K_{19}d_3\cos\phi_3(SBA - h_3SBB - SBC)$$
$$- K_{20}d_3\cos\phi_3(SBA + h_3SBB - SBC) + KP_{35}(\phi_3 - \phi_5)$$
$$+ C_{17}d_3\cos\phi_3(SBD - h_3SBE + SBF) + C_{18}d_3\cos\phi_3(SBD + h_3SBE + SBF)$$
$$- C_{19}d_3\cos\phi_3(SBD - h_3SBE - SBF) - C_{20}d_3\cos\phi_3(SBD + h_3SBE$$
$$- SBF) = 0$$

For $q_{14} = \alpha_3$

$$I_{z_3}(\ddot{\phi}_3\sin\psi_3 + \dot{\phi}_3\cos\psi_3\dot{\psi}_3 + \ddot{\alpha}_3) - I_{x_3}(-\dot{\phi}_3\cos\psi_3\sin\alpha_3$$
$$+ \dot{\psi}_3\cos\alpha_3)(\dot{\phi}_3\cos\psi_3\cos\alpha_3 + \dot{\psi}_3\sin\alpha_3) - I_{y_3}(-\dot{\psi}_3\sin\alpha_3$$
$$- \dot{\phi}_3\cos\psi_3\cos\alpha_3)(\dot{\psi}_3\cos\alpha_3 - \dot{\phi}_3\cos\psi_3\sin\alpha_3) - (K_{17}L$$
$$+ K_{18}L)d_3\cos\alpha_3SCG + (K_{19}L + K_{20}L)d_3\cos\alpha_3SCH + KT_{35}(\alpha_3 - \alpha_5)$$
$$- (C_{17}L + C_{18}L)d_3\cos\alpha_3SCC + (C_{19}L + C_{20}L)d_3\cos\alpha_3SCD = 0$$

For $q_{15} = x_4$

$$m_4\ddot{x}_4 + K_{21}L(x_4 - SBO - SBP + R_9) + K_{22}L(x_4 - SBO - SBP - R_{10})$$
$$+ K_{23}L(x_4 + SBO - SBP + R_{11}) + K_{24}L(x_4 + SBO - SBP - R_{12})$$
$$- (C_{13}L + C_{14}L)SCA - (C_{15}L + C_{16}L)SCB - KGIB\delta_5(SAF - GIB)$$
$$- KGIB\delta_6(SAF + GIB) = 0$$

<u>For $q_{16} = z_4$</u>

$$m_4\ddot{z}_4 - K_{13}(SAM - h_2SAO + SAQ) - K_{14}(SAM + h_2SAO + SAQ) - K_{15}(SAM - h_2SAO - SAQ) - K_{16}(SAM + h_2SAO - SAQ) + K_{21}(z_4 - SBM + SBN - V_1) + K_{22}(z_4 + h_4 \sin\psi_4 + SBN - V_2) + K_{23}(z_4 - SBM - SBN - V_3) + K_{24}(z_4 + SBM - SBN - V_4) - KBOM\delta_9(SAM - h_2SAO + TL) - KBOM\delta_{10}(SAM + h_2SAO + TL) + m_4g - C_{13}(SAN - h_2SAP + SAR) - C_{14}(SAN + h_2SAP + SAR) - C_{15}(SAN - h_2SAP - SAR) - C_{16}(SAN + h_2SAP - SAR) = 0$$

<u>For $q_{17} = \psi_4$</u>

$$I_{x_4} \cos\alpha_4\dot{\alpha}_4(\dot{\phi}_4 \cos\psi_4 \cos\alpha_4 + \dot{\psi}_4 \sin\alpha_4)$$
$$+ I_{x_4} \sin\alpha_4(\ddot{\phi}_4 \cos\psi_4 \cos\alpha_4 - \dot{\phi}_4 \cos\alpha_4 \sin\psi_4\dot{\psi}_4$$
$$- \dot{\phi}_4 \cos\psi_4 \sin\alpha_4\dot{\alpha}_4 + \ddot{\psi}_4 \sin\alpha_4 + \dot{\psi}_4 \cos\alpha_4\dot{\alpha}_4)$$
$$+ I_{y_4} \sin\alpha_4\dot{\alpha}_4(\dot{\psi}_4 \cos\alpha_4 - \dot{\phi}_4 \cos\psi_4 \sin\alpha_4)$$
$$+ I_{y_4} \cos\alpha_4(\ddot{\psi}_4 \cos\alpha_4 - \dot{\psi}_4 \sin\alpha_4\dot{\alpha}_4 - \ddot{\phi}_4 \cos\psi_4 \sin\alpha_4$$
$$+ \dot{\phi}_4 \sin\alpha_4 \sin\psi_4\dot{\psi}_4 - \dot{\phi}_4 \cos\psi_4 \cos\alpha_4\dot{\alpha}_4) +$$
$$+ I_{x_4} \sin\psi_4\dot{\phi}_4 \cos\alpha_4(\dot{\phi}_4 \cos\psi_4 \cos\alpha_4 + \dot{\psi}_4 \sin\alpha_4)$$
$$- I_{y_4}\sin\psi_4\dot{\phi}_4 \sin\alpha_4(\dot{\psi}_4 \cos\alpha_4 - \dot{\phi}_4 \cos\psi_4 \sin\alpha_4)$$
$$- I_{z_4}\dot{\phi}_4 \cos\psi_4(\dot{\phi}_4 \sin\psi_4 + \dot{\alpha}_4) + K_{13}h_2 \cos\psi_4(SAM - h_2SAO + SAQ)$$
$$- K_{14}h_2 \cos\psi_4(SAM + h_2SAO + SAQ) + K_{15}h_2 \cos\psi_4(SAM - h_2SAO - SAQ) - K_{16}h_2 \cos\psi_4(SAM + h_2SAO - SAQ) - (K_{13}L + K_{14}L)r_4 \cos\psi_4SCE - (K_{15}L + K_{16}L)r_4 \cos\psi_4SCF - K_{21}h_4 \cos\psi_4(z_4 - SBM + SBN - V_1) + K_{22}h_4 \cos\psi_4(z_4 + SBM + SBN - V_2)$$
$$- K_{23}h_4 \cos\psi_4(z_4 - SBM - SBN - V_3) + K_{24}h_4 \cos\psi_4(z_4 + SBM - SBN - V_4) - K_{21}LR \cos\psi_4(x_4 - SBO - SBP + R_9) - K_{22}LR \cos\psi_4(x_4 - SBO - SBP - R_{10}) - K_{23}LR \cos\psi_4(x_4 + D \sin\alpha_4 - R \sin\psi_4 + R_{11})$$
$$- K_{24}LR \cos\psi_4(x_4 + D \sin\alpha_4 - R \sin\psi_4 - R_{12})$$
$$+ KBOM\delta_9h_2 \cos\psi_4(SAM - h_2SAO + TL) - KBOM\delta_{10}h_2 \cos\psi_4(SAM$$

$+ h_2SAO + TL) + C_{13}h_2 \cos\psi_4(SAN - h_2SAP + SAR) - C_{14}h_2 \cos\psi_4(SAN$
$+ h_2SAP + SAR) + C_{15}h_2 \cos\psi_4(SAN - h_2SAP - SAR)$
$- C_{16}h_2 \cos\psi_4(SAN + h_2SAP - SAR) - (C_{13}L + C_{14}L)r_4 \cos\psi_4 SCA$
$- (C_{15}L + C_{16}L)r_4 \cos\psi_4 SCB = 0$

<u>For $q_{18} = \phi_4$</u>

$$
\begin{aligned}
&- I_{x_4} \cos\alpha_4 \sin\psi_4\dot\psi_4(\dot\phi_4 \cos\psi_4 \cos\alpha_4 + \dot\psi_4 \sin\alpha_4)\\
&- I_{x_4} \cos\psi_4 \sin\alpha_4\dot\alpha_4(\dot\phi_4 \cos\psi_4 \cos\alpha_4 + \dot\psi_4 \sin\alpha_4)\\
&+ I_{x_4} \cos\psi_4 \cos\alpha_4(\ddot\phi_4 \cos\psi_4 \cos\alpha_4 - \dot\phi_4 \cos\alpha_4 \sin\psi_4\dot\psi_4\\
&- \dot\phi_4 \cos\psi_4 \sin\alpha_4\dot\alpha_4 + \ddot\psi_4 \sin\alpha_4 + \dot\psi_4 \cos\alpha_4\dot\alpha_4)\\
&+ I_{y_4} \sin\alpha_4 \sin\psi_4\dot\psi_4(\dot\psi_4 \cos\alpha_4 - \dot\phi_4 \cos\alpha_4 \sin\alpha_4)\\
&- I_{y_4} \cos\psi_4 \cos\alpha_4\dot\alpha_4(\dot\psi_4 \cos\alpha_4 - \dot\phi_4 \cos\psi_4 \sin\alpha_4)\\
&- I_{y_4} \cos\psi_4 \sin\alpha_4(\ddot\psi_4 \cos\alpha_4 - \dot\psi_4 \sin\alpha_4\dot\alpha_4 - \ddot\phi_4 \cos\psi_4 \sin\alpha_4\\
&+ \dot\phi_4 \sin\alpha_4 \sin\psi_4\dot\psi_4 - \dot\phi_4 \cos\psi_4 \cos\alpha_4\dot\alpha_4) + I_{z_4} \cos\psi_4\dot\psi_4(\dot\phi_4 \sin\psi_4\\
&+ \dot\alpha_4) + I_{z_4} \sin\psi_4(\ddot\phi_4 \sin\psi_4 + \dot\phi_4 \cos\psi_4\dot\psi_4 + \ddot\alpha_4) - K_{13}d_4 \cos\phi_4(SAM\\
&- h_2SAO + SAQ) - K_{14}d_4 \cos\phi_4(SAM + h_2SAO + SAQ) + K_{15}d_4 \cos\phi_4(SAM\\
&- h_2SAO - SAQ) + K_{16}d_4 \cos\phi_4(SAM + h_2SAO - SAQ) + K_{21}D \cos\phi_4(z_4\\
&- SBM + SBN - V_1) + K_{22}D \cos\phi_4(z_4 + SBM + SBN - V_2)\\
&- K_{23}D \cos\phi_4(z_4 - SBM - SBN - V_3) - K_{24}D \cos\phi_4(z_4 + SBM - SBN\\
&- V_4) - KP_{24}(\phi_2 - \phi_4) - C_{13}d_4 \cos\phi_4(SAN - h_2SAP + SAR)\\
&- C_{14}d_4 \cos\phi_4(SAN + h_2SAP + SAR) + C_{15}d_4 \cos\phi_4(SAN - h_2SAP - SAR)\\
&+ C_{16}d_4 \cos\phi_4(SAN + h_2SAP - SAR) = 0
\end{aligned}
$$

<u>For $q_{19} = \alpha_4$</u>

$$
\begin{aligned}
&I_{z_4}(\ddot\phi_4 \sin\psi_4 + \dot\phi_4 \cos\psi_4\dot\psi_4 + \ddot\alpha_4) - I_{x_4}(- \dot\phi_4 \cos\psi_4 \sin\alpha_4\\
&+ \dot\psi_4 \cos\alpha_4)(\dot\phi_4 \cos\psi_4 \cos\alpha_4 + \dot\psi_4 \sin\alpha_4) - I_{y_4}(- \dot\psi_4 \sin\alpha_4\\
&- \dot\phi_4 \cos\psi_4 \cos\alpha_4)(\dot\psi_4 \cos\alpha_4 - \dot\phi_4 \cos\psi_4 \sin\alpha_4) + (K_{13}L +\\
&K_{14}L)d_4 \cos\alpha_4 SCE - (K_{15}L + K_{16}L)d_4 \cos\alpha_4 SCF - K_{21}LD \cos\alpha_4(x_4\\
&- SBO - SBP + R_9) - K_{22}LD \cos\alpha_4(x_4 - SBO - SBP - R_{10})
\end{aligned}
$$

$+ K_{23}LD \cos \alpha_4 (x_4 + SBO - SBP + R_{11}) + K_{24}LD \cos \alpha_4 (x_4 + SBO$

$- SBP - R_{12}) - KT_{24}(\alpha_2 - \alpha_4) + (C_{13}L + C_{14}L)d_4 \cos \alpha_4 SCA$

$- (C_{15}L + C_{16}L)d_4 \cos \alpha_4 SCB = 0$

<u>For $q_{20} = x_5$</u>

$m_5\ddot{x}_5 - (K_{17}L + K_{18}L)SCG - (K_{19}L + K_{20}L)SCH + K_{25}L(x_5 - D \sin \alpha_5$

$- R \sin \psi_5 + R_{13}) + K_{26}L(x_5 - D \sin \alpha_5 - R \sin \psi_5 - R_{14})$

$+ K_{27}L(x_5 + D \sin \alpha_5 - R \sin \psi_5 + R_{15}) + K_{28}L(x_5 + D \sin \alpha_5$

$- R \sin \psi_5 - R_{16}) - KGIB\delta_7(SAU - GIB) - KGIB\delta_8(SAU + GIB)$

$- (C_{17}L + C_{18}L)SCC - (C_{19}L + C_{20}L)SCD = 0$

<u>For $q_{21} = z_5$</u>

$m_5\ddot{z}_5 - K_{17}(SBA - h_3SBB + SBC) - K_{18}(SBA + h_3SBB + SBC) - K_{19}(SBA$

$- h_3SBB - SBC) - K_{20}(SBA + h_3SBB - SBC) + K_{25}(z_5 - SBQ + SBR - V_5)$

$+ K_{26}(z_5 + SBQ + SBR - V_6) + K_{27}(z_5 - SBQ - SBR - V_7) + K_{28}(z_5$

$+ SBQ - SBR - V_8) + m_5g - KBOM\delta_{11}(SBA - h_3SBB + TL)$

$- KBOM\delta_{12}(SBA + h_3SBB + TL) - C_{17}(SBD - h_3SBE + SBF) - C_{18}(SBD$

$+ h_3SBE + SBF) - C_{19}(SBD - h_3SBE - SBF) - C_{20}(SBD + h_3SBE - SBF) = 0$

<u>For $q_{22} = \psi_5$</u>

$(I_{x_5} \sin^2 \alpha_5 + I_{y_5} \cos^2 \alpha_5)\ddot{\psi}_5 + (I_{x_5} \sin \alpha_5 \cos \psi_5 \cos \alpha_5$

$- I_{y_5} \cos \alpha_5 \cos \psi_5 \sin \alpha_5)\ddot{\phi}_5 + I_{x_5}\cos \alpha_5\dot{\alpha}_5(\dot{\phi}_5 \cos \psi_5 \cos \alpha_5$

$+ \dot{\psi}_5\sin \alpha_5) + I_{x_5} \sin \alpha_5(- \dot{\phi}_5 \cos \alpha_5 \sin \psi_5\dot{\psi}_5 - \dot{\phi}_5 \cos \psi_5 \sin \alpha_5\dot{\alpha}_5$

$+ \dot{\psi}_5 \cos \alpha_5\dot{\alpha}_5) - I_{y_5} \sin \alpha_5\dot{\alpha}_5(\dot{\psi}_5 \cos \alpha_5 - \dot{\phi}_5 \cos \psi_5 \sin \alpha_5)$

$+ I_{y_5} \cos \alpha_5(- \dot{\psi}_5 \sin \alpha_5\dot{\alpha}_5 + \dot{\phi}_5 \sin \alpha_5 \sin \psi_5\dot{\psi}_5 - \dot{\phi}_5 \cos \psi_5 \cos \alpha_5\dot{\alpha}_5)$

$+ I_{x_5}\dot{\phi}_5 \cos \alpha_5 \sin \psi_5(\dot{\phi}_5 \cos \psi_5 \cos \alpha_5 + \dot{\psi}_5 \sin \alpha_5)$

$- I_{y_5}\dot{\phi}_5 \sin \alpha_5 \sin \psi_5(\dot{\psi}_5 \cos \alpha_5 - \dot{\phi}_5 \cos \psi_5 \sin \alpha_5)$

$- I_{z_5}\dot{\phi}_5 \cos \psi_5(\dot{\phi}_5 \sin \psi_5 + \dot{\alpha}_5) + K_{17}h_3 \cos \psi_5(SBA - h_3SBB + SBC)$

$- K_{18}h_3 \cos \psi_5(SBA + h_3SBB + SBC) + K_{19}h_3 \cos \psi_5(SBA - h_3SBB$

$- SBC) - K_{20}h_3 \cos \psi_5(SBA + h_3SBB - SBC) - (K_{17}L + K_{18}L)r_5 \cos \psi_5SCG$

$$- (K_{19}L + K_{20}L)r_5 \cos\psi_5 SCH - K_{25}h_5 \cos\psi_5(z_5 - SBQ + SBR - V_5)$$
$$+ K_{26}h_5 \cos\psi_5(z_5 + SBQ + SBR - V_6) - K_{27}h_5 \cos\psi_5(z_5 - SBQ - SBR$$
$$- v_7) + K_{28}h_5 \cos\psi_5(z_5 + SBQ - SBR - V_8) - K_{25}LR \cos\psi_5(x_5 - SBS$$
$$- SBT + R_{13}) - K_{26}LR \cos\psi_5(x_5 - SBS - SBT - R_{14}) - K_{27}LR \cos\psi_5(x_5$$
$$+ SBS - SBT + R_{15}) - K_{28}LR \cos\psi_5(x_5 + SBS - SBT - R_{16})$$
$$+ KBOM\delta_{11}h_3(SBA - h_3SBB + TL) - KBOMh_3(z_3 - z_5 + h_3SBB + TL)$$
$$+ C_{17}h_3 \cos\psi_5(SBD - h_3SBE + SBF) - C_{18}h_3 \cos\psi_5(SBD + h_3SBE$$
$$+ SBF) + C_{19}h_3 \cos\psi_5(SBD - h_3SBE - SBF) - C_{20}h_3 \cos\psi_5(SBD$$
$$+ h_3SBE - SBF) - (C_{17}L + C_{18}L)r_5 \cos\psi_5 SCC - (C_{19}L$$
$$+ C_{20}L)r_5 \cos\psi_5 SCD = 0$$

