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FREQUENCY DOMAIN COMPUTER PROGRAMS  
FOR PREDICTION AND ANALYSIS OF RAIL VEHICLE DYNAMICS  
Volume I: Technical Report

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FINAL REPORT

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02-Track-Train Dynamics

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16. Abstract  <p>Frequency domain computer programs developed or acquired by TSC for the analysis of rail vehicle dynamics are described in two volumes.</p> <p>Volume I defines the general analytical capabilities required for computer programs applicable to single rail vehicles and presents a detailed description of frequency domain programs developed at TSC in terms of analytical capabilities, model description, equations of motion, solution procedure, input/output parameters, and program control logic. Descriptions of programs FULL, FLEX, LATERAL, and HALF are presented. TSC programs for assessing lateral dynamic stability of single rail vehicles are also described.</p> <p>Volume II contains program listings including subroutines and card-by-card descriptions for inputting data for the four TSC frequency domain programs described in Volume I.</p>					
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## PREFACE

The Federal Railroad Administration (FRA) is conducting research and development programs to provide improved safety, performance, speed, reliability, and maintainability of rail transportation systems at reduced life cycle costs. A major portion of these efforts is related to improvement of the dynamic characteristics of rail vehicles, track structures, and train consists.

The Transportation Systems Center (TSC) is developing and maintaining a center for resources to be applied to programs for improved passenger service, most cost-effective freight service, and improved safety. As part of this effort, TSC is developing and identifying computer programs which have the capability to provide realistic predictions of rail system dynamic performance under field conditions.

This report describes frequency domain computer programs which are operational at TSC and their applicability to rail vehicle dynamic problems. Applications include prediction of the influence of passenger and freight vehicle design parameters on vehicle performance, based on response to various track irregularities. The influence of vehicle configuration (e.g. location of large suspended masses), suspensions, flexural modes, as well as track and roadbed parameters, may also be assessed. The track irregularities modeled include sinusoidal and random representations of surface, alignment, and cross-level track geometries.

This report has been assembled from several TSC working papers and from program documentation technical briefs which were produced by Dr. D. Sheldon and Messrs. Schweinhart, Luongo and Squires of Kentron Hawaii, Ltd., under ADP Support Services Project, Contract DOT-TSC-297. Messrs. Picardi and Kurzweil of TSC have also contributed to the material contained herein. The work described here was conducted under the RR-515 Rail Systems Dynamics Project, in support of the Federal Railroad Administration. The TSC project manager for this project is Dr. Herbert Weinstock. The FRA program manager for RR-515 is Ms. Grace Fay.

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## 1. INTRODUCTION

The objectives of this report include:

- a) Definition of the general analytical capabilities required for computer programs applicable to the dynamics of single rail vehicles and relatively short trains; and
- b) Presentation of a detailed description of computer programs developed at or acquired by TSC to date, including their usefulness in studying the dynamics of rail vehicles.

The programs developed by TSC and described herein are relatively simple and inexpensive to use. Collectively, they provide the following predictive capabilities for the dynamics of a single-rail vehicle on straight track: (1) vertical or lateral responses to sinusoidal or random representation of track surface, alignment or crosslevel irregularities; (2) the influence of track structure on vertical response, wheel-rail interaction forces, and track deflections; and (3) the influence of car body bending flexibilities and suspended masses on vertical response.

Other programs acquired by TSC, which are in various degrees of operational readiness, have the capability to predict vehicle lateral dynamic stability for a relatively simple single rail vehicle or truck component or for a multiple-vehicle system of up to 50 degrees of freedom. In addition these programs provide for more detailed representation of vehicle structure, and an alternative computation of frequency response by modal summation.

Additional analytic capabilities are being developed to provide: (a) unsteady dynamic behavior on curved track; (b) modeling of specific components in more exacting detail; and (c) non-linear effects. No general purpose program exists or is, perhaps, even desirable. In general, programs are complementary with specific application appropriate to specific needs.

## 2. ANALYTICAL REQUIREMENTS

### 2.1 DISCUSSION SCOPE

The scope of this discussion considers the dynamics of single-rail vehicles and relatively short consists. While such analyses are useful for all types of rail vehicles, their primary application is to passenger vehicles. The longitudinal dynamics of long consists, although not addressed here, is an important topic and is being pursued by the International Government Industry Program on Track/Train Dynamics administered by the Association of American Railroads (AAR).

The current level of analysis for the dynamics of single rail vehicles can be considered as three relatively distinct areas for which computer programs can provide a useful base for the design, evaluation, and understanding of vehicle behavior and performance.

### 2.2 STABILITY OF LATERAL DYNAMICS

Truck hunting, a lateral dynamic instability, is the critical limitation on safe high speed operation of conventional rail vehicles. In addition, reduction of low speed or body hunting effects is a major constraint on the rational design of suspension systems, trucks, and wheel assemblies.

Computer programs for lateral stability assessment simulate the dynamics of a vehicle moving at constant velocity on straight, ideal track. Lateral stability models include provisions for details of vehicle geometry, structure, and suspension. Description of the creep, friction, and gravity effect forces resulting from interaction of profiled wheels rolling on rails is the key to the validity of such analyses. Existing programs have sufficient capacity for prediction of the lateral motion of wheel-sets, trucks, single car vehicles, and multiple car trains.

### 2.3 RESPONSE TO TRACK IRREGULARITIES

For vehicles designed to ensure lateral stability in their range of operation, dynamic response to vertical and lateral irregularities of an ideal track structure determines the vibration

environment. These vibration levels provide measures of safety such as wheel/rail forces, sway and roll amplitudes, as well as indicate passenger comfort, freight security, and component life and reliability.

Programs which predict dynamic response to track irregularities should have the capacity for modeling subsystems, single vehicles, and consists. For response in a vertical plane, less detail of wheel/rail interaction forces is necessary than for a lateral stability analysis. However, models for this response analysis require detailed descriptions of suspension nonlinearities and of distributions of vehicle mass and structure.

Irregularities in vertical or lateral track geometry profiles determined from either measured or prescribed data