<u>For $q_{23} = \phi_5$</u>

$$(I_{x_5} - I_{y_5})\sin\alpha_5\ddot{\psi}_5 + (I_{x_5} + I_{y_5}\sin^2\alpha_5 + I_{z_5}\sin^2\psi_5)\ddot{\phi}_5$$
$$+ I_{z_5}\sin\psi_5\ddot{\alpha}_5 - I_{x_5}\cos\alpha_5\sin\psi_5\dot{\psi}_5(\dot{\phi}_5\cos\psi_5\cos\alpha_5 + \dot{\psi}_5\sin\alpha_5)$$
$$- I_{x_5}\cos\psi_5\sin\alpha_5\dot{\alpha}_5(\dot{\phi}_5\cos\psi_5\cos\alpha_5 + \dot{\psi}_5\sin\alpha_5)$$
$$+ I_{x_5}\cos\psi_5\cos\alpha_5(-\dot{\phi}_5\cos\alpha_5\sin\psi_5\dot{\psi}_5 - \dot{\phi}_5\cos\psi_5\sin\alpha_5\dot{\alpha}_5$$
$$+ \dot{\psi}_5\cos\alpha_5\dot{\alpha}_5) + I_{y_5}\sin\alpha_5\sin\psi_5\dot{\psi}_5(\dot{\psi}_5\cos\alpha_5 - \dot{\phi}_5\cos\psi_5\sin\alpha_5)$$
$$- I_{y_5}\cos\psi_5\cos\alpha_5\dot{\alpha}_5(\dot{\psi}_5\cos\alpha_5 - \dot{\phi}_5\cos\psi_5\sin\alpha_5)$$
$$- I_{y_5}\cos\psi_5\sin\alpha_5(-\dot{\psi}_5\sin\alpha_5\dot{\alpha}_5 + \dot{\phi}_5\sin\alpha_5\sin\psi_5\dot{\psi}_5$$
$$- \dot{\phi}_5\cos\psi_5\cos\alpha_5\dot{\alpha}_5) + I_{z_5}\cos\psi_5\dot{\psi}_5(\dot{\phi}_5\sin\psi_5 + \dot{\alpha}_5)$$
$$+ I_{z_5}\sin\psi_5\dot{\phi}_5\cos\psi_5\dot{\psi}_5 - K_{17}d_5\cos\phi_5(SBA - h_3SBB + SBC)$$
$$- K_{18}d_5\cos\phi_5(SBA + h_3SBB + SBC) + K_{19}d_5\cos\phi_5(SBA - h_3SBB$$
$$- SBC) + K_{20}d_5\cos\phi_5(SBA + h_3SBB - SBC) + K_{25}D\cos\phi_5(z_5 - SBQ$$
$$+ SBR - V_5) + K_{26}D\cos\phi_5(z_5 + SBQ + SBR - V_6) - K_{27}D\cos\phi_5(z_5$$
$$- SBQ - SBR - V_7) - K_{28}D\cos\phi_5(z_5 + SBQ - SBR - V_8) - KP_{35}(\phi_3$$
$$- \phi_5) - C_{17}d_5\cos\phi_5(SBD - h_3SBE + SBF) - C_{18}d_5\cos\phi_5(SBD$$
$$+ h_3SBE + SBF) + C_{19}d_5\cos\phi_5(SBD - h_3SBE - SBF)$$
$$+ C_{20}d_5\cos\phi_5(SBD + h_3SBE - SBF) = 0$$

For $q_{24} = \alpha_5$

$$I_{z_5}(\ddot{\phi}_5 \sin\psi_5 + \dot{\phi}_5 \cos\psi_5\dot{\psi}_5 + \ddot{\alpha}_5) - I_{x_5}(- \dot{\phi}_5 \cos\psi_5 \sin\alpha_5$$
$$+ \dot{\psi}_5 \cos\alpha_5)(\dot{\phi}_5 \cos\psi_5 \cos\alpha_5 + \dot{\psi}_5 \sin\alpha_5) - I_{y_5}(- \dot{\psi}_5 \sin\alpha_5$$
$$- \dot{\phi}_5 \cos\psi_5 \cos\alpha_5)(\dot{\psi}_5 \cos\alpha_5 - \dot{\phi}_5 \cos\psi_5 \sin\alpha_5)$$
$$- K_{25}LD \cos\alpha_5(x_5 - SBS - SBT + R_{13}) - K_{26}LD \cos\alpha_5(x_5 - SBS$$
$$- SBT - R_{14}) + K_{27}LD \cos\alpha_5(x_5 + SBS - SBT + R_{15})$$
$$+ K_{28}LD \cos\alpha_5(x_5 + D \sin\alpha_5 - R \sin\psi_5 - R_{16}) + (K_{17}L$$
$$+ K_{18}L)d_5 \cos\alpha_5 SCG - (K_{19}L + K_{20}L)d_5 \cos\alpha_5 SCH - KT_{35}(\alpha_3 - \alpha_5)$$
$$+ (C_{17}L + C_{18}L)\dot{d}_5 \cos\alpha_5 SCC - (C_{19}L + C_{20}L)\dot{d}_5 \cos\alpha_5 SCD = 0$$

For $q_{25} = x_6$

$$m_6\ddot{x}_6 + KF(x_6 - x_1) + CF(\dot{x}_6 - \dot{x}_1) = 0$$

For $q_{26} = y_6$

$$m_6\ddot{y}_6 + KF(y_6 - y_1) + CF(\dot{y}_6 - \dot{y}_1) = 0$$

For $q_{27} = z_6$

$$m_6\ddot{z}_6 + KF(z_6 - z_1) + CF(\dot{z}_6 - \dot{z}_1) = 0$$

The above 27 equations describe the motion of the system we have modelled. Options like centerplate extension pads, lateral springs at the centerplates can easily be introduced to the model and the corresponding potential and dissipation energy terms can be derived by the same method.

APPENDIX B

COMPUTER PROGRAM LISTINGS

```
WELT,ILD T.ELT
ELT007 RLIB70 04/23-10:45:37-(,0)
000   C
000   C
000   C      MAIN PROGRAM
000   C
000   C
000   C      *************************************************************@***
000   C      THE FOLLOWING ARE SYSTEM PARAMETERS
000   C
000   C      SPRING STIFFNESS (LB/IN)
000   C
000   C      K1,K2,K7,K8: CENTER PLATE SPRING STIFFNESS
000   C      K5,K6,K11,K12:VERTICAL STIFFNESS OF SIDE BEARINGS OF FRONT AND
000   C      REAR BOLSTERS
000   C      K13 TO K20: VERTICAL SUSPENSION SPRING STIFFNESS
000   C      K13L TO K16L:LATERAL SPRING CONSTANTS AT POINTS 13,14,15,16
000   C      K17L TO K20L:LATERAL SPRING CONSTANTS AT POINTS 17,18,19,20.
000   C      K21 TO K28: TRACK VERTICAL SPRINGS
000   C      K21L TO K28L: TRACK LATERAL SPRINGS
000   C      KGIB: GIB SPRING
000   C      KBOM: BOTTOMED SPRING
000   C
000   C      EQUIVALENT VISCOUS DAMPING COEFFICIENTS FOR THE SYSTEM: (LB-SEC/IN)
000   C
000   C      C13 TO C16:VERTICAL DAMPING COEFFICIENTS FOR ELEMENTS AT POINTS
000   C      13,14,15,16 BETWEEN FRONT BOLSTER AND SID5 6R1M5
000   C      C13L TO C16L:LATERAL DAMPING COEFFICIENTS FOR ELEMENTS AT POINTS
000   C      13,14,15,16 BETWEEN FRONT BOLSTER AND TRUCK
000   C      C17 TO C20:VERTICAL DAMPING COEFFICIENTS AT POINTS 17,18,19,20
000   C      BETWEEN REAR BOLSTER AND TRUCK
000   C      C17L TO C20L:LATERAL DAMPING COEFFICIENTS AT POINTS 17,18,19,20
000   C      BETWEEN REAR BOLSTER AND TRUCK
000   C
000   C      MOMENT OF INERTIA OF THE BODIES IN THE SYSTEM: (IN-LB-SEC**2)
000   C
000   C      CIX1,CIY1 AND CIZ1:MOMENT OF INERTIA OF CAR BODY WITH RESPECT TO
000   C      ITS CENTER OF MASS ABOUT ITS LATERAL AXIS,LONGITUDINAL AXIS AND
000   C      VERTICAL AXIS RESPECTIVELY
000   C      BIX2,BIY2 AND BIZ2:MOMENT OF INERTIA OF FRONT BOLSTER WITH RESPECT
000   C      TO ITS CENTER OF MASS ABOUT ITS LATERAL,LONGITUDINAL AND VERTICAL AXES
000   C      RESPECTIVELY
000   C      BIX3,BIY3 AND BIZ3:MOMENT OF INERTIA OF REAR BOLSTER WITH RESPECT
000   C      TO ITS CENTER OF MASS ABOUT ITS LATERAL,LONGITUDINAL AND VERTICAL
000   C      AXES RESPECTIVELY
000   C      TIX4,TIY4 AND TIZ4:MOMENT OF INERTIA OF FRONT TRUCK WITH RESPECT
000   C      TO ITS CENTER OF MASS ABOUT LATERAL,LONGITUDINAL AND VERTICAL AXES
000   C      RESPECTIVELY
000   C      TIX5,TIY5 AND TIZ5:MOMENT OF INERTIA OF REAR TRUCK WITH RESPECT TO
000   C      ITS CENTER OF MASS ABOUT ITS LATERAL,LONGITUDINAL AND VERTICAL AXES
000   C      RESPECTIVELY
000   C
000   C      MASS OF THE BODIES OF THE SYSTEM: (LB-SEC**2/IN)
000   C
000   C      CM1:MASS OF THE CAR BODY
000   C      BM2:MASS OF THE FRONT BOLSTER
000   C      BM3:MASS OF THE REAR BOLSTER
```

```
000056   000   C     TM4:MASS OF THE FRONT TRUCK
000057   000   C     TM5:MASS OF THE REAR TRUCK
000058   000   C     FM1: MASS OF FREIGHT ELEMENT
000059   000   C
000060   000   C     GEOMETRIC PARAMETERS (INCH)
000061   000   C
000062   000   C     R1: DIST. OF C.G. OF CAR BODY ABOVE BOLSTER CENTER PLATE.
000063   000   C     R2: VERTICAL DIST. BETWEEN C.G. OF FRONT BOLSTER AND CENTER PLATE
000064   000   C     R3: VERTICAL DIST. BETWEEN C.G. OF REAR BOLSTER & CENTER PLATE.
000065   000   C     R4,R5: VERTICAL DIST. BETWEEN C.G. OF FRONT AND REAR OF SIDE
000066   000   C     FRAME & BOTTOM SEAT OF SUSPENSION SPRINGS.
000067   000   C     C:VERTICAL DISTANCE BETWEEN CENTER OF MASSES OF BOLSTER AND TRUCK
000068   000   C     B:VERTICAL DISTANCE BETWEEN CENTER OF MASSES OF CAR BODY AND THE
000069   000   C     BOLSTERS
000070   000   C     D:TRUCK CENTER DISTANCE
000071   000   C     E:RADIUS OF THE CENTER PLATE
000072   000   C     G1:DISTANCE BETWEEN CENTER LINE OF BOLSTER AND SIDE BEARING
000073   000   C     GAP:SIDE BEARING CLEAREANCE
000074   000   C     G1B:G1B CLEAREANCE
000075   000   C     D1:DISTANCE BETWEEN C.G. OF CAR BODY TO CENTER LINE OF CENTER PLATE
000076   000   C     D2:DISTANCE BETWEEN C.G. OF FRONT BOLSTER TO POINTS OF SPRING
000077   000   C     ATTACHMENTS FOR SPRING GROUPS (K13 TO K16)
000078   000   C     D3:DISTANCE BETWEEN C.G. OF REAR BOLSTER TO POINTS OF SPRING
000079   000   C     ATTACHMENT FOR SPRING GROUP (K17 TO K20)
000080   000   C     D4:DISTANCE BETWEEN C.G. OF FRONT TRUCK TO SUSPENSION SPRING
000081   000   C     ATTACHMENT
000082   000   C     D5:DISTANCE BETWEEN C.G. OF REAR TRUCK TO SUSPENSION SPRING
000083   000   C     ATTACHMENT
000084   000   C     B1:WHEEL CENTER DISTANCE FOR TRUCK
000085   000   C     RL:RAIL LENGTH
000086   000   C
000087   000   C     OTHER PARAMETERS
000088   000   C
000089   000   C     ACCELERATION DUE TO GRAVITY (IN/SEC**2)
000090   000   C     SPEED:SPEED OF THE TRAIN (MPH)
000091   000   C     S:MAXIMUM CROSS LEVEL DIFFERENCE (INCH)
000092   000   C     DT:TIME STEP OF INTEGRATION (SEC)
000093   000   C     TMAX:MAXIMUM TIME FOR COMPUTER SIMULATION (SEC)
000094   000   C     T:TIME AT ANY INSTANT (SEC)
000095   000   C     PTIME: PRINTING TIME (SEC)
000096   000   C
000097   000   C     INPUT RAIL DISPLACEMENTS (INCH)
000098   000   C
000099   000   C     V1 TO V8: VERTICAL TRACK INPUT
000100   000   C     R9 TO R16:HORIZONTAL INPUTS OF THE TRACK
000101   000   C
000102   000   C     PROGRAM OUTPUT:
000103   000   C
000104   000   C     DD(1):LATERAL DISPLACEMENT OF C.G. OF CAR BODY
000105   000   C     DD(2):LONGITUDINAL
000106   000   C     DD(3):VERTICAL
000107   000   C     DD(4):ROLL OF THE CAR BODY
000108   000   C     DD(5):PITCH
000109   000   C     DD(6):YAW
000110   000   C     DD(7):LATERAL DISPLACEMENT OF FRONT BOLSTER
000111   000   C     DD(8):LONGITUDINAL
000112   000   C     DD(9):VERTICAL
```

```
000   C    DD(10):ROLL           @
000   C    DD(11):PITCH
000   C    DD(12):YAW
000   C    DD(13):LATERAL DISPLACEMENT OF REAR BOLSTER
000   C    DD(14):LONGITUDINAL
000   C    DD(15):VERTICAL
000   C    DD(16):ROLL OF REAR BOLSTER
000   C    DD(17):PITCH
000   C    DD(18):YAW
000   C    DD(19):LATERAL DISPLACEMENT OF FRONT TRUCK
000   C    DD(20):LONGITUDINAL
000   C    DD(21):VERTICAL          @
000   C    DD(22):ROLL OF THE FRONT TRUCK
000   C    DD(23):PITCH          @
000   C    DD(24):YAW          @
000   C    DD(25):LATERAL DISPLACEMENT OF REAR TRUCK
000   C    DD(26):LONGITUDINAL           @
000   C    DD(27):VERTICAL          @
000   C    DD(28):ROLL OF THE REAR TRUCK
000   C    DD(29):PITCH          @
000   C    DD(30):YAW          @
000   C    DD(31) TO DD(33): DISPLACEMENTS OF FREIGHT ELEMENT IN X,Y,AND Z DIRECTIONS
000   C    VV(1) TO VV(33) ARE THE CORRESPONDING VELOCITY TERMS
000   C    AA(1) TO AA(33) ARE CORRESPONDING ACCELERATION TERMS
000   C    ***********************************************************************
000        COMMON/ACCELN/
000       1SI,CMGA,D3,PHI1,PHI2,PHI3,PHI4,PHI5,DT,T,D2,D4,K6,D5,K1L,
000       1K2L,CM1,K7L,K8L,E,K3,F1,D1,K5,GAP,BM2,K9,K10,K11,K12,G,CIX1,
000       2CIY1,CIZ1,K13L,K4,K15L,K16L,C14L,K13S,GIB,K14S,K15S,K16S,C13L,
000       3K14L,C15L,C16L,C13,C14,C15,C16,BIX2,BIY2,BIZ2,K17L,K18L,K19L,
000       4BM3,K17S,K18S,K19S,K20S,C17L,C18L,C19L,C20L,C17,C18,C19,C20,
000       5BIX3,BIY3,BIZ3,K20L,TM4,TIX4,TIZ4,TIY4,TM5,TIX5,TIY5,TIZ5,TMAX
000       6,R1,R2,R3,R4,R5,K13,K14,K15,K16,K17,K18,K19,K20,SAM,SAO,SAQ,SBA,
000       7SBB,SBC,V1,V2,V3,V4,V5,V6,V7,V8,H2,H3,K1,K2,K7,K8,SAA,SAC,GIBC,
000       8GIBD,GIBE,GIBF,GIBG,GIBH,GIBJ,GIBK,SAG,SAH,SAJ,SAU,SAV,SAW,R9,R10,
000       9R11,R12,R13,R14,R15,R16,DEL4,DEL6,DEL7,DEL8,DEL9,DEL10,DEL11,DEL12
000       1,GAPC,GAPD,SAB,SAD,G1,DEL1,DEL2,SAI,DEL3,DEL5,H4,H5
000       2,K21,K22,K23,K24,K25,K26,K27,K28,K21L,K22L,K23L,K24L,K25L,K26L,
000       3K27L,K28L,CONS10,CONS11,CONS12,CONS16,CONS17,CONS18,KT24,KT35
000       4,PTIME,KP24,KP35,KGIB,KBOM,TL,FM1,KF,CF,KC,CC
000        REAL K1,K2,K3,K4,K5,K6,K7,K8,K9,K10,K11,K12,K13,K14,K15,K16,K17,
000       1K18,K19,K20,K1L,K2L,K7L,K8L,K13L,K14L,K15L,K16L,K17L,K18L,K19L,K20
000       2L,K13S,K14S,K15S,K16S,K17S,K18S,K19S,K20S
000       3,K21,K22,K23,K24,K25,K26,K27,K28,K21L,K22L,K23L,K24L,K25L,K26L,
000       4K27L,K28L,KT24,KT35,KP24,KP35,KGIB,KBOM,KF,KC
000        DIMENSION DD(33),VV(33),AA(33),F(66),Y(66),AC(33),YC(33)
000   C    THE FOLLWOING DATA IS FOR A 70-TON BOX CAR
000        CIX1=16650000.
000        CIY1=1288800.
000        CIZ1=16416000.
000        BIX2=2640.
000        BIY2=12000.
000        BIZ2=12000.
000        TIX4=120000.
000        TIY4=19200.
000        TIZ4=120000.
000        B=74.4
```

```
      C=7.2
      D=474.
      E=6.5
      F1=14.25
      R1=72.5
      R2=4.5
      R3=4.5
      R4=9.5
      R5=R4
      H2=39.
      H3=39.
      H4=39.
      H5=39.
      G1=25.
      D1=237.
      D2=6.
      D3=6.
      D4=6.
      D5=6.
      B1=68.004
      GIB=0.375
      GAP=.3273273
      RL=468.
C     SPRING STIFFNESS
      K1=666000.
      K2=K1
      K7=K1
      K8=K1
      K5=666000.
      K6=666000.
      K11=666000.
      K12=666000.
      K13=10420.
      K14=K13
      K15=K13
      K16=K13
      K17=K13
      K18=K13
      K19=K13
      K20=K13
      K13L=4425.
      K14L=4425.
      K15L=4425.
      K16L=4425.
      K17L=4425.
      K18L=4425.
      K19L=4425.
      K20L=4425.
      KGIB=666000.*2
      KBOM=666000.
      K21=105000.
      K22=K21
      K23=K21
      K24=K21
      K25=K21
      K26=K21
      K27=K21
```

```
000227    000          K28=K21
000228    000          K21L=K21*2./3.
000229    000          K22L=K21L
000230    000          K23L=K21L
000231    000          K24L=K21L
000232    000          K25L=K21L
000233    000          K26L=K21L
000234    000          K27L=K21L
000235    000          K28L=K21L
000236    000          KT24=K13L*39**2
000237    000          KT35=KT24
000238    000          KP24=4200000.
000239    000          KP35=4200000.
000240    000          KF=200.
000241    000          KC=0.
000242    000     C    DEFINE OTHER PARAMETERS:
000243    000          CF=18.
000244    000          CC=0.
000245    000          FM1=150./386.4
000246    000          CM1=206000./386.4
000247    000          BM2=1150./(32.2*12.)
000248    000          TM4=7300./(32.2*12.)
000249    000          BIX3=BIX2
000250    000          BIY3=BIY2
000251    000          BIZ3=BIZ2
000252    000          TIX5=TIX4
000253    000          TIY5=TIY4
000254    000          TIZ5=TIZ4
000255    000          BM3=BM2
000256    000          TM5=TM4
000257    000          SPEED=60.0
000258    000          T=0.
000259    000          DT=0.00025
000260    000          TMAX=3.0003
000261    000          PT=400.
000262    000          PI=ACOS(-1.)
000263    000          OMGA=PI*SPEED*17.6/RL
000264    000          PHI1=PI/2.0
000265    000          PHI2=PI*B1/RL
000266    000          PHI3=PI*D/RL
000267    000          PHI4=PHI1+PHI3
000268    000          PHI5=PHI2+PHI3
000269    000          TL=3.6875
000270    000          G=32.2*12.
000271    000          GAPC=GAP
000272    000          GAPD=GAP
000273    000          S1=2.
000274    000          N=66
000275    000          L=1
000276    000          GF=386.4
000277    000          DF=57.29528
000278    000          WRITE (6,71) SPEED
000279    000          WRITE (6,72) DT
000280    000          WRITE (6,70) TMAX
000281    000          WRITE (6,73) PT
000282    000          WRITE (6,75) K1
000283    000          WRITE (6,77) K5
```

```
000          WRITE (6,78) K13
000          WRITE (6,79) K13L
000          WRITE (6,81) K21
000          WRITE (6,82) K21L
000          WRITE (6,67) CIX1
000          WRITE (6,68) CIY1
000          WRITE (6,69) CIZ1
000          WRITE (6,83)
000          WRITE (6,621)
000       70 FORMAT (//,10X,'MAX.SIMULATION TIME (SEC.) =',F10.4)
000       71 FORMAT (//,10X,'CAR SPEED IN MPH=',F5.1)
000       72 FORMAT (//,10X,'ITERATION TIME STEP SIZE (SEC.) =',F10.5)
000       73 FORMAT (//,10X,'ITERATION/PRINT INTERVAL=',F7.1)
000       67 FORMAT (//,10X,'CAR BODY MOMENT OF INERTIA ABOUT LAT. AXIS',F12.1)
000       68 FORMAT (/,10X,'CAR BODY MOMENT OF INERTIA ABOUT LONG. AXIS',F12.1)
000       69 FORMAT (/,10X,'CAR BODY MOMENT OF INERTIA ABOUT VERT. AXIS',F12.1)
000       75 FORMAT (//,10X,'CENTER PLATE STIFFNESS=',F10.1)
000       77 FORMAT (//,10X,'SIDE BEARING STIFFNESS=',F10.1)
000       78 FORMAT (//,10X, 'SUSPENSION SPRING VERT. STIFFNESS (8 IN TOTAL)=',
000         1F10.1)
000       79 FORMAT (//,10X,'SUSPENSION SPRING LAT. STIFFNESS =',F10.1)
000       81 FORMAT (//,10X,'VERT. TRACK STIFFNESS=',F10.1)
000       82 FORMAT (//,10X,'LAT. TRACK STIFFNESS=',F10.1)
000       83 FORMAT (///,10X,'***  ALL STIFFNESSES IN UNITS OF LB/IN.  ***')
000      621 FORMAT (///,10X,'*** ALL REACTIONS AND WHEEL LOADS IN 1000LB ***')
000          WRITE (6,284)
000      284 FORMAT (///,10X,'*** ALL INERTIA IN LB-IN-SEC**2 ***')
000          WRITE (6,198)
000      198 FORMAT (1H1,10X,'CAR BODY ACCELERATION (G OR DEG./SEC**2)',
000         123X,'CAR BODY DISPLACEMENT (INCH OR DEG.)')
000          WRITE (6,998)
000      998 FORMAT (11X,'*********************',42X,'*********************')
000    C     INITIALIZE THE VARIABLES
000          DO 1 I=1,33
000          DD(I)=0.
000          VV(I)=0.
000        1 CONTINUE
000    C     THE CAR BODY IS INITIALLY ASSIGNED A VERTICAL DISPLACEMENT
000    C     DOWNWARDS DUE TO ITS OWN WT. COMPRESSING ONTO THE CENTER PLATE
000    C     SPRINGS.
000    C     SIMILARLY, THE FRONT AND REAR BOLSTERS ARE ASSIGNED A VERTICAL
000    C     DISPLACEMENT DOWNWARDS DUE TO THE TOTAL WT. OF THE CAR BODY AND
000    C     THE TWO BOLSTERS COMPRESSING ON THE EIGHT SUSPENSION SPRINGS.
000          DD(3)=-2.3728333
000          DD(9)=-2.2894713
000          DD(21)=-0.2228929
000          DD(33)=-3.1228333
000          DD(15)=DD(9)
000          DD(27)=DD(21)
000          JT=0
000    C     ASSIGN DISPLACEMENTS AND VELOCITIES INTO 'Y' TERMS AS FOLLOWS::
000          DO 301 I=1,33
000          Y(I)=DD(I)
000          Y(I+33)=VV(I)
000      301 CONTINUE
000    C     SOLUTION OF 30 EQUATIONS OF MOTION FOR ACCELERATIONS
000      842 CALL ACCELN(DD,VV,AA,N)
```

```
000  C      CALL RUNG FOR NUMERICAL INTEGRATION.
000     212 CALL RUNG(N,F,T,Y,DT,JT)
000         IF (JT) 9,10,9
000  C      LIST OF ALL THE DERIVATIVES
000       9 DO 302 I=1,33
000         F(I)=Y(I+33)
000     302 CONTINUE
000         DO 303 I=34,66
000         F(I)=AA(I-33)
000     303 CONTINUE
000         GO TO 212
000      10 IF(T-TMAX)999,17,17
000     999 PTIME=L*PI*DT
000         IF (T-PTIME) 53,54,54
000      54 DO 529 I=1,3
000         AC(I)=AA(I)/GF
000     529 YC(I)=Y(I)
000         DO 240 I=4,6
000         AC(I)=AA(I)*DF
000     240 YC(I)=Y(I)*DF
000         DO 530 I=7,9
000         AC(I)=AA(I)/GF
000     530 YC(I)=Y(I)
000         DO 241 I=10,12
000         AC(I)=AA(I)*DF
000     241 YC(I)=Y(I)*DF
000         DO 531 I=13,15
000         AC(I)=AA(I)/GF
000     531 YC(I)=Y(I)
000         DO 242 I=16,18
000         AC(I)=AA(I)*DF
000     242 YC(I)=Y(I)*DF
000         DO 532 I=19,21
000         AC(I)=AA(I)/GF
000     532 YC(I)=Y(I)
000         DO 243 I=22,24
000         AC(I)=AA(I)*DF
000     243 YC(I)=Y(I)*DF
000         DO 533 I=25,27
000         AC(I)=AA(I)/GF
000     533 YC(I)=Y(I)
000         DO 244 I=28,30
000         AC(I)=AA(I)*DF
000     244 YC(I)=Y(I)*DF
000         DO 534 I=31,33
000         AC(I)=AA(I)/GF
000     534 YC(I)=Y(I)
000         WRITE (6,397) T
000     397 FORMAT (//,10X,'T=',F10.5)
000         WRITE (6,398) (AC(I),I=1,30)
000         WRITE (6,398) (AC(I),I=31,33)
000         WRITE (6,398) (YC(I),I=1,30)
000         WRITE (6,398) (YC(I),I=31,33)
000     398 FORMAT (/,        )
000     555 L=L+1
000         REASSIGN Y(I) TO DD(I) & VV(I) FOR THE NEXT TIME STEP.
000      53 DO 551 I=1,33
```

```
      DD(I)=Y(I)
  551 VV(I)=Y(I+33)
      GO TO 842
   17 STOP
      E N D
C
C
C     SUBROUTINE DELGAP
C
C
C     ******************************************************************
C     SUBROUTINE DELGAP CHECKS THE SIDE BEARING CLEAREANCE AT EACH TIME
C     STEP. IF THE CLEAREANCE IS EQUAL TO ZERO THEN THE SIDE BEARING
C     COMES INTO THE ACTION
C     ******************************************************************
C
      SUBROUTINE DELGAP (GAPC,GAPD,SAA,SAB,SAC,SAD,G1,DEL1,DEL2,SAI,
     1DEL3,DEL4,T,PTIME,GAP,K5,K6,K11,K12)
C
      REAL K5,K6,K11,K12
C
      GAP1=(SAA+SAI-G1*SAB)
      IF(GAPC+GAP1)1,2,2
    1 DEL1=1.0
      GO TO 4
    2 DEL1=0.
      GAP2=(SAA+SAI+G1*SAB)
      IF(GAPD+GAP2)3,4,4
    3 DEL2=1.0
      GO TO 12
    4 DEL2=0.
   12 GAP3=(SAC-SAI-G1*SAD)
      IF(GAPC+GAP3)9,6,6
    9 DEL3=1.0
      GO TO 8
    6 DEL3=0.
      GAP4=(SAC-SAI+G1*SAD)
      IF(GAPD+GAP4)7,8,8
    7 DEL4=1.0
      GO TO 14
    8 DEL4=0.
   14 IF (T-PTIME) 53,54,54
   54 SF1=K5*DEL1*(SAA-G1*SAB+GAP+SAI)
      SF2=K6*DEL2*(SAA+G1*SAB+GAP+SAI)
      SF3=K11*DEL3*(SAC-G1*SAD+GAP-SAI)
      SF4=K12*DEL4*(SAC+G1*SAD+GAP-SAI)
      RXN=-(SF1+SF2+SF3+SF4) /1000.
      WRITE (6,16) RXN
   16 FORMAT (/,10X,'SIDEBEARING VERT. RXN=',F7.1)
   53 RETURN
      END
C     ******************************************************************
C
C
C     SUBROUTINE RUNG
C
C
```

```
000  C      THIS SUBROUTINE INTEGRATES EQUATIONS OF MOTION BY RUNGE-KUTTA
000  C      INTEGRATION METHOD
000  C      *****************************************************************
000  C
000         SUBROUTINE RUNG(N,F,T,Y,DT,JT)
000  C
000         DIMENSION SAVEY (66),PHI(66),Y(66),F(66)
000  C
000         JT=JT+1
000         GO TO(1,2,3,4,5),JT
000       1 RETURN
000       2 DO 6 J=1,N
000         SAVEY(J)=Y(J)
000         PHI(J)=F(J)
000       6 Y(J)=SAVEY(J)+.5*DT*F(J)
000         T=T+.5*DT
000         RETURN
000       3 DO 7 J=1,N
000         PHI(J)=PHI(J)+2.0*F(J)
000       7 Y(J)=SAVEY(J)+.5*DT*F(J)
000         RETURN
000       4 DO 8 J=1,N
000         PHI(J)=PHI(J)+2.0*F(J)
000       8 Y(J)=SAVEY(J)+DT*F(J)
000         T=T+.5*DT
000         RETURN
000       5 DO 9 J=1,N
000       9 Y(J)=SAVEY(J)+(PHI(J)+F(J))*DT/6.
000         JT=0
000         RETURN
000         END
000  C
000  C
000  C      SUBROUTINE SPRING
000  C
000  C
000  C      *****************************************************************
000  C      SUBROUTINE SPRING CHECKS FOR THE BOTTOMING OF SUSPENSION SPRINGS.
000  C      IF THE SPRINGS BOTTOM OUT, A SPRING OF MUCH GREATER STIFFNESS IS
000  C      ADDED IN PARALLEL TO THE SUSPENSION SPRING GROUPS.
000  C      *****************************************************************
000  C
000         SUBROUTINE SPRING (SAM,SAO,SBA,SBB,H2,H3,TL,T,PTIME,DEL9,DEL10,
000        1DEL11,DEL12)
000  C
000         IF (SAM-H2*SAO+TL) 1,2,2
000       1 DEL9=1.
000         GO TO 11
000       2 DEL9=0.
000      11 IF (SAM+H2*SAO+TL) 3,4,4
000       3 DEL10=1.
000         GO TO 12
000       4 DEL10=0.
000      12 IF (SBA-H3*SBB+TL) 5,6,6
000       5 DEL11=1.
000         GO TO 13
000       6 DEL11=0.
```

```
000    13 IF (SBA+H3*SBB+TL) 7,8,8
000     7 DEL12=1.
000       GO TO 14
000     8 DEL12=0.
000    14 IF (T-PTIME) 53,54,54
000    54 WRITE (6,16) DEL9,DEL10,DEL11,DEL12
000    16 FORMAT (/,10X,'DEL9=',F4.1,10X,'DEL10=',F4.1,10X,'DEL11=',F4.1,10X
000      1,'DEL12=',F4.1)
000    53 RETURN
000       END
000  C    ****************************************************************
000  C    SUBROUTINE DELGIB CHECKS THE GIB (LAT. CLEAREANCE BETWEEN THE
000  C    BOLSTER AND SIDEFRAME ) EACH TIME IT IS CALLED.  IF GIB IS ZERO
000  C    ,THE GIB SPRINGS ARE ADDED IN PARALLEL TO THE LAT. SPRING GROUPS.
000  C
000  C
000  C    SUBROUTINE DELGIB
000  C
000  C
000  C    ****************************************************************
000  C
000       SUBROUTINE DELGIB (SAF,SAU,GIB,DEL5,DEL6,DEL7,DEL8,T,PTIME)
000  C
000       IF (SAF-GIB) 1,1,2
000     2 DEL5=1.
000       GO TO 4
000     1 DEL5=0.
000       IF (SAF+GIB) 3,4,4
000     3 DEL6=1.
000       GO TO 9
000     4 DEL6=0.
000     9 IF (SAU-GIB) 5,5,6
000     6 DEL7=1.
000       GO TO 8
000     5 DEL7=0.
000       IF (SAU+GIB)7,8,8
000     7 DEL8=1.
000       GO TO 14
000     8 DEL8=0.
000    14 IF (T-PTIME) 53,54,54
000    54 WRITE (6,16) DEL5,DEL6,DEL7,DEL8
000    16 FORMAT (/,10X,'DEL5=',F4.1,10X,'DEL6=',F4.1,10X,'DEL7=',F4.1,10X,
000      1'DEL8=',F4.1)
000    53 RETURN
000       END
000  C
000  C
000  C    SUBROUTINE SGNFUN
000  C
000  C
000  C    ****************************************************************
000  C    SUBROUTINE SGNFUN COMPUTES THE RELATIVE VEL. BETWEEN THE
000  C    BOLSTER AND THE SIDEFRAME COLUMN AND ASSIGNS THE PROPER SIGN
000  C    TO THE DAMPING FORCE SUCH THAT THE DAMPING FORCE ALWAYS
000  C    OPPOSES THE MOTION.
000  C    ****************************************************************
000  C
```

```
000   C
000         SUBROUTINE SGNFUN (SCA,SCB,SCC,SCD,SAN3,SAN4,SAN5,SAN6,
000        1SBD7,SBD8,SBD9,SBD0)
000   C
000   C
000         IF (SCA) 1,2,3
000       1 SCA=-1.
000         GO TO 2
000       3 SCA=1.
000       2 IF (SCB) 4,5,6
000       4 SCB=-1.
000         GO TO 5
000       6 SCB=1.
000       5 IF (SCC) 7,8,9
000       7 SCC=-1.
000         GO TO 8
000       9 SCC=1.
000       8 IF (SCD) 10,11,12
000      10 SCD=-1.
000         GO TO 11
000      12 SCD=1.
000      11 IF (SAN3) 13,14,15
000      13 SAN3=-1.
000         GO TO 14
000      15 SAN3=1.
000      14 IF (SAN4) 16,17,18
000      16 SAN4=-1.
000         GO TO 17
000      18 SAN4=1.
000      17 IF (SAN5) 19,20,21
000      19 SAN5=-1.
000         GO TO 20
000      21 SAN5=1.
000      20 IF (SAN6) 22,23,24
000      22 SAN6=-1.
000         GO TO 23
000      24 SAN6=1.
000      23 IF (SBD7) 25,26,27
000      25 SBD7=-1.
000         GO TO 26
000      27 SBD7=1.
000      26 IF (SBD8) 28,29,30
000      28 SBD8=-1.
000         GO TO 29
000      30 SBD8=1.
000      29 IF (SBD9) 31,32,33
000      31 SBD9=-1.
000         GO TO 32
000      33 SBD9=1.
000      32 IF (SBD0) 34,35,36
000      34 SBD0=-1.
000         GO TO 35
000      36 SBD0=1.
000      35 RETURN
000         END
000   C
000   C
```

```
000  C      SUBROUTINE CPLATE
000  C
000  C
000  C      *****************************************************************
000  C      SUBROUTINE CPLATE COMPUTES THE VERT. REACTIONS OF THE FRONT AND
000  C      REAR CENTER PLATES.
000  C      *****************************************************************
000  C
000         SUBROUTINE CPLATE (SAA, SAB,SAI,SAC,SAD,E,K1,K2,K7,K8)
000  C
000         REAL K1,K2,K7,K8
000  C
000         SP12=K1*(SAA-E*SAB+SAI)+K2*(SAA+E*SAB+SAI)
000         SP78= K7*(SAC-E*SAD-SAI)+K8*(SAC+E*SAD-SAI)
000         RXNFT=-SP12/1000.
000         RXNRR=-SP78/1000.
000         IF (RXNFT) 1,1,2
000       1 RXNFT=0.
000       2 IF (RXNRR) 3,3,4
000       3 RXNRR=0.
000       4 WRITE (6,16) RXNFT,RXNRR
000      16 FORMAT (/,10X,'FRONT CENTER PLATE VERT. RXN=',F7.1,/,10X,
000        1'REAR CENTER PLATE VERT. RXN=',F7.1)
000         RETURN
000         END
000  C
000  C
000  C      SUBROUTINE CAL
000  C
000  C
000  C      *****************************************************************
000  C      SUBROUTINE CAL COMPUTES THE FRONT BOLSTER LAT. RXN AND WHEEL LOADS.
000  C      *****************************************************************
000  C
000         SUBROUTINE CAL (K13L,K14L,K15L,K16L,KGIB,K21,K23,K26,K28,
000        1SCE,SCF,SAF,SBM,SBN,SBQ,SBR,GIB,DEL5,DEL6,V1,V3,V6,V8,DD)
000  C
000  C      **************************
000  C      FRONT BOLSTER LATERAL RXN.
000  C      **************************
000  C
000         DIMENSION DD(33)
000         REAL K13L,K14L,K15L,K16L,K21,K23,K26,K28,KGIB
000         SPFORL=(K13L+K14L)*SCE+(K15L+K16L)*SCF
000        1+KGIB*DEL5*(SAF-GIB)+KGIB*DEL6*(SAF+GIB)
000         RXNL=-SPFORL/1000.
000         WRITE (6,1)     RXNL
000       1 FORMAT (/,10X,'FRONT BOLSTER LAT. RXN=',F7.1)
000  C
000  C      ***********
000  C      WHEEL LOADS
000  C      ***********
000  C
000         SPFR21=K21*(DD(21)-SBM+SBN-V1)
000         SPFR23=K23*(DD(21)-SBM-SBN-V3)
000         WHLDLF=-(SPFR21+SPFR23)/1000.
000         SPFR26=K26*(DD(27)+SBQ+SBR-V6)
```

```
000           SPFR28=K28*(DD(27)+SBQ-SBR-V8)
0Q0           WHLDRR=-(SPFR26+SPFR28)/1000.
000           IF (WHLDLF) 3,3,4
000         3 WHLDLF=0.
000         4 IF (WHLDRR) 5,5,6
000         5 WHLDRR=0.
000         6 WRITE (6,2) WHLDLF,WHLDRR
000         2 FORMAT (/,10X,'WHEEL LOAD LEFT FRONT =',F7.1,/,10X,
000          1'WHEEL LOAD RIGHT REAR =',F7.1)
000           RETURN
000           END
000    C      ****************************************************************
000    C      SUBROUTINE T5000 COMPUTES THE VERT. ACCELN AND THE LAT. ACCELN
000    C      ON THE ROOF AND FLOOR OF THE CAR BODY AT BOTH ENDS.
000    C      ****************************************************************
000    C
000    C
000    C
000    C      SUBROUTINE T5000
000    C
000    C
000           SUBROUTINE T5000 (AA)
000    C
000           DIMENSION AA(33)
000           GF=386.4
000           XM=61.
000           XN=70.
000           XL=331.2
000    C
000           VAAN=(AA(3)+XL*AA(5))/GF
000           VABN=(AA(3)-XL*AA(5))/GF
000           ARAN=(AA(1)+XM*AA(4)-XL*AA(6))/GF
000           ARBN=(AA(1)+XM*AA(4)+XL*AA(6))/GF
000           AFAN=(AA(1)-XN*AA(4)-XL*AA(6))/GF
000           AFBN=(AA(1)-XN*AA(4)+XL*AA(6))/GF
000           WRITE (6,16)
000        16 FORMAT (//,43X,'*** A END ***',7X,'*** B END ***')
000           WRITE (6,17) VAAN,VABN
000        17 FORMAT (/,10X,'VERT. ACCELN',20X,F10.5,10X,F10.5)
000           WRITE (6,18)  ARAN, ARBN
000        18 FORMAT (/,10X,'LAT. ROOF ACCELN',16X,F10.5,10X,F10.5)
000           WRITE (6,19)  AFAN, AFBN
000        19 FORMAT (/,10X,'LAT. FLOOR ACCELN',15X,F10.5,10X,F10.5)
000           RETURN
000           END
000    C
000    C
000    C      SUBROUTINE ACCELN
000    C
000    C
000    C      ****************************************************************
000    C      SUBROUTINE ACCELN
000    C      THIS SUBROUTINE COMPUTES THE ACCELERATION VALUES
000    C      ****************************************************************
000    C
000           SUBROUTINE ACCELN(DD,VV,AA,N)
0Q0    C
```

```
000          COMMON/ACCELN/
000          1SI,OMGA,D3,PHI1,PHI2,PHI3,PHI4,PHI5,DT,T,D2,D4,K6,D5,K1L,
000          1K2L,CM1,K7L,K8L,E,K3,F1,D1,K5,GAP,BM2,K9,K10,K11,K12,G,CIX1,
000          2CIY1,CIZ1,K13L,K4,K15L,K16L,C14L,K13S,GIB,K14S,K15S,K16S,C13L,
000          3K14L,C15L,C16L,C13,C14,C15,C16,BIX2,BIY2,BIZ2,K17L,K18L,K19L,
000          4BM3,K17S,K18S,K19S,K20S,C17L,C18L,C19L,C20L,C17,C18,C19,C20,
000          5BIX3,BIY3,BIZ3,K20L,TM4,TIX4,TIZ4,TIY4,TM5,TIX5,TIY5,TIZ5,TMAX
000          6,R1,R2,R3,R4,R5,K13,K14,K15,K16,K17,K18,K19,K20,SAM,SAO,SAQ,SBA,
000          7SBB,SBC,V1,V2,V3,V4,V5,V6,V7,V8,H2,H3,K1,K2,K7,K8,SAA,SAC,GIBC,
000          8GIBD,GIBE,GIBF,GIBG,GIBH,GIBJ,GIBK,SAG,SAH,SAJ,SAU,SAV,SAW,R9,R10,
000          9R11,R12,R13,R14,R15,R16,DEL4,DEL6,DEL7,DEL8,DEL9,DEL10,DEL11,DEL12
000          1,GAPC,GAPD,SAB,SAD,G1,DEL1,DEL2,SAI,DEL3,DEL5,H4,H5
000          2,K21,K22,K23,K24,K25,K26,K27,K28,K21L,K22L,K23L,K24L,K25L,K26L,
000          3K27L,K28L,CONS10,CONS11,CONS12,CONS16,CONS17,CONS18,KT24,KT35
000          4,PTIME,KP24,KP35,KGIB,KBOM,TL,FM1,KF,CF,KC,CC
000           REAL K1,K2,K3,K4,K5,K6,K7,K8,K9,K10,K11,K12,K13,K14,K15,K16,K17,
000          1K18,K19,K20,K1L,K2L,K7L,K8L,K13L,K14L,K15L,K16L,K17L,K18L,K19L,K20
000          2L,K13S,K14S,K15S,K16S,K17S,K18S,K19S,K20S
000          3,K21,K22,K23,K24,K25,K26,K27,K28,K21L,K22L,K23L,K24L,K25L,K26L,
000          4K27L,K28L,KT24,KT35,KP24,KP35,KGIB,KBOM,KF,KC
000           DIMENSION A1(4),A2(3),A3(3),A4(3),A5(3),AA(33),VV(33),DD(33),IR(3)
000          1,JC(3),AM1(4,5),AM2(3,4),AM3(3,4),AM4(3,4),AM5(3,4),IRR(4),JCC(4)
000     C
000           IF (S-1.5)   321,432,432
000      321 S=SI*T
000          IF(S-1.5) 542,542,432
000      432 S=1.5
000      542 IF(T-1.7728)543,543,1
000      543 V1=S*ABS(SIN(OMGA*T))
000          V2=V1
000          V3=S*ABS(SIN(OMGA*T-PHI2))
000          V4=V3
000          V5=S*ABS(SIN(OMGA*T-PHI3))
000          V6=V5
000          V7=S*ABS(SIN(OMGA*T-PHI5))
000          V8=V7
000          R9=0.
000          R11=0.
000          R13=0.
000          R15=0.
000          R10=0.
000          R12=0.
000          R14=0.
000          R16=0.
000     C
000          GO TO 2
000        1 V1=0.
000          V2=0.
000          V3=0.
000          V4=0.
000          V5=0.
000          V6=0.
000          V7=0.
000          V8=0.
000          R9=0.
000          R10=0.
000          R11=0.
```

```
000              R12=0.
000              R13=0.
000              R14=0.
000              R15=0.
000              R16=0.
000            2 A=0.
000              YS=A*SIN(OMGA*T)
000              DYS=A*OMGA*COS(OMGA*T)
000              R=16.5
000              H=28.2
000              D=34.
000              XM1=48.
000              XL1=300.
000              C13=261.
000              C14=C13
000              C15=C13
000              C16=C13
000              C17=C13
000              C18=C13
000              C19=C13
000              C20=C13
000              C3=0.
000              C4=0.
000              C9=0.
000              C10=0.
000              C13L=C13
000              C14L=C13L
000              C15L=C13L
000              C16L=C13L
000              C17L=C13L
000              C18L=C13L
000              C19L=C13L
000              C20L=C13L
000    C
000    C         GROUP VARIABLES INTO SYMBOLS
000    C
000              SAA=DD(3)-DD(9)
000              SAB=DD(4)-DD(10)
000              SAC=DD(3)-DD(15)
000              SAD=DD(4)-DD(16)
000              SAE=D1*VV(5)
000              SAH=D2*DD(12)-D4*DD(24)
000              SAI=D1*DD(5)
000              SAJ=R2*DD(10)+R4*DD(22)
000              SAK=D2*VV(12)-D4*VV(24)
000              SAL=R2*VV(10)+R4*VV(22)
000              SAM=DD(9)-DD(21)
000              SAN=VV(9)-VV(21)
000              SAO=DD(10)-DD(22)
000              SAP=VV(10)-VV(22)
000              SAQ=D2*DD(11)-D4*DD(23)
000              SAR=D2*VV(11)-D4*VV(23)
000              SAS=DD(4)-DD(21)
000              SAT=D2*DD(11)-D4*DD(23)
000              SAV=D3*DD(18)-D5*DD(30)
000              SAW=R3*DD(16)+R5*DD(28)
000              SAY=D3*VV(18)-D5*VV(30)
```

```
000854    000          SAZ=R3*VV(16)+R5*VV(28)
000855    000          SBA=DD(15)-DD(27)
000856    000          SBB=DD(16)-DD(28)
000857    000          SBC=D3*DD(17)-D5*DD(29)
000858    000          SBD=VV(15)-VV(27)
000859    000          SBE=VV(16)-VV(28)
000860    000          SBF=D3*VV(17)-D5*VV(29)
000861    000          SBM=H*DD(22)
000862    000          SBN=D*DD(23)
000863    000          SBO=D*DD(24)
000864    000          SBP=R*DD(22)
000865    000          SBQ=H*DD(28)
000866    000          SBR=D*DD(29)
000867    000          SBS=D*DD(30)
000868    000          SBT=R*DD(28)
000869    000          SAF=DD(1)-R1*DD(4)-D1*DD(6)-DD(19)
000870    000          SAU=DD(1)-R1*DD(4)+D1*DD(6)-DD(25)
000871    000          SAX=VV(1)-R1*VV(4)+D1*VV(6)-VV(25)
000872    000          SBJ=VV(1)-R1*VV(4)-D1*VV(6)-VV(19)
000873    000          SCA=SBJ-SAK-SAL
000874    000          SCB=SBJ+SAK-SAL
000875    000          SCC=SAX-SAY-SAZ
000876    000          SCD=SAX+SAY-SAZ
000877    000          SCE=SAF-SAH-SAJ
000878    000          SCF=SAF+SAH-SAJ
000879    000          SCG=SAU-SAV-SAW
000880    000          SCH=SAU+SAV-SAW
000881    000          SAN3=SAN-H2*SAP+SAR
000882    000          SAN4=SAN+H2*SAP+SAR
000883    000          SAN5=SAN-H2*SAP-SAR
000884    000          SAN6=SAN+H2*SAP-SAR
000885    000          SBD7=SBD-H3*SBE+SBF
000886    000          SBD8=SBD+H3*SBE+SBF
000887    000          SBD9=SBD-H3*SBE-SBF
000888    000          SBD0=SBD+H3*SBE-SBF
000889    000    C
000890    000    C     CHECK SIDE BEARING CLEARANCE
000891    000    C
000892    000          CALL       DELGAP (GAPC,GAPD,SAA,SAB,SAC,SAD,G1,DEL1,DEL2,SAI,
000893    000         1DEL3,DEL4,T,PTIME,GAP,K5,K6,K11,K12)
000894    000    C
000895    000    C     CHECK GIB CLEARANCE
000896    000    C
000897    000          CALL       DELGIB (SAF,SAU,GIB,DEL5,DEL6,DEL7,DEL8,T,PTIME)
000898    000    C
000899    000    C     CHECK SPRING STIFFNESS
000900    000    C
000901    000          CALL       SPRING (SAM,SAO,SBA,SBB,H2,H3,TL,T,PTIME,DEL9,DEL10,
000902    000         1DEL11,DEL12)
000903    000          IF (T-PTIME) 589,599,599
000904    000    C
000905    000    C     AT PRINTING TIME,COMPUTE CENTER PLATE LOADINGS
000906    000    C
000907    000      599 CALL       CPLATE (SAA, SAB,SAI,SAC,SAD,E,K1,K2,K7,K8)
000908    000    C
000909    000    C     CALCULATE THE BOLSTER LATER LOADINGS AND WHEEL LOADINGS
000910    000    C
```

```
000           CALL       CAL (K13L,K14L,K15L,K16L,KGIB,K21,K23,K26,K28,
000          1SCE,SCF,SAF,SBM,SBH,SBQ,SBR,GIB,DEL5,DEL6,V1,V3,V6,V8,DD)
000     C
000     C     SOLVE FOR THE ACCELERATION VALUES FROM THE EQN. OF MOTION
000     C
000       589 CON1A=((K13L+K14L)*SCE+(K15L+K16L)*SCF+(K17L+K18L)*SCG
000          1+(K19L+K20L)*SCH)/-(CM1+BM2+BM3)
000           CON1B=((C13L+C14L)*SCA+(C15L+C16L)*SCB+(C17L+C18L)*SCC
000          1+(C19L+C20L)*SCD)/-(CM1+BM2+BM3)
000           CON1C=2*BM2*R1*DD(4)*VV(4)**2/-(CM1+BM2+BM3)
000           CON1D=KGIB*(DEL5*(SAF-GIB)+DEL6*(SAF+GIB)+DEL7*(SAU-GIB)+DEL8*
000          1(SAU+GIB))/-(CM1+BM2+BM3)
000           CON1E=(-KF*(DD(31)-DD(1))-CF*(VV(31)-VV(1)))/-(CM1+BM2+BM3)
000           CONS1=CON1A+CON1B+CON1C+CON1D+CON1E
000           CONS2=(KC*(YS-DD(2))+CC*(DYS-VV(2)))/CM1
000           CON3A   =(K1*(SAA-E*SAB+SAI) +K2*(SAA+E*SAB+SAI) +K3*(SAA-F1*SAB
000          1+SAI) +K4*(SAA+F1*SAB+SAI) +K5*DEL1*(SAA-G1* SAB+GAP +SAI)
000          2+K6*DEL2*(SAA+G1* SAB+GAP +SAI)
000          3+K7*(SAC-E*SAD-SAI) +K8*(SAC+E*SAD-SAI) +K9*(SAC-F1*SAD-SAI))/-CM1
000           CON3B=(
000          1 K10*(SAC+F1*SAD-SAI) +K11*DEL3*(SAC-G1* SAD+GAP -SAI)
000          5+K12*DEL4*(SAC+G1* SAD+GAP -SAI) +C3*(VV(3)-VV(9)-F1*(VV(4)-
000          6VV(10))+D1*VV(5)))/-CM1
000           CON3C=          (C4*(VV(3)-VV(9)+F1*(VV(4)-VV(10))+D1*VV(5))
000          7+C9*(VV(3)-VV(15)-F1*(VV(4)-VV(16))-D1*VV(5))+C10*(VV(3)-VV(15)
000          8+F1*(VV(4)-VV(16))-D1*VV(5)))/-CM1-G
000           CON3D=(-KF*(DD(33)-DD(3))-CF*(VV(33)-VV(3)))/-CM1
000           CONS3=CON3A+CON3B+CON3C+CON3D
000           CON4A=(CIX1*(VV(5)+VV(4)*DD(6))*VV(6)+CIX1*DD(6)*(-VV(5)*VV(4)
000          1*DD(4)
000          1-VV(5)*DD(6)*VV(6)+VV(4)*VV(6))-CIY1*DD(6)*VV(6)*(VV(4)-VV(5)*
000          2DD(6))+CIY1*(-VV(4)*DD(6)*VV(6)+VV(5)*DD(6)*DD(4)*VV(4)-VV(5)*VV(
000          26)))/-(CIX1*DD(6)**2+CIY1+2*BM2*R1**2)
000           CON4B=(
000          1 CIX1*VV(5)*DD(4)*(VV(5)+VV(4)*DD(6))-CIY1*VV(5)*DD(6)*DD(4)*
000          4(VV(4)-VV(5)*DD(6))-CIZ1*VV(5)*(VV(5)*DD(4)+VV(6))-K1*E*(SAA-E*
000          5SAB+SAI)+K2*E*(SAA+E*SAB+SAI)-K3*F1*(SAA-F1*SAB+SAI)+K4*F1*(SAA
000          6+F1*SAB+SAI))/-(CIX1*DD(6)**2+CIY1+2*BM2*R1**2)
000           CON4C=(-K5*DEL1*G1*(SAA-G1*SAB+GAP+SAI)+K6*DEL2*G1*(SAA+G1*
000          7SAB+GAP+SAI)-K7*E*(SAC-E*SAD-SAI)+K8*E*(SAC+E*SAD-SAI)-K9*F1*(
000          8SAC-F1*SAD-SAI)+K10*F1*(SAC+F1*SAD-SAI)-K11*DEL3*G1*(SAC-G1* SAD+
000          8GAP -SAI))/-(CIX1*DD(6)**2+CIY1+2*BM2*R1**2)
000           CON4D=K12*DEL4*G1*(SAC+G1*SAD+GAP-SAI)/-(CIX1*DD(6)**2+CIY1+
000          12*BM2*R1**2)
000           CON4E=R1*(-(K13L+K14L)*SCE-(K15L+K16L)*SCF-(K17L+K18L)*SCG
000          1-(K19L+K20L)*SCH)/-(CIX1*DD(6)**2+CIY1+2*BM2*R1**2)
000           CON4F=(-C3*F1*(VV(3)-VV(9)-F1*(VV(4)
000          4-VV(10))+SAE)+C4*F1*(VV(3)-VV(9)+F1*(VV(4)-VV(10))+SAE)
000          5-C9*F1*(VV(3)-VV(15)-F1*(VV(4)-VV(16))-SAE)+C10*F1*(VV(3)-VV(15)
000          6+F1*(VV(4)-VV(16))-SAE))/-(CIX1*DD(6)**2+CIY1+2*BM2*R1**2)
000           CON4G=R1*(-(C13L+C14L)*SCA-(C15L+C16L)*SCB-(C17L+C18L)*SCC
000          1-(C19L+C20L)*SCD)/-(CIX1*DD(6)**2+CIY1+2*BM2*R1**2)
000           CON4H=-2*BM2*R1*(DD(4)*VV(1)*VV(4)-2*R1*DD(4)*VV(4)**2)
000          1/-(CIX1*DD(6)**2+CIY1+2*BM2*R1**2)
000           CON4I=-KGIB*R1*(DEL5*(SAF-GIB)+DEL6*(SAF+GIB)+DEL7*(SAU-GIB)+DEL8
000          1*(SAU+GIB))/-(CIX1*DD(6)**2+CIY1+2*BM2*R1**2)
000           CONS4=CON4A+CON4B+CON4C+CON4D+CON4E+CON4F+CON4G+CON4H+CON4I
```

```
000968   000   CON5A  =(-CIX1*DD(4)*VV(4)*(VV(5)+VV(4)*DD(6))-CIX1*DD(6)*VV(6)*
000969   000  1(VV(5)+VV(4)*DD(6))+CIX1*(-VV(5)*DD(4)*VV(4)-VV(5)*DD(6)*VV(6)
000970   000  2+VV(4)*VV(6))+CIY1*DD(6)*DD(4)*VV(4)*(VV(4)-VV(5)*DD(6))-CIY1
000971   000  3*VV(6)*(VV(4)-VV(5)*DD(6)))/-(CIX1+CIY1*DD(6)**2+CIZ1*DD(4)**2)
000972   000   CON5B=                    (-CIY1*DD(6)*(-VV(4)*DD(6)*VV(6)+VV(5)*DD
000973   000  4(6)*DD(4)*VV(4)-VV(5)*VV(6))+CIZ1*VV(4)*(VV(5)*DD(4)+VV(6))
000974   000  5+CIZ1*DD(4)*VV(5)*VV(4)
000975   000  5+K1*D1*(SAA-E*SAB+SAI)+K2*D1*(SAA+E*SAB+SAI)
000976   000  6+K3*D1*(SAA-F1*SAB+SAI))/-(CIX1+CIY1*DD(6)**2+CIZ1*DD(4)**2)
000977   000   CON5C=                    (K4*D1*(SAA+F1*SAB+SAI)+K5*DEL1*D1*(SAA-
000978   000  7 G1*SAB+GAP +SAI)+K6*DEL2*D1*(SAA+ G1*SAB+GAP +SAI)-K7*D1*(SAC-E*S
000979   000  8AD-SAI)-K8*D1*(SAC+E*SAD-SAI)-K9*D1*(SAC-F1*SAD-SAI)-K10*D1*(SAC
000980   000  9+F1*SAD-SAI))/-(CIX1+CIY1*DD(6)**2+CIZ1*DD(4)**2)
000981   000   CON5D=      (-K11*DEL3*D1*(SAC- G1*SAD+GAP -SAI)-K12*DEL4*D1*(SAC
000982   000  1+ G1*SAD+GAP -SAI)+C3*D1*(VV(3)-VV(9)-F1*(VV(4)-VV(10))+SAE)+
000983   000  1C4*D1*(VV(3)-VV(9)+F1*(VV(4)-VV(10))+SAE)-C9*D1*(VV(3)-VV(15)-F1*
000984   000  2(VV(4)-VV(16))-SAE)-C10*D1*(VV(3)-VV(15)+F1*(VV(4)-VV(16))-SAE))
000985   000  3/-(CIX1+CIY1*DD(6)**2+CIZ1*DD(4)**2)
000986   000   CONS5=CON5A+CON5B+CON5C+CON5D
000987   000   CON6A  =(CIZ1*(VV(5)*VV(4))-CIX1*(VV(5)+VV(4)*DD(6))*(-VV(5)*DD(6)
000988   000  1+VV(4))-CIY1*(-VV(4)*DD(6)-VV(5))*(VV(4)-VV(5)*DD(6)))
000989   000  2/-(CIZ1+2*BM2*D1**2)
000990   000   CON6B=D1*(-(K13L+K14L)*SCE-(K15L+K16L)*SCF+(K17L+K18L)*SCG
000991   000  1+(K19L+K20L)*SCH)/-(CIZ1+2*BM2*D1**2)
000992   000   CON6C=D1*(-(C13L+C14L)*SCA-(C15L+C16L)*SCB+(C17L+C18L)*SCC
000993   000  1+(C19L+C20L)*SCD)/-(CIZ1+2*BM2*D1**2)
000994   000   CON6D=(-4*BM2*D1**2*DD(6)*VV(6)**2+2*BM2*D1**2*DD(6)*VV(6)**2)/
000995   000  1-(CIZ1+2*BM2*D1**2)
000996   000   CON6E=KGIB*D1*(-DEL5*(SAF-GIB)-DEL6*(SAF+GIB)+DEL7*(SAU-GIB)
000997   000  1+DEL8*(SAU+GIB))/-(CIZ1+2*BM2*D1**2)
000998   000   CONS6=CON6A+CON6B+CON6C+CON6D+CON6E
000999   000   CONS8=CONS2
001000   000   CON9A  =(-K1*(SAA-E*SAB+SAI)-K2*(SAA+E*SAB+SAI)-K3*(SAA-F1*SAB+SAI
001001   000  1)-K4*(SAA+F1*SAB+SAI)-K5*DEL1*(SAA- G1*SAB+GAP +SAI)-K6*DEL2*(SAA
001002   000  2+G1* SAB+GAP +SAI)+K13*(SAM-H2*SAO+SAQ    )+K14*(SAM+H2*SAO+SAQ    )
001003   000  3+K15*(SAM-H2*SAO-SAQ    ))/-BM2
001004   000   CON9B=                    (K16*(SAM+H2*SAO-SAQ    )+BM2*G-C3*(VV(3)
001005   000  4-VV(9)-F1*(VV(4)-VV(10))+D1*VV(5))-C4*(VV(3)-VV(9)+F1*(VV(4)-VV(10
001006   000  4))+D1*VV(5)))/-BM2
001007   000   CON9C=(C13*SAN3+C14*SAN4+C15*SAN5+C16*SAN6)/-BM2
001008   000   CON9D=KBOM*(DEL9*(SAM-H2*SAO+TL)+DEL10*(SAM+H2*SAO+TL))/-BM2
001009   000   CONS9=CON9A+CON9B+CON9C +CON9D
001010   000   CON10A=(BIX2*VV(12)*(VV(11)+VV(10)*DD(12))+BIX2*DD(12)*(-VV(11)*
001011   000  1DD(10)*VV(10)-VV(11)*DD(12)*VV(12)+VV(10)*VV(12))-BIY2*DD(12)*
001012   000  2VV(12)*(VV(10)-VV(11)*DD(12))+BIY2*(-VV(10)*DD(12)*VV(12)+VV(11)*
001013   000  3DD(12)*DD(10)*VV(10)-VV(11)*VV(12)))/-(BIX2*DD(12)**2+BIY2)
001014   000   CON10C=                   (BIX2*VV(11)*DD(10)*(VV(11)+
001015   000  4VV(10)*DD(12))-BIY2*VV(11)*DD(12)*DD(10)*(VV(10)-VV(11)*DD(12))-
001016   000  5BIZ2*VV(11)*(VV(11)*DD(10)+VV(12))+K1*E*(SAA-E*SAB+SAI)-K2*E*(SAA+
001017   000  6E*SAB+SAI))/-(BIX2*DD(12)**2+BIY2)
001018   000   CON10D=    (K3*F1*(SAA-F1*SAB+SAI)-K4*F1*(SAA+F1*SAB+SAI)+K5*DEL1*
001019   000  7G1*(SAA- G1*SAB+GAP +SAI)-K6*DEL2*G1*(SAA+G1*SAB+GAP+SAI)
001020   000  7-K13*H2*(SAM-H2*SAO+SAT    )+K14*H2*(SAM+H2*SAO+SAT    )-K15*H2*(SAM
001021   000  8-H2*SAO-SAT    ))/-(BIX2*DD(12)**2+BIY2)
001022   000   CON10E= K16*H2*(SAM+H2*SAO-SAT    )/-(BIX2*DD(12)**2+BIY2)
001023   000   CON10F=R2*((K1L+K2L)*(D2*DD(12)+R2*DD(10))-(K13L+K14L)*SCE
001024   000  1-(K15L+K16L)*SCF)/-(BIX2*DD(12)**2+BIY2)
```

```
001025   000        CON10G=(C3*F1*(VV(3)-VV(9)-F1*(VV(4)-VV(10))+SAE)-C4*F1*(VV(3)-
001026   000       1VV(9)+F1*(VV(4)-VV(10)+SAE)))/-(BIX2*DD(12)**2+BIY2)
001027   000        CON10H=(-C13*H2*SAN3+C14*H2*SAN4-C15*H2*SAN5+C16*H2*SAN6
001028   000       3-R2*(C13L+C14L)*SCA-R2*(C15L+C16L)*SCB)/-(BIX2*DD(12)**2+BIY2)
001029   000        CON10I=KBOM*H2*(-DEL9*(SAM-H2*SAO+TL)+DEL10*(SAM+H2*SAO+TL))/
001030   000       1-(BIX2*DD(12)**2+BIY2)
001031   000        CONS10=CON10A+CON10C+CON10D+CON10E+CON10F+CON10G+CON10H+CON10I
001032   000        CON11A  =(-BIX2*DD(10)*VV(10)*(VV(11)+VV(10)*DD(12))-BIX2*DD(12)*
001033   000       1VV(12)*(VV(11)+VV(10)*DD(12))+BIX2*(-VV(11)*DD(10)*VV(10)-VV(11)*
001034   000       2DD(12)*VV(12)+VV(10)*VV(12))
001035   000       3+BIY2*DD(12)*DD(10)*VV(10)*(VV(10)-VV(11)*DD(12)))/
001036   000       2-(BIX2+BIY2*DD(12)**2+BIZ2*DD(10)**2)
001037   000        CON11B=(KP24*(DD(11)-DD(23))
001038   000       3-BIY2*VV(12)*(VV(10)-VV(11)*DD(12))-BIY2*DD(12)*(-VV(10)
001039   000       3*DD(12)*VV(12)+VV(11)*DD
001040   000       4(12)*DD(10)*VV(10)-VV(11)*VV(12))+BIZ2*VV(10)*(VV(11)*DD(10)+VV(12
001041   000       5))+BIZ2*DD(10)*(VV(11)*VV(10)))/-(BIX2+BIY2*DD(12)**2
001042   000       1+BIZ2*DD(10)**2)
001043   000        CON11D=(K13*D2*(SAM-H2*SAO+SAT)+K14*D2*(SAM+H2*SAO+SAT)
001044   000       1-K15*D2*(SAM-H2*SAO-SAT)-K16*D2*(SAM+H2*SAO-SAT))/-
001045   000       2(BIX2+BIY2*DD(12)**2+BIZ2*DD(10)**2)
001046   000        CON11E=(C13*D2*(SAN3)+C14*D2*SAN4-C15*D2*SAN5-C16*D2*SAN6)/-(BIX2
001047   000       1+BIY2 *DD(12)**2+BIZ2*DD(10)**2)
001048   000        CONS11=CON11A+CON11B+CON11D+CON11E
001049   000        CON12A=  (BIZ2*(VV(11)*VV(10))-BIX2*(-VV(11)*DD(12)+VV(10))
001050   000       1*(VV(11)+VV(10)*DD(12))-BIY2*(-VV(10)*DD(12)-VV(11))*
001051   000       1(VV(10)-VV(11)*DD(12))+KT24*(DD(12)-DD(24)))/-BIZ2
001052   000        CON12B=D2*((K1L+K2L)*(D2*DD(12)+R2*DD(10))-(K13L+K14L)*SCE
001053   000       1+(K15L+K16L)*SCF-(C13L+C14L)*SCA+(C15L+C16L)*SCB)/-BIZ2
001054   000        CONS12=CON12A+CON12B
001055   000        CONS14=CONS2
001056   000        CON15A   =(-K7*(SAC-E*SAD-SAI)-K8*(SAC+E*SAD-SAI)-K9*(SAC-F1*SAD-
001057   000       1SAI)-K10*(SAC+F1*SAD-SAI)-K11*DEL3*(SAC- G1*SAD+GAP -SAI)-K12*
001058   000       2DEL4*(SAC+G1*SAD+GAP-SAI)+K17*(SBA-H3*SBB+SBC    )+K18*(SBA+H3*SBB+
001059   000       3SBC    )+K19*(SBA-H3*SBB-SBC    )+K20*(SBA+H3*SBB-SBC    ))/-BM3
001060   000        CON15B=(BM3*G
001061   000       4-C9*(VV(3)-VV(15)-F1*(VV(4)-VV(16))-SAE)-C10*(VV(3)-VV(15)+F1*(
001062   000       5VV(4)-VV(16))-SAE)+C17*SBD7+C18*SBD8+C19*SBD9+C20*SBD0)/-BM3
001063   000        CON15C=KBOM*(DEL11*(SBA-H3*SBB+TL)+DEL12*(SBA+H3*SBB+TL))/-BM3
001064   000        CONS15=CON15A+CON15B+CON15C
001065   000         CON16A   =(BIX3*VV(18)*(VV(17)+VV(16)*DD(18))+BIX3*DD(18)*(-VV(17)
001066   000       1*DD(16)*VV(16)-VV(17)*DD(18)*VV(18)+VV(16)*VV(18))
001067   000       1-BIY3*DD(18)*VV(18)*
001068   000       2(VV(16)-VV(17)*DD(18))+BIY3*(-VV(16)*DD(18)*VV(18)+VV(17)*DD(18)*D
001069   000       3D(16)*VV(16)-VV(17)*VV(18)))/-(BIX3*DD(18)**2+BIY3)
001070   000        CON16B=(BIX3*VV(17)*DD(16)*(VV(17)+VV(16)*DD
001071   000       4(18))-BIY3*VV(17)*DD(18)*DD(16)*(VV(16)-VV(17)*DD(18))-BIZ3*VV(17)
001072   000       5*(VV(17)*DD(16)+VV(18))+K7*E*(SAC-E*SAD-SAI)-K8*E*(SAC+E*SAD-SAI)
001073   000       6+K9*F1*(SAC-F1*SAD-SAI))/-(BIX3*DD(18)**2+BIY3)
001074   000        CON16C=(-K10*F1*(SAC+F1*SAD-SAI)+K11*DEL3*G1*(SAC-
001075   000       7G1*SAD+GAP -SAI)-K12*DEL4*G1*(SAC+G1*SAD+GAP-SAI)-K17*H3*(SBA-H3*
001076   000       8SBB+SBC    )+K18*H3*(SBA+H3*SBB+SBC    )-K19*H3*(SBA-H3*SBB-SBC    )
001077   000       9+K20*H3*(SBA+H3*SBB-SBC    ))/-(BIX3*DD(18)**2+BIY3)
001078   000        CON16D=R3*((K7L+K8L)*(-D3*DD(18)+R3*DD(16))
001079   000       1-(K17L+K18L)*SCG-(K19L+K20L)*SCH)/-(BIX3*DD(18)**2+BIY3)
001080   000        CON16E=(C9*F1*(VV(3)-VV(15)-F1*(DD(4)-DD(16))-SAE)-
001081   000       5C10*F1*(VV(3)-VV(15)+F1*(DD(4)-DD(16))-SAE)-C17*H3*SBD7+C18*H3*
```

95

```
001082   000   1SBD8-C19*H3*SBD9+C20*H3*SBD0)/-(BIX3*DD(18)**2+BIY3)
001083   000    CON16F=R3* (-(C17L+C18L)*SCC-(C19L+C20L)*SCD)/-(BIX3*DD(18)**2
001084   000   2+BIY3)
001085   000    CON16G=KBOM*H3*(-DEL11*(SBA-H3*SBB+TL)+DEL12*(SBA+H3*SBB+TL))/
001086   000   -(BIX3*DD(18)**2+BIY3)
001087   000    CONS16=CON16A+CON16B+CON16C+CON16D+CON16E+CON16F+CON16G
001088   000    CON17A=(- BIX3*DD(16)*VV(16)*(VV(17)+VV(16)*DD(18))-BIX3*DD(18)*
001089   000   1VV(18)*(VV(17)+VV(16)*DD(18))+BIX3*(-VV(17)*DD(18)*VV(18)-VV(17)
001090   000   2*DD(16)*VV(16)+VV(16)*VV(18))
001091   000   2+BIY3*DD(16)*DD(18)*VV(16)*(VV(16)-VV(17)
001092   000   3*DD(18)))/-(BIX3+BIY3*DD(18)**2+BIZ3*DD(16)**2)
001093   000    CON17B=(-BIY3*VV(18)*(VV(16)-VV(17)*DD(18))-BIY3*DD(18)*(-VV(16)
001094   000   4*DD(18)*VV(18)+VV(17)*DD(18)*DD(16)*VV(16)-VV(17)*VV(18))+BIZ3*
001095   000   5VV(16)*(VV(17)*DD(16)+VV(18))+BIZ3*DD(16)*(VV(17)*VV(16)))
001096   000   5/-(BIX3+BIY3*DD(18)**2+BIZ3*DD(16)**2)
001097   000    CON17D=(K17*D3*(SBA-H3*SBB+SBC)+K18*D3*(SBA+H3*
001098   000   9SBB+SBC    )-K19*D3*(SBA-H3*SBB-SBC    )-K20*D3*(SBA+H3*SBB-SBC)
001099   000   9+KP35*(DD(17)-DD(29)))/-(BIX3+BIY3*DD(18)**2+BIZ3*DD(16)**2)
001100   000    CON17F=(C17*D3*SBD7+C18*D3*SBD8-C19*D3*SBD9-C20*D3*(SBD0
001101   000   3     ))/-(BIX3+BIY3*DD(18)**2+BIZ3*DD(16)**2)
001102   000    CONS17=CON17A+CON17B+CON17D+CON17F
001103   000    CON18A   =(BIZ3*VV(17)*VV(16)-BIX3*(-VV(17)*DD(18)+VV(16))*(VV(17)
001104   000   1+VV(16)*DD(18))-BIY3*(-VV(16)*DD(18)-VV(17))*(VV(16)-VV(17)*DD(18)
001105   000   2)+KT35*(DD(18)-DD(30)))/-BIZ3
001106   000    CON18B=D3* (-(K7L+K8L)*(-D3*DD(18)+R3*DD(16))
001107   000   1-(K17L+K18L)*SCG+(K19L+K20L)*SCH-(C17L+C18L)*SCC+(C19L+C20L)*SCD
001108   000   1)/-BIZ3
001109   000    CONS18=CON18A+CON18B
001110   000    CON19A=(-(K13L+K14L)*SCE-(K15L+K16L)*SCF-(C13L+C14L)*SCA
001111   000   1-(C15L+C16L)*SCB)/-TM4
001112   000    CON19B=(K21L*(DD(19)-SBO-SBP+R9)+K22L*(DD(19)-SBO-SBP-R10)+K23L*
001113   000   1(DD(19)+SBO-SBP+R11)+K24L*(DD(19)+SBO-SBP-R12))/-TM4
001114   000    CON19C=-KGIB*(DEL5*(SAF-GIB)+DEL6*(SAF+GIB))/-TM4
001115   000    CONS19=CON19A+CON19B+CON19C
001116   000    CONS20=CONS2
001117   000    CON21A=(-K13*(SAM-H2*SAO+SAQ    )-K14*(SAM+H2*SAO+SAQ    )-K15*(SAM-
001118   000   1H2*SAO-SAQ    )-K16*(SAM+H2*SAO-SAQ    ))/-TM4
001119   000    CON21B=(TM4*G-C13*SAN3-C14*SAN4-C15*SAN5-C16*SAN6)/-TM4
001120   000    CON21C=(K21*(DD(21)-SBM+SBN-V1)+K22*(DD(21)+SBM+SBN-V2)+K23*(
001121   000   1DD(21)-SBM-SBN-V3)+K24*(DD(21)+SBM-SBN-V4))/-TM4
001122   000    CON21D=-KBOM*(DEL9*(SAM-H2*SAO+TL)+DEL10*(SAM+H2*SAO+TL))/-TM4
001123   000    CONS21=CON21A+CON21B+CON21C+CON21D
001124   000    CON22A   =(TIX4*VV(24)*(VV(23)+VV(22)*DD(24))+TIX4*DD(24)*(-VV(23)*
001125   000   1DD(22)*VV(22)-VV(23)*DD(24)*VV(24)+VV(22)*VV(24))-TIY4*DD(24)*
001126   000   2VV(24)*(VV(22)-VV(23)*DD(24))+TIY4*(-VV(22)*DD(24)*VV(24)+VV(23)*
001127   000   3DD(24)*DD(22)*VV(22)-VV(23)*VV(24)))/-(TIX4*DD(24)**2+TIY4)
001128   000    CON22B=(TIX4*VV(23)*DD(22)*(VV(23)+VV
001129   000   4(22)*DD(24))-TIY4*VV(23)*DD(24)*DD(22)*(VV(22)-VV(23)*DD(24))
001130   000   5-TIZ4*VV(23)*(VV(23)*DD(22)+VV(24))+K13*H2*(SAM-H2*SAO+SAQ    )
001131   000   6-K14*H2*(SAM+H2*SAO+SAQ    ))/-(TIX4*DD(24)**2+TIY4)
001132   000    CON22C=(K15*H2*(SAM-H2*SAO-SAQ)-K16*H2*(SAM+H2*SAO-SAQ))/-
001133   000   1(TIX4*DD(24)**2+TIY4)
001134   000    CON22D=(-R4*(K13L+K14L)*SCE-R4*(K15L+K16L)*SCF
001135   000   1+C13*H2*SAN3-C14*H2*SAN4+C15*H2*SAN5-C16*H2*SAN6
001136   000   2         -R4*(C13L+C14L)*SCA-R4*(C15L+C16L)*SCB)/-(TIX4*DD(24)
001137   000   3**2+TIY4)
001138   000    CON22E=(-K21*H4*(DD(21)-SBM+SBN-V1)+K22*H4*(DD(21)+SBM+SBN-V2)
```

```
001139   000   1-K23*H4*(DD(21)-SBM-SBN-V3)  +K24*H4*(DD(21)+SBM-SBN-V4)
001140   000   2-K21L*R *(DD(19)-SBO-SBP+R9)-K22L*R *(DD(19)-SBO-SBP-R10)-K23L*
001141   000   3R *(DD(19)+SBO-SBP+R11)-K24L*R *(DD(19)+SBO-SBP-R12))/-(TIX4*DD
001142   000   3(24)**2+TIY4)
001143   000    CON22F=KBOM*H2*(DEL9*(SAM-H2*SAO+TL)-DEL10*(SAM+H2*SAO+TL))
001144   000   1/-(TIX4*DD(24)**2+TIY4)
001145   000    CONS22=CON22A+CON22B+CON22C+CON22D+CON22E+CON22F
001146   000    CON23A  =(-TIX4*VV(22)*DD(22)
001147   000   1                       *(VV(23)+VV(22)*DD(24))-TIX4*DD(24)*VV(24)*
001148   000   1(VV(23)+VV(22)*DD(24))+TIX4*(-VV(23)*DD(22)*VV(22)-VV(23)*DD(24)*
001149   000   2VV(24)+VV(22)*VV(24)))/-(TIX4+TIY4*DD(24)**2+TIZ4*DD(22)**2)
001150   000    CON23B=(TIY4*DD(24)*DD(22)*VV(22)*(VV(22)-VV(23)*
001151   000   3DD(24))-TIY4*VV(24)*(VV(22)-VV(23)*DD(24))-TIY4*DD(24)*(-VV(22)*
001152   000   4DD(24)*VV(24)+VV(23)*DD(24)*DD(22)*VV(22)-VV(23)*VV(24)))
001153   000   4/-(TIX4+TIY4*DD(24)**2+TIZ4*DD(22)**2)
001154   000    CON23C=(TIZ4*
001155   000   5VV(22)*(VV(23)*DD(22)+VV(24))+TIZ4*DD(22)*(VV(23)*VV(22))-K13*D4*
001156   000   6(SAM-H2*SAO+SAQ    )-K14*D4*(SAM+H2*SAO+SAQ    )+K15*D4*(SAM-H2*
001157   000   7SAO-SAQ    ))/-(TIX4+TIY4*DD(24)**2+TIZ4*DD(22)**2)
001158   000    CON23D=(K16*D4*(SAM+H2*SAO-SAQ    )-C13*D4*SAN3-C14*D4*SAN4
001159   000   1+C15*D4*SAN5+C16*D4*SAN6)/-(TIX4+TIY4*DD(24)**2+TIZ4*DD(22)
001160   000   1**2)
001161   000    CON23E=(K21*D *(DD(21)-SBM+SBN-V1)+K22*D *(DD(21)+SBM+SBN-V2)
001162   000   1-K23*D *(DD(21)-SBM-SBN-V3)-K24*D *(DD(21)+SBM-SBN-V4)
001163   000   2-KP24*(DD(11)-DD(23)))/-(TIX4+TIY4*DD(24)**2+TIZ4*DD(22)**2)
001164   000    CONS23=CON23A+CON23B+CON23C+CON23D+CON23E
001165   000    CON24A  =(TIZ4*(VV(23)*VV(22))+TIX4*VV(23)*DD(24)*(VV(23)+VV(22)*
001166   000   1DD(24))-TIX4*VV(22)*(VV(23)+VV(22)*DD(24))-TIY4*(-VV(22)*DD(24)-V
001167   000   2V(23))*(VV(22)-VV(23)*DD(24))-KT24*(DD(12)-DD(24)))/-TIZ4
001168   000    CON24B=D4*((K13L+K14L)*SCE-(K15L+K16L)*SCF
001169   000   1+(C13L+C14L)*SCA-(C15L+C16L)*SCB)/-TIZ4
001170   000    CON24C=D *(-K21L*(DD(19)-SBO-SBP+R9)-K22L*(DD(19)-SBO-SBP-R10)
001171   000   1+K23L*(DD(19)+SBO-SBP+R11)+K24L*(DD(19)+SBO-SBP-R12))/-TIZ4
001172   000    CONS24=CON24A+CON24B+CON24C
001173   000    CON25A=(-(K17L+K18L)*SCG-(K19L+K20L)*SCH-(C17L+C18L)*SCC
001174   000   1-(C19L+C20L)*SCD)/-TM5
001175   000    CON25B=(K25L*(DD(25)-SBS-SBT+R13)+K26L*(DD(25)-SBS-SBT-R14)+K27L*
001176   000   1(DD(25)+SBS-SBT+R15)+K28L*(DD(25)+SBS-SBT-R16))/-TM5
001177   000    CON25C=-KGIB*(DEL7*(SAU-GIB)+DEL8*(SAU+GIB))/-TM5
001178   000    CONS25=CON25A+CON25B+CON25C
001179   000    CONS26=CONS2
001180   000    CON27A=   (-K17*(SBA-H3*SBB+SBC     )-K18*(SBA+H3*SBB+SBC     )
001181   000   1-K19*(SBA-H3*SBB-SBC     )-K20*(SBA+H3*SBB-SBC     )+TM5*G
001182   000   2-C17*SBD7-C18*SBD8-C19*SBD9-C20*SBD0)/-TM5
001183   000    CON27B=(K25*(DD(27)-SBQ+SBR-V5)+K26*(DD(27)+SBQ+SBR-V6)+K27*
001184   000   1(DD(27)-SBQ-SBR-V7)+K28*(DD(27)+SBQ-SBR-V8))/-TM5
001185   000    CON27C=-KBOM*(DEL11*(SBA-H3*SBB+TL)+DEL12*(SBA+H3*SBB+TL))/-TM5
001186   000    CONS27=CON27A+CON27B+CON27C
001187   000    CON28A  =(TIX5*VV(30)*(VV(29)+VV(28)*DD(30))+TIX5*DD(30)*(-VV(29)
001188   000   1*DD(28)*VV(28)-VV(29)*DD(30)*VV(30)+VV(28)*VV(30))-TIY5*DD(30)*
001189   000   2VV(30)*(VV(28)-VV(29)*DD(30))+TIY5*(-VV(28)*DD(30)*VV(30)+VV(29)*
001190   000   3DD(30)*DD(28)*VV(28)-VV(29)*VV(30)))/-(TIX5*DD(30)**2+TIY5)
001191   000    CON28B=(TIX5*VV(29)*DD(28)*(VV(29)+
001192   000   4VV(28)*DD(30))-TIY5*VV(29)*DD(30)*DD(28)*(VV(28)-VV(29)*DD(30))
001193   000   5-TIZ5*VV(29)*(VV(29)*DD(28)+VV(30))+K17*H3*(SBA-H3*SBB+SBC     )
001194   000   6-K18*H3*(SBA+H3*SBB+SBC     ))/-(TIX5*DD(30)**2+TIY5)
001195   000    CON28C=(K19*H3*(SBA-H3*SBB-SBC)-K20*H3*(SBA+H3*SBB-SBC))/
```

```
000            1-(TIX5*DD(30)**2+TIY5)
000             CON28D=-R5*((K17L+K18L)*SCG+(K19L+K20L)*SCH+(C17L+C18L)*SCC
000            1+(C19L+C20L)*SCD)/-(TIX5*DD(30)**2+TIY5)
000             CON28E=(-K25*H5*(DD(27)-SBQ+SBR-V5)+K26*H5*(DD(27)+SBQ+SBR-V6)
000            1-K27*H5*(DD(27)-SBQ-SBR-V7)+K28*H5*(DD(27)+SBQ-SBR-V8)
000            2-K25L*R *(DD(25)-SBS-SBT+R13)-K26L*R *(DD(25)-SBS-SBT-R14)
000            3-K27L*R *(DD(25)+SBS-SBT+R15)-K28L*R *(DD(25)+SBS-SBT-R16))/
000            4-(TIX5*DD(30)**2+TIY5)
000             CON28F=H3*(C17*SBD7-C18*SBD8+C19*SBD9-C20*SBD0)/-(TIX5*DD(30)**2
000            1+TIY5)
000             CON28G=KBOM*H3*(DEL11*(SBA-H3*SBB+TL)-DEL12*(SBA+H3*SBB+TL))
000            1/-(TIX5*DD(30)**2+TIY5)
000             CONS28=CON28A+CON28B+CON28C+CON28D+CON28E+CON28F+CON28G
000             CON29A  =(-TIX5*DD(28)*VV(28)*(VV(29)+VV(28)*DD(30))-TIX5*DD(30)*
000            1VV(30)*(VV(29)+VV(28)*DD(30))+TIX5*(-VV(29)*DD(28)*VV(28)-VV(29)*
000            2DD(30)*VV(30)+VV(28)*VV(30)))/-(TIX5+TIY5*DD(30)**2+TIZ5*DD(28)**2
000            2)
000             CON29B=(TIY5*DD(30)*DD(28)*VV(28)*(VV(28)-
000            3VV(29)*DD(30))-TIY5*VV(30)*(VV(28)-VV(29)*DD(30))-TIY5*DD(30)*(
000            4-VV(28)*DD(30)*VV(30)+VV(29)*DD(30)*DD(28)*VV(28)-VV(29)*VV(30)))
000            4/-(TIX5+TIY5*DD(30)**2+TIZ5*DD(28)**2)
000             CON29C=(TIZ5*VV(28)*(VV(29)*DD(28)+VV(30)))/-(TIX5+TIY5*DD(30)**2
000            5+TIZ5*DD(28)**2)
000             CON29D=(TIZ5*DD(28)*(VV(29)*VV(28))
000            6-K17*D5*(SBA-H3*SBB+SBC    )-K18*D5*(SBA+H3*SBB+SBC    )+K19*D5*(
000            7SBA-H3*SBB-SBC    )+K20*D5*(SBA+H3*SBB-SBC    ))/-(TIX5+TIY5*DD(30)*
000            7*2+TIZ5*DD(28)**2)
000             CON29E=(-C17*D5*SBD7-C18*D5*SBD8+C19*D5*SBD9+C20*D5*SBD0)
000            1/-(TIX5+TIY5*DD(30)**2+TIZ5*DD(28)**2)
000             CON29F=(K25*D *(DD(27)-SBQ+SBR-V5)+K26*D *(DD(27)+SBQ+SBR-V6)
000            1-K27*D *(DD(27)-SBQ-SBR-V7)-K28*D *(DD(27)+SBQ-SBR-V8)
000            2-KP35*(DD(17)-DD(29)))/-(TIX5+TIY5*DD(30)**2+TIZ5*DD(28)**2)
000             CONS29=CON29A+CON29B+CON29C+CON29D+CON29E+CON29F
000             CON30A=(TIZ5*VV(29)*VV(28)-TIX5*(-VV(29)*DD(30)+VV(28))*(VV(29)
000            1+VV(28)*DD(30))-TIY5*(-VV(28)*DD(30)-VV(29))*(VV(28)-VV(29)*DD(30)
000            2)-KT35*(DD(18)-DD(30)))/-TIZ5
000             CON30B=D5*((K17L+K18L)*SCG-(K19L+K20L)*SCH+(C17L+C18L)*SCC
000            1-(C19L+C20L)*SCD)/-TIZ5
000             CON30C=(-K25L*D *(DD(25)-SBS-SBT+R13)-K26L*D *(DD(25)-SBS-SBT-R14)
000            1+K27L*D *(DD(25)+SBS-SBT+R15)+K28L*D *(DD(25)+SBS-SBT-R16))/-TIZ5
000             CONS30=CON30A+CON30B+CON30C
000             CONS31=(KF*(DD(31)-DD(1)+XL1*DD(6))+CF*(VV(31)-VV(1)+XL1*VV(6)))/
000            1-FM1
000             CONS32=(KF*(DD(32)-DD(2))+CF*(VV(32)-VV(2)))/-FM1
000             CONS33=(KF*(DD(33)-DD(3)-XL1*DD(5))+CF*(VV(33)-VV(3)-XL1*VV(5)))/
000            1-FM1
000      C
000      C      COMPILE THE 16 COUPLED EQNS. INTO 5 MATRICES:
000      C
000             DO 167 I=1,4
000             AM1(I,I)=1.
000        167  CONTINUE
000             DO 101 I=1,3
000             AM2(I,I)=1.
000             AM3(I,I)=1.
000             AM4(I,I)=1.
000             AM5(I,I)=1.
```

```
000      101 CONTINUE
000          AM1(1,2)=-2*BM2*R1/(CM1+BM2+BM3)
000          AM1(1,5)=CONS1
000          AM1(2,1)=-2*BM2*R1/(CIX1*DD(6)**2+CIY1+2*BM2*R1**2)
000          AM1(2,3)=(CIX1-CIY1)*DD(6)/(CIX1*DD(6)**2+CIY1+2*BM2*R1**2)
000          AM1(2,5)=CONS4
000          AM1(3,2)=(CIX1-CIY1)*DD(6)/(CIX1+CIY1*DD(6)**2+CIZ1*DD(4)**2)
000          AM1(3,4)=CIZ1*DD(4)/(CIX1+CIY1*DD(6)**2+CIZ1*DD(4)**2)
000          AM1(3,5)=CONS5
000          AM1(4,3)=CIZ1*DD(4)/(CIZ1+2*BM2*D1**2)
000          AM1(4,5)=CONS6
000          AM2(1,2)=(BIX2-BIY2)*DD(12)/(BIX2*DD(12)**2+BIY2)
000          AM2(1,4)=CONS10
000          AM2(2,1)=(BIX2-BIY2)*DD(12)/(BIX2+BIY2*DD(12)**2+BIZ2*DD(10)**2)
000          AM2(2,3)=BIZ2*DD(10)/(BIX2+BIY2*DD(12)**2+BIZ2*DD(10)**2)
000          AM2(2,4)=CONS11
000          AM2(3,2)=DD(10)
000          AM2(3,4)=CONS12
000          AM3(1,2)=(BIX3-BIY3)*DD(18)/(BIX3*DD(18)**2+BIY3)
000          AM3(1,4)=CONS16
000          AM3(2,1)=(BIX3-BIY3)*DD(18)/(BIX3+BIY3*DD(18)**2+BIZ3*DD(16)**2)
000          AM3(2,3)=BIZ3*DD(16)/(BIX3+BIY3*DD(18)**2+BIZ3*DD(16)**2)
000          AM3(2,4)=CONS17
000          AM3(3,2)=DD(16)
000          AM3(3,4)=CONS18
000          AM4(1,2)=(TIX4-TIY4)*DD(24)/(TIX4*DD(24)**2+TIY4)
000          AM4(1,4)=CONS22
000          AM4(2,1)=(TIX4-TIY4)*DD(24)/(TIX4+TIY4*DD(24)**2+TIZ4*DD(22)**2)
000          AM4(2,3)=TIZ4*DD(22)/(TIX4+TIY4*DD(24)**2+TIZ4*DD(22)**2)
000          AM4(2,4)=CONS23
000          AM4(3,2)=DD(22)
000          AM4(3,4)=CONS24
000          AM5(1,2)=(TIX5-TIY5)*DD(30)/(TIX5*DD(30)**2+TIY5)
000          AM5(1,4)=CONS28
000          AM5(2,1)=(TIX5-TIY5)*DD(30)/(TIX5+TIY5*DD(30)**2+TIZ5*DD(28)**2)
000          AM5(2,3)=TIZ5*DD(28)/(TIX5+TIY5*DD(30)**2+TIZ5*DD(28)**2)
000          AM5(2,4)=CONS29
000          AM5(3,2)=DD(28)
000          AM5(3,4)=CONS30
000      C
000      C   CALL LSIMEQ TO SOLVE THE MATRIX:
000      C
000          CALL LSIMEQ (AM1,4,IRR,JCC,4,0.01,A1,IERR1)
000          CALL LSIMEQ (AM2,3,IR,JC,3,0.01,A2,IERR1)
000          CALL LSIMEQ (AM3,3,IR,JC,3,0.01,A3,IERR1)
000          CALL LSIMEQ (AM4,3,IR,JC,3,0.01,A4,IERR1)
000          CALL LSIMEQ (AM5,3,IR,JC,3,0.01,A5,IERR1)
000      C
000      C   ASSIGN SOLUTION TO CORRESPONDING ACCELN. VARIABLES
000      C
000          AA(1)=A1(1)
000          AA(2)=CONS2
000          AA(3)=CONS3
000          AA(4)=A1(2)
000          AA(5)=A1(3)
000          AA(6)=A1(4)
000          AA(7)=0.
```

```
000          AA(8)=CONS8
000          AA(9)=CONS9
000          AA(10)=A2(1)
000          AA(11)=A2(2)
000          AA(12)=A2(3)
000          AA(13)=0.
000          AA(14)=CONS14
000          AA(15)=CONS15
000          AA(16)=A3(1)
000          AA(17)=A3(2)
000          AA(18)=A3(3)
000          AA(19)=CONS19
000          AA(20)=CONS20
000          AA(21)=CONS21
000          AA(22)=A4(1)
000          AA(23)=A4(2)
000          AA(24)=A4(3)
000          AA(25)=CONS25
000          AA(26)=CONS26
000          AA(27)=CONS27
000          AA(28)=A5(1)
000          AA(29)=A5(2)
000          AA(30)=A5(3)
000          AA(31)=CONS31
000          AA(32)=CONS32
000          AA(33)=CONS33
000          IF (T-PTIME) 63,64,64
000    C
000    C     GENERATE INFORMATION FOR COMPARING WITH THE 5000 MILE ROAD TEST DATA
000    C
000       64 CALL T5000 (AA)
000    C
000    C     COMPUTE SPRING DEFLECTIONS AT PRINT TIME
000    C
000          DP35=SAM-H2*SAO
000          DP46=SAM+H2*SAO
000          DP79=SBA-H3*SBB
000          DP80=SBA+H3*SBB
000          WRITE (6,343) DP35,DP79,DP46,DP80
000      343 FORMAT (/,10X,'RF',F10.3,2X,'RR',F10.3,2X,'LF',F10.3,2X,'LR',
000         1F10.3)
000       63 RETURN
000          END

END ELT.
```

```
WDH006*TPF$.DFSRIE
            SUBROUTINE DFSRIE(NP,NH,Y,A,B)                                  DFSRI
            DIMENSION Y(NP),A(1),B(1)                                       DFSRI
      C                 ---------------------------------------------------
      C                 NUMBER OF HARMONICS  MUST NOT EXCEED HALF
      C                 THE NUMBER OF DATA POINTS INPUTED.
      C                 ---------------------------------------------------
            NNH=MIN0(NH,NP/2)
      C-----------------------------------------------------------------------
      C     INITIALIZATION AND CONSTANTS.                                   DFSRI
      C-----------------------------------------------------------------------
            SP=0.                                                           DFSRI
            CP=1.                                                           DFSRI
            RN=2./NP                                                        DFSRI
            B(1)=0.                                                         DFSRI
            A(1)=0.                                                         DFSRI
            ARG=RN*3.14159265                                               DFSRI
            C=COS(ARG)                                                      DFSRI
            S=SIN(ARG)                                                      DFSRI
      C-----------------------------------------------------------------------
      C     COMPUTE A FOR THE ZEROTH HARMONIC                               DFSRI
      C-----------------------------------------------------------------------
            DO10I=1,NP                                                      DFSRI
       10   A(1)=A(1)+Y(I)                                                  DFSRI
            A(1)=RN*A(1)                                                    DFSRI
      C-----------------------------------------------------------------------
      C     MAIN LOOP.                                                      DFSRI
      C-----------------------------------------------------------------------
             DO 100 K=1,NNH
            X=C*CP-S*SP                                                     DFSRI
            SP=C*SP+S*CP
            CP=X                                                            DFSRI
            U=0.                                                            DFSRI
            V=0.                                                            DFSRI
      C     COMPUTE RECURSIVE U,S                                           DFSRI
            DO20II=2,NP                                                     DFSRI
            J=NP-II+2                                                       DFSRI
            W=Y(J)+2.*CP*V-U                                                DFSRI
            U=V                                                             DFSRI
       20   V=W                                                             DFSRI
            A(K+1)=RN*(Y(1)+CP*V-U)                                         DFSRI
      100   B(K+1)=RN*SP*V                                                  DFSRI
            RETURN                                                          DFSRI
            END                                                             DFSRI

WFIN                                                      NIF@

     LOAD 0285   12/0 0285        -1 X1E05

   0-X1E05*MSG: TAPE: 0285 7 TRK RING OUT

   0 WJB.

     X1E05  FIN  WDH006
```

```
@FOR,S LSIMEQ
FOR S1A-04/09/75-15:32:04 (1,)

   SUBROUTINE LSIMEQ    ENTRY POINT 000323

   STORAGE USED: CODE(1) 000357; DATA(0) 000060; BLANK COMMON(2) 000000

   EXTERNAL REFERENCES (BLOCK, NAME)

    0003   NERR3$

   STORAGE ASSIGNMENT  (BLOCK, TYPE, RELATIVE LOCATION, NAME)

    0001   000031 106G      0001   000043 113G      0001   000054 116G      0001   000065 123G      0001   000075 130G
    0001   000170 157G      0001   000200 164G      0001   000230 172G      0001   000110 2L        0001   000257 202G
    0001   000125 3L        0001   000142 4L        0001   000235 7L        0000 R 000012 AJCK      0000 R 000003 BIGA
    0000 I 000004 I         0000 I 000006 II        0000   000016 INJP$     0000 I 000013 IRI       0000 I 000010 IRK
    0000 I 000005 J         0000 I 000014 JCI       0000 I 000011 JCK       0000 I 000007 JJ        0000 I 000001 K
    0000 I 000002 KM1       0000 I 000000 NP1


 00101      1*           SUBROUTINE LSIMEQ (A,NMAX,IR,JC,N,EPS1,X,IERR1)        LSIMEQ
 00101      2*      C-----------------------------------------------------------LSIMEQ
 00101      3*      C    THIS SUBROUTINE COMPUTES THE SOLUTION OF A SET OF SIMULTANEOUS   LSIMEQ
 00101      4*      C    LINEAR ALGEBRAIC EQUATIONS USING THE GAUSS-JORDAN REDUCTION     LSIMEQ
 00101      5*      C    SCHEME WITH THE MAXIMUM PIVOT CRITERION.                        LSIMEQ
 00101      6*      C-----------------------------------------------------------LSIMEQ
 00103      7*           DIMENSION A(NMAX,N),IR(N),JC(N),X(N)                   LSIMEQ
 00103      8*      C-----------------------------------------------------------LSIMEQ
 00103      9*      C    BEGIN THE ELIMINATION PROCEDURE.                       LSIMEQ
 00103     10*      C-----------------------------------------------------------LSIMEQ
 00104     11*           NP1=N+1                                                LSIMEQ
 00105     12*           DO 7 K=1,N                                             LSIMEQ
 00110     13*           KM1=K-1                                                LSIMEQ
 00110     14*      C-----------------------------------------------------------LSIMEQ
 00110     15*      C    BEGIN THE SEARCH FOR THE MAXIMUM PIVOT ELEMENT.        LSIMEQ
 00110     16*      C-----------------------------------------------------------LSIMEQ
 00111     17*           BIGA=0.0                                               LSIMEQ
 00112     18*           DO 3 I=1,N                                             LSIMEQ
 00113     19*           DO 3 J=1,N                                             LSIMEQ
 00120     20*           IF (KM1.EQ.0) GO TO 2                                  LSIMEQ
 00120     21*      C-----------------------------------------------------------LSIMEQ
 00120     22*      C    CHECK ROW AND COLUMN PIVOT SUBSCRIPTS ALREADY USED.    LSIMEQ
 00120     23*      C-----------------------------------------------------------LSIMEQ
 00122     24*           DO 1 II=1,KM1                                          LSIMEQ
 00125     25*           IF (I.EQ.IR(II)) GO TO 3                               LSIMEQ
 00127     26*           DO 1 JJ=1,KM1                                          LSIMEQ
 00132     27*           IF (J.EQ.JC(JJ)) GO TO 3                               LSIMEQ
 00134     28*         1 CONTINUE                                               LSIMEQ
 00137     29*         2 IF (.NOT.ABS(A(I,J)).GT.BIGA) GO TO 3
 00141     30*           BIGA=ABS(A(I,J))                                       LSIMEQ
```

```
00142     31*            IR(K)=I                                                 LSIMEQ
00143     32*            JC(K)=J                                                 LSIMEQ
00144     33*          3 CONTINUE                                                LSIMEQ
00144     34*      C-------------------------------------------------------------LSIMEQ
00144     35*      C     CHECK TO SEE IF THE MAXIMUM PIVOT ELEMENT IS GREATER THAN EPS1.    LSIMEQ
00144     36*      C-------------------------------------------------------------LSIMEQ
00147     37*            IF (.NOT.BIGA.LT.EPS1) GO TO 4                          LSIMEQ
00151     38*            IERR1=-1                                                LSIMEQ
00152     39*            RETURN                                                  LSIMEQ
00152     40*      C-------------------------------------------------------------LSIMEQ
00152     41*      C     NORMALIZE THE PIVOT ELEMENT.                            LSIMEQ
00152     42*      C-------------------------------------------------------------LSIMEQ
00153     43*          4 IRK=IR(K)                                               LSIMEQ
00154     44*            JCK=JC(K)                                               LSIMEQ
00155     45*            BIGA=A(IRK,JCK)                                         LSIMEQ
00156     46*            DO 5 J=1,NP1                                            LSIMEQ
00161     47*          5 A(IRK,J)=A(IRK,J)/BIGA                                  LSIMEQ
00161     48*      C-------------------------------------------------------------LSIMEQ
00161     49*      C     ELIMINATE THE NON ZERO ELEMENTS IN THE JC(K) TH COLUMN.  LSIMEQ
00161     50*      C-------------------------------------------------------------LSIMEQ
00163     51*            DO 7 I=1,N                                              LSIMEQ
00166     52*            IF (I.EQ.IRK) GO TO 7                                   LSIMEQ
00170     53*            AJCK=A(I,JCK)                                           LSIMEQ
00171     54*            DO 6 J=1,NP1                                            LSIMEQ
00174     55*          6 A(I,J)=A(I,J)-AJCK*A(IRK,J)                             LSIMEQ
00176     56*          7 CONTINUE                                                LSIMEQ
00176     57*      C-------------------------------------------------------------LSIMEQ
00176     58*      C     REORDER THE SOLUTION AND TRANSFER IT TO THE X ARRAY.    LSIMEQ
00176     59*      C-------------------------------------------------------------LSIMEQ
00201     60*            DO 8 I=1,N                                              LSIMEQ
00204     61*            IRI=IR(I)                                               LSIMEQ
00205     62*            JCI=JC(I)                                               LSIMEQ
00206     63*          8 X(JCI)=A(IRI,NP1)                                       LSIMEQ
00206     64*      C-------------------------------------------------------------LSIMEQ
00206     65*      C     SUCCESSFUL RETURN.                                      LSIMEQ
00206     66*      C-------------------------------------------------------------LSIMEQ
00210     67*            IERR1=+1                                                LSIMEQ
00211     68*            RETURN                                                  LSIMEQ
00212     69*            END                                                     LSIMEQ

      END OF COMPILATION:        NO  DIAGNOSTICS.

@FIN                                                   NIFQ

      LOAD 0265   12/4 0265        -1 XIEU5

   1-XIEU5*MSG: TAPE: 0265 7 TRK RING OUT

   1 IM

     XIEU5  FIN  WDH006
```

```
@FOR,IS FREQ
FOR S11A-04/07/75-10:30:46 (,0)

 MAIN PROGRAM

 STORAGE USED: CODE(1) 000114; DATA(0) 001347; BLANK COMMON(2) 000000


 EXTERNAL REFERENCES (BLOCK, NAME)

  0003   DFSRIE
  0004   NINTR$
  0005   NRDU$
  0006   NI01$
  0007   NI02$
  0010   NWDU$
  0011   XPRR
  0012   NSTOP$


 STORAGE ASSIGNMENT  (BLOCK, TYPE, RELATIVE LOCATION, NAME)

  0000   001314 10F      0001   000011 105G     0000   001311 111F     0001   000045 124G     0001   000110 18L
  0000   001330 30F      0001   000002 91L      0000 R 000311 A        0000 R 000623 AMP      0000 R 000456 B
  0000 R 001307 DELFCY   0000 R 000770 FREQ     0000 R 001306 FREQCY   0000 I 001302 I        0000 I 001310 J
  0000 I 001303 NH       0000 I 001304 NP       0000 R 001305 PERIOD   0000 R 001135 PSD      0000 R 000000 Y



00101      1*            DIMENSION Y(201),A(101),B(101),AMP(101),FREQ(101),PSD(101)
00103      2*         91 READ (5,111,END=18) (Y(I),I=1,201)
00111      3*        111 FORMAT (6X,10F6.3,14X)
00112      4*            NH=100
00113      5*            NP=201
00114      6*            PERIOD=4.5
00115      7*            FREQCY=1/PERIOD
00116      8*            DELFCY=FREQCY
00117      9*            CALL DFSRIE (NP,NH,Y,A,B)
00120     10*            WRITE (6,10)
00122     11*         10 FORMAT (///,6X,'FREQUENCY (HZ) ',10X,'AMPLITUDE (G) ',10X,
00122     12*           1'G**2/HZ')
00123     13*            DO 20 I=2,101
00126     14*            J=I-1
00127     15*            AMP(I)=(A(I)**2+B(I)**2)**0.5
00130     16*            FREQ(J)=J*FREQCY
00131     17*            PSD(J)=AMP(I)**2/DELFCY
00132     18*         20 WRITE (6,30) FREQ(J),AMP(I),PSD(J)
00140     19*         30 FORMAT (/,11X,F10.4,13X,F6.3,13X,F6.3)
00141     20*            GO TO 91
00142     21*         18 STOP
00143     22*            END

     END OF COMPILATION:       NO  DIAGNOSTICS.
```

APPENDIX C

C-1. EQUIVALENT VISCOUS DAMPING COEFFICIENT

C-2. DETERMINATION OF SUSPENSION SPRING STIFFNESS FOR A TYPICAL TRUCK

C-3. BENDING STIFFNESS OF A TYPICAL TRUCK BOLSTER

C-4. DESCRIPTIVE DATA FOR A 70-TON BOX CAR

C-5. TRACK INPUT EQUATIONS

C-1. Equivalent Viscous Damping Coefficient

An equivalent viscous damping coefficient is derived on the principle of equivalent energy dissipation for the coulomb friction losses and losses by linear viscous damping.

For a non-linear damper, let the friction force be $F(x, \dot{x})$ with motion assumed to be $x = A \sin \omega t$. The energy dissipated per cycle in this model then becomes

$$\begin{aligned} E_{NL} &= \int F(x,\dot{x})\, dx \\ &= \int F(x,\dot{x}) \frac{dx}{dt}\, dt \\ &= \int_0^\tau F(x,\dot{x})\dot{x}\, dt \\ &= A\omega \int_0^\tau F(x,\dot{x}) \cos \omega t\, dt \end{aligned}$$

where τ = periodic time = $\frac{2\pi}{\omega}$ and

A = amplitude of the assumed motion.

For a linear damper, $F = c\dot{x}$, where c is the coefficient of the viscous damping system, and the energy dissipated per cycle, for this case is given by

$$\begin{aligned} E_L &= \int F\, dx \\ &= \int c\dot{x}\, dx \\ &= \int_0^\tau c\, A\omega \cos \omega t \left(\frac{dx}{dt}\right) dt \\ &= A^2\omega^2 c \int_0^\tau \cos^2 \omega t\, dt \\ &= A^2\omega^2 c\, \frac{\tau}{2} \\ &= \pi A^2 \omega c. \end{aligned}$$

It is assumed here that both systems, the linear and the nonlinear model, have the same amplitude A, and the same periodic time. This model assumption is shown to hold good at a later stage, when the model is compared with test data. Therefore, for $E_L = E_{NL}$, we have

$$C_{equiv.} = \frac{1}{\pi A} \int_0^\tau F(x,\dot{x}) \cos \omega t\, dt \tag{1}$$

Analysis of a Typical Truck Damping System

Refer to the free body diagram (Fig. C-1) of the friction shoes in the upward stroke, $\sum F_x = 0$ gives

$$N_1 \cos \theta - F_1 \sin \theta - N_2 = 0 \tag{2}$$

$\sum F_y = 0$ gives

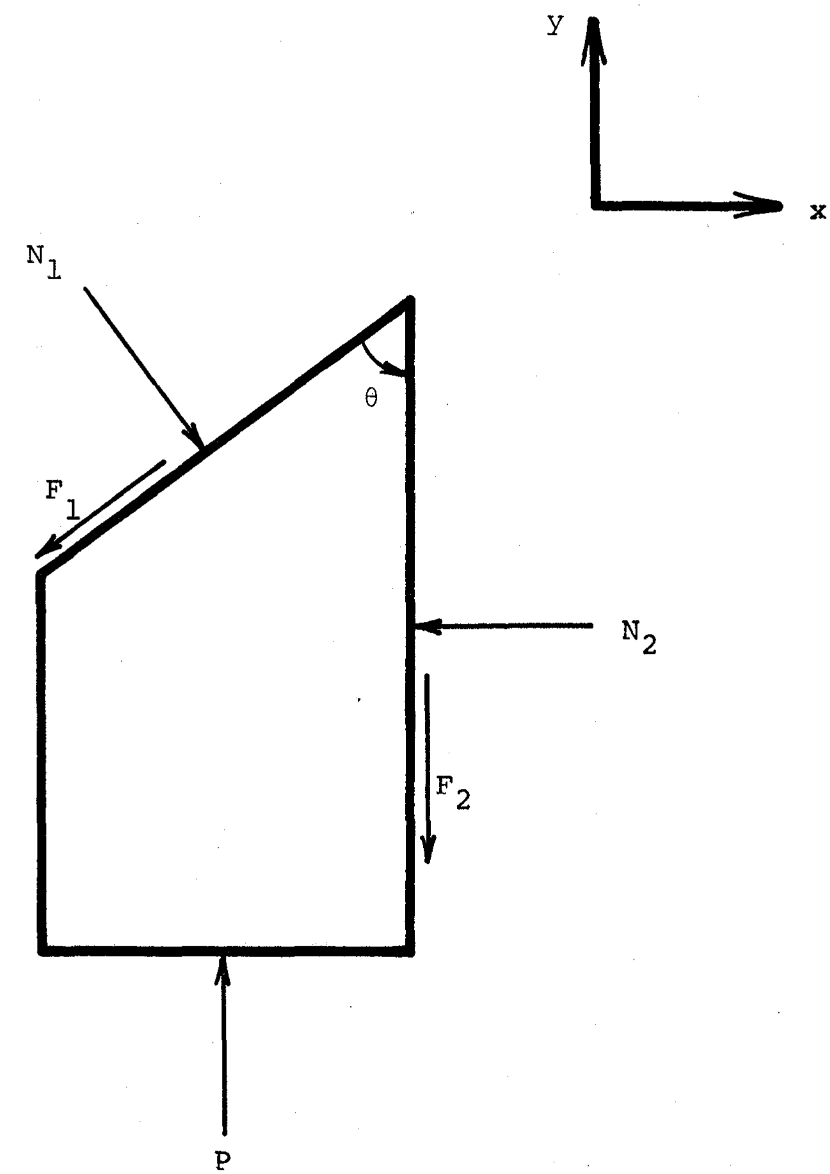


FIG. C-1. FREE BODY DIAGRAM OF A TYPICAL FRICTION SHOE DURING UPSTROKE

$$P - F_2 - N_1 \sin\theta - F_1 \cos\theta = 0 \tag{3}$$

with $F_1 = N_1\mu_1$ (4)

$$F_2 = N_2\mu_2 \tag{5}$$

we have four equations with four unknowns, F_1, F_2, N_1, N_2.

Applying specific values, based on data supplied by a leading truck manufacturer, where $\mu_1 = 0.5$, $\mu_2 = 0.37$, $\theta = 35°$

$$P = 3220 \text{ lb}$$

we can solve for N_1 and N_2, giving

$$N_1 = 2730 \text{ lb}$$

$$N_2 = 1452 \text{ lb}$$

So, $F_1 \cos\theta = .5 \times 2730 \times .819 \text{ lb} = 1118 \text{ lb}$

and $F_2 = .37 \times 1452 \text{ lb} = 537 \text{ lb}$

Therefore, the total vertical frictional force exerting on the wedge-shaped friction shoe on the upward stroke

$$F_u = F_1 + F_2 = 1655 \text{ lb}$$

Based on experimental studies performed by a truck manufacturer on a typical truck, it was concluded that the upward damping stroke dissipates 65% of the total energy in the cycle. Therefore, the total damping force per cycle

$$F = 1655/.65 \text{ lb} = 2546 \text{ lb}$$

This is the quantity assumed in the earlier analysis (equation (1) for the equivalent viscous damping) to be $F = f(x,\dot{x})$. It is seen that in the case of this typical truck, the force F can be modeled as a constant in this analysis.

The coefficient of viscous damping then becomes

$$C = \frac{2546}{\pi A}\int_0^{\tau} \cos\omega t \, dt = \frac{10184}{\pi\omega A}$$

This model holds good provided that the basic assumptions relating the linear and nonlinear motions are not violated.

Putting $\omega = 2\pi f$, assuming f approximately equal to the frequency of excitation, the amplitude of damping oscillation

A = 1/2(suspension group travel length + static deflection on suspension group)

$$= 1/2(3.6875 + 2.29) \text{ in} \simeq 3.0 \text{ in}$$

So, $C = \frac{10184}{2\pi^2 fA}$

$$C = \frac{172}{f}$$

This value of C is the equivalent viscous damping coefficient [lb-sec/in] for the typical truck damping system.

C-2. Determination of Suspension Spring Stiffnesses for a Typical Truck

The typical truck considered has three series of suspension springs classified by spring travels. The type with 3 11/16 in. travel is chosen to conform with the suspension type in the test truck of the 5000 Mile Box Car Vibration test. There are two suspension groups/truck and each group has the following stiffnesses:

Spring Designation	Spring Rate (lb/in)	No. of Springs	Total Spring Rate (lb/in)
Outer D5	2140	7	14980
Inner D5	1070	3	3210
Side Spring Outer 4143-2	984	2	1968
Side Spring Inner 4143-3	439	2	878
Total suspension stiffness/group = 21036			

This suspension group stiffness is represented by two springs in the mathematical model, and hence each one has the stiffness of 10518 lb/in. When the truck bolster bending stiffness is incorporated into these springs in the model, their stiffness value will be slightly lower.

C-3. Bending Stiffness of a Typical Truck Bolster

Before computing the bending stiffness of a typical truck bolster, the area moment of inertia of the bolster has to be estimated. Based on the mechanical drawing on the typical truck bolster on p. 831, CAR AND LOCOMOTIVE CYCLOPEDIA, 1966, it is assumed that sections I, II and III are uniform enough to have a constant area moment of inertia of its own (refer to Fig. C-2a).

The area of moment of inertia about its own centroidal axis is determined for each section and they are respectively

$$I_{Y-Y(I)} = 501.57 \text{ in}^4$$

$$I_{Y-Y(II)} = 811.3 \text{ in}^4$$

$$I_{Y-Y(III)} = 1431.3 \text{ in}^4$$

Assume the truck bolster is simple supported with a concentrated load in the center as shown, the area moments of inertia for each section is plotted in Figure C-2b and the corresponding shear and bending moment diagrams plotted in Figures C-2c and C-2d.

Applying Castigliano's strain energy method, and neglecting strain energy due to torsional and axial loading, the total energy in the truck bolster,

$$U = \frac{1}{2}\int_0^L \frac{M^2}{EI}\,dx + \frac{K}{2}\int_0^L \frac{Q^2}{AG}\,dx,$$

and so the deflection at P becomes

$$\Delta_P = \frac{\partial U}{\partial P} = \int_0^L \frac{M}{EI}\frac{\partial M}{\partial P}\,dx + K\int_0^L \frac{Q}{AG}\frac{\partial Q}{\partial P}\,dx$$

To find the deflection at the mid-span of the typical truck bolster, the following table is constructed:

Load Type	Section I		Section II		Section III	
	Load	$\frac{\partial \text{ Load}}{\partial p}$	Load	$\frac{\partial \text{ Load}}{\partial p}$	Load	$\frac{\partial \text{ Load}}{\partial p}$
Bending	$\frac{p}{2}(a-x)$	$\frac{1}{2}(a-x)$	$\frac{p}{2}(b-x)$	$\frac{1}{2}(b-x)$	$\frac{p}{2}(c-x)$	$\frac{1}{2}(c-x)$
Transverse Shear	$\frac{p}{2}$	$\frac{1}{2}$	$\frac{p}{2}$	$\frac{1}{2}$	$\frac{p}{2}$	$\frac{1}{2}$

Therefore, deflection

$$\begin{aligned}
\Delta_p = {} & \frac{2}{EI_1}\int_0^a \left(\frac{p}{2}(6.75 - x_1)\frac{1}{2}(6.75 - x_1)\right)dx_1 \\
& + \frac{2}{EI_2}\int_0^{(b-a)} \left(\frac{p}{2}(28.75 - x_2)\frac{1}{2}(28.75 - x_2)\right)dx_2 \\
& + \frac{2}{EI_3}\int_0^{(c-b)} \left(\frac{p}{2}(38.75 - x_3)\frac{1}{2}(38.75 - x_3)\right)dx_3 \\
& + \frac{2K}{A_1G}\int_0^a \left(\frac{p}{2}\right)\left(\frac{1}{2}\right)dx_1 + \frac{2K}{A_2G}\int_0^{(b-a)} \left(\frac{p}{2}\right)\left(\frac{1}{2}\right)dx_2 \\
& + \frac{2K}{A_3G}\int_0^{(c-b)} \left(\frac{p}{2}\right)\left(\frac{1}{2}\right)dx_3
\end{aligned}$$

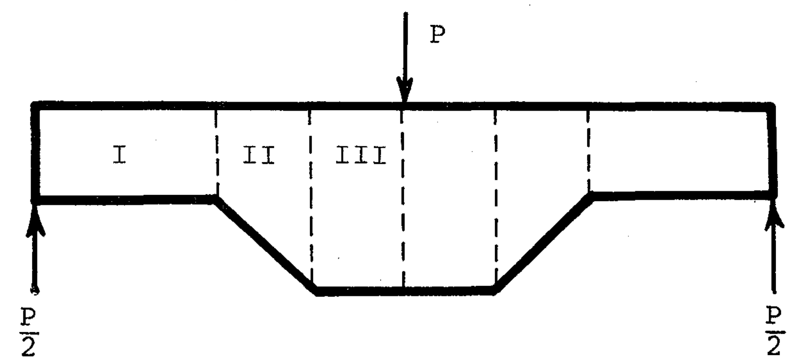


FIG. C-2A. CROSS-SECTION OF A TYPICAL TRUCK BOLSTER

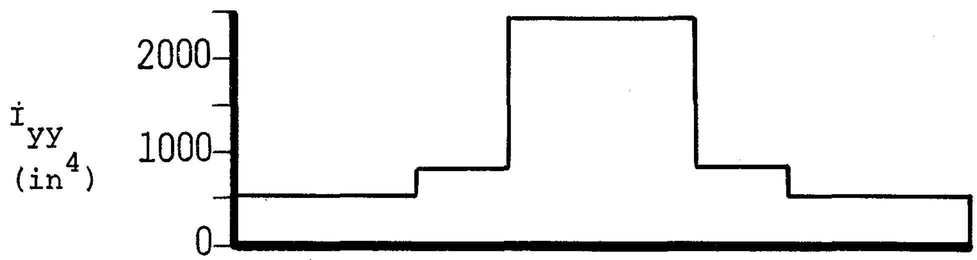


FIG. C-2B. AREA MOMENTS OF INERTIA

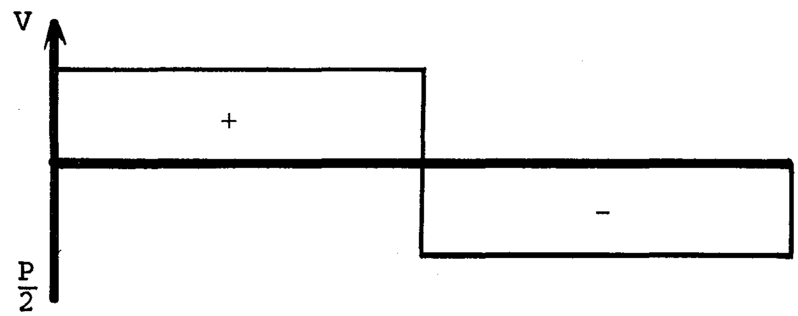


FIG. C-2C. TRANSVERSE SHEAR

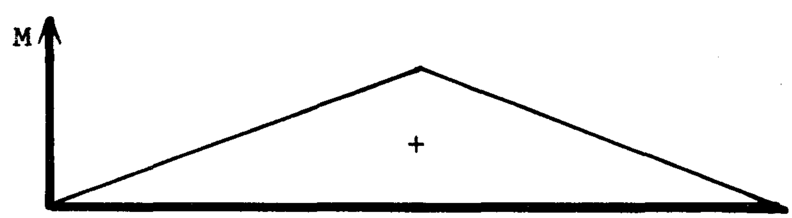


FIG. C-2D. BENDING MOMENT DIAGRAM

Putting a = 6.75", b = 28.75", c = 38.75", A_1 = 37.3 in^2, A_2 = 46.0 in^2, A_3 = 51.8 in^2, I_1 = 501.57 in^4, I_2 = 811.3 in^4, I_3 = 1431.3 in^4 and assuming for steel $E = 30 \times 10^6$ psi, $G = 12 \times 10^6$ psi, K = .12 in the last equation and after some evaluation,

$$\Delta_p = .3395 \times 10^{-6}\ p$$

Bending stiffness of the bolster

$$K_{Bol.} = \frac{p}{\Delta p} = 2{,}945{,}000 \text{ lb/in.}$$

Bolster Bending Stiffness Added in Series with the Suspension Springs

We have modelled the suspension group per truck as 4 springs each with stiffness 10518 lb/in. Incorporating the bending effect of the bolster, the stiffness value for the suspension springs in the model becomes

$$K_{truck} = 1/(1/2{,}945{,}000) + (1/4x10518) = 41666.67 \text{ lb/in.}$$

and the stiffness for each spring,

$$K = K_{truck}/4 = 10420 \text{ lb/in.}$$

With the bolster bending stiffness considered, the suspension spring stiffness is reduced by approximately 1%. Hence, based on this analysis, the bolster bending mode is not too significant for this particular truck type.

C-4. Descriptive Data for a 70-Ton Box Car

WEIGHTS AND INERTIAS		SYMBOLS
Total car weight on rail	223,050 lb	W
Loaded car body weight	206,000 lb	W_1*
Bolster weight per truck	1,150 lb	W_2, W_3
Side frame axle and wheel set weight per truck	7,300 lb	W_4, W_5
Freight element weight	150 lb	FM_1g
Car body rotational inertia about longitudinal axis	1,288,800 lb in sec^2	I_{y1}
Car body rotational inertia about lateral axis	16,650,000 lb in sec^2	I_{x1}
Car body rotational inertia about vertical axis	16,416,000 lb in sec^2	I_{z1}

---

*$W_1 = m_1 g$, where g = 386.4 in/sec^2

Truck bolster rotational inertia about longitudinal axis	12,000 lb in sec^2	I_{y2}, I_{y3}
Truck bolster rotational inertia about lateral axis	2,640 lb in sec^2	I_{x2}, I_{x3}
Truck bolster rotational inertia about vertical axis	12,000 lb in sec^2	I_{z2}, I_{z3}
Axle-side frame-wheel set rotational inertia about longitudinal axis	19,200 lb in sec^2	I_{y4}, I_{y5}
Axle-side frame-wheel set rotational inertia about lateral axis	120,000 lb in sec^2	I_{x4}, I_{x5}
Axle-side frame-wheel set rotational inertia about vertical axis	120,000 lb in sec^2	I_{z4}, I_{z5}

DIMENSIONS

Loaded car body center of gravity height above rail	98.5 in	
Loaded car body center of gravity height above center plate	72.5 in	R_1
Center plate radius	7 in	E
Side bearing spacing from car centerline	25 in	G_1
Spring group spacing from car centerline	39 in	H_2 to H_5
Side bearing clearance (static)	1/4 in	GAP
Spring travel to solid	3.69 in	TL
Bolster gib clearance	0.375 in	GIB
Truck center distance	39.5 ft	D
Axle centers in each truck	68 in	B_1
Distance between rail surface variations	39 ft	RL
Rail surface variation, max. vertical changes, rocking mode	3/4 in	S

Height of center plate above bolster center of gravity	4.5 in	R_2
Longitudinal distance between car body center of gravity and centerline of front bolster	237 in	D_1
Longitudinal distance between bolster center of gravity and suspension spring	6 in	D_2 to D_5
Car body center of gravity above bolster center of gravity	74.4 in	B

STIFFNESSES AND DAMPING CHARACTERISTICS

Suspension spring vertical stiffness	10420 lb/in	K_{13} to K_{20}
Suspension spring lateral stiffness	4425 lb/in	$K_{13}L$ to $K_{20}L$
Center plate and side bearing stiffness	666,000 lb/in	K_1, K_2, K_7, K_8
Gib spring stiffness	666,000 lb/in	KGIB
Bottoming spring stiffness	666,000 lb/in	KBOM
Bolster and truck torsional spring stiffness	6,730,425 lb/rad.	KT_{24}, KT_{35}
Bolster and truck pitching spring stiffness	4,200,000 lb/rad.	KP_{24}, KP_{35}
Freight cushioning stiffness	200 lb/in	KF
Freight cushion damping coefficient	18 lb sec/in	CF
Track vertical stiffness	105,000 lb/in	K_{21} to K_{28}
Track lateral stiffness	70,000 lb/in	$K_{21}L$ to $K_{28}L$

C-5. Track Input Equations

Two types of track profiles are currently being used for the computer simulations of rocking and bounce modes.

For the rocking mode, a half-staggered, rectified sine wave is used as the track profile. The equations which describe the track inputs are (refer to Fig. C-3a) as follows:

(a) For vertical track profiles:

(a) For vertical track profiles:

$$V_1 = S|\sin(\omega t)|$$

$$V_2 = S|\sin(\omega t - \phi_1)|$$

$$V_3 = S|\sin(\omega t - \phi_2)|$$

$$V_4 = S|\sin(\omega t - \phi_2 - \phi_1)|$$

$$V_5 = S|\sin(\omega t - \phi_3)|$$

$$V_6 = S|\sin(\omega t - \phi_4)|$$

$$V_7 = S|\sin(\omega t - \phi_5)|$$

$$V_8 = S|\sin(\omega t - \phi_5 - \phi_1)|$$

where $\phi_1 = \pi/2$, $\phi_2 = \pi B_1/RL$, $\phi_3 = \pi D/RL$, $\phi_4 = \phi_1 + \phi_3$,

$\phi_5 = \phi_2 + \phi_3$ and,

$\omega = \pi VEL/RL$,

RL = rail length,

B_1 = Distance between axle centers,

S = Maximum rail surface variations,

D = Truck center distance

(b) The corresponding lateral rail profiles adopted presently are:

$$R_9 = V_1/2$$

$$R_{10} = 0$$

$$R_{11} = V_3/2$$

$$R_{12} = 0$$

$$R_{13} = V_5/2$$

$$R_{14} = 0$$

$$R_{15} = V_7/2$$

$$R_{16} = 0$$

For the bounce mode, in order to decouple the responses somewhat, the two tracks are zero staggered, i.e. the rail joints on the opposite tracks are in phase, the following equations describe the track profiles:

(a) For vertical track profiles (refer to Fig. C-3b):

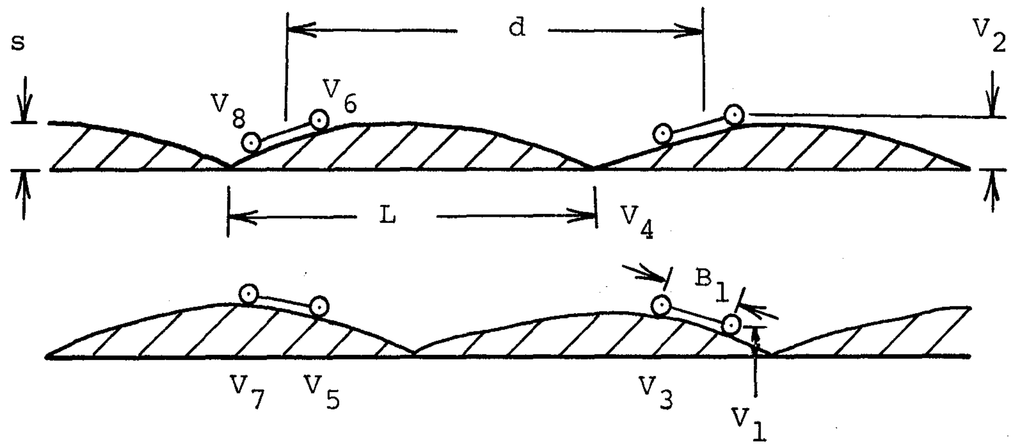


FIG. C-3A. ROCKING MODE VERTICAL TRACK PROFILE - HALF-STAGGERED

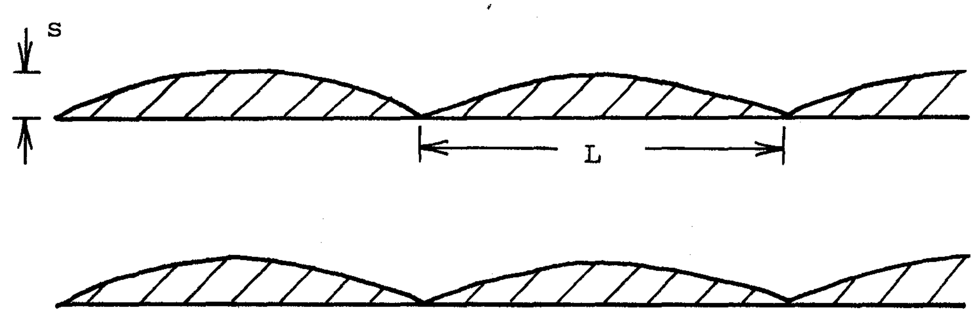


FIG. C-3B. BOUNCE MODE VERTICAL TRACK PROFILE - RAIL JOINTS IN PHASE

FIG. C-3C. LATERAL TRACK PROFILE

$$V_1 = S|\sin(\omega t)|$$

$$V_2 = V_1$$

$$V_3 = S|\sin(\omega t - \phi_2)|$$

$$V_4 = V_3$$

$$V_5 = S|\sin(\omega t - \phi_3)|$$

$$V_6 = V_5$$

$$V_7 = S|\sin(\omega t - \phi_5)|$$

$$V_8 = V_7$$

(b) The tracks are assumed to be parallel and no lateral variations.

REFERENCES AND BIBLIOGRAPHY

1-1. Circular No. FCD-2309, May 1, 1974, Association of American Railroads, "Freight Loss and Damage 1973".

1-2. Publication of Association of American Railroads: "Loading Pamphlets" and "General Information Series".

1-3. S. R. Pierce, "The Effects of Impact on Packaging from Consumer's Viewpoint," Rail Transportation Proceedings, 70-RR-3, ASME 1970.

1-4. Matsui, "Theoretical Study on the Forced Vertical Vibration of Bogie Cars," 1962.

1-5. Maucin and Toly, "Study of Vertical Suspension of Railway Vehicle (1968)," Monthly Bull. of International Rly. Congress Assn., Oct. 1968.

1-6. Liepens, A. A., "Digital Computer Simulation of Railroad Freight Car Rocking," J. Eng. for Industry, Nov. 1968, Paper No. 68-RR-3.

1-7. Monselle, D., "Truck-Bolster Dynamic Loadings Measured Under Harmonic Roll Conditions," ASME Tech. Paper, 71-WA/RT-6, 1971.

1-8. Cooperider, N. K., "The Hunting Behavior of Conventional Railway Trucks," ASME Tech. Paper 70-WA/RR-2, 1970.

1-9. Diboll, W. B. Jr. and Bieniecki, H. S., "Suspension Dynamics by Computer Simulation," ASME/IEEE Conference, Chicago, 1968.

1-10. Wiebe, D., "Damping Requirements to Control Vertical and Roll Motion of Freight Cars," presented at the Rail Transportation Division, Annual Meeting ASME, Nov. 20, 1974.

1-11. Tse, Y. H., "Method of Analysis for the Dynamic Behavior of a Flexible Body Railroad Freight Car," MMAE Dept., Illinois Institute of Technology, Chicago, Illinois.

1-12. Patel, R., "Method of Analysis of the Generalized Motions of a Railroad Freight Car," Mechanical and Aerospace Engineering Department, Illinois Institute of Technology, Chicago, Illinois.

2-1. Shum, K. and Willis, T., "Freight Damage Study, Report No. 1," May, 1974, IIT publication.

2-2. Guins, S. G. and Olson, L. L., "5000 Mile Box Car Vibration Test," prepared for a Panel Discussion on Freight Car Dynamics at the Rail Transportation Division of the American Society of Mechanical Engineers, Winter Annual Meeting, New York, New York, Nov. 19-20, 1974.

2-3. Meacham, H. C., "Analysis of Track Structures," presented at AAR Conf. on Track Train Dynamics Interaction, Dec. 1971, Chicago.

3-1. Keller, W. M. and Olson, L. L., "Final Report on the Evaluation of Cushioned Underframes," Report No. MR-443, Aug. 1965, Association of American Railroads.

7-1. Newton, R., "Fragility Assessment Theory and Test Procedures," Monterey Research Lab Inc., Monterey, Cal., 1968.

☆U.S. GOVERNMENT PRINTING OFFICE: 1978-261-264/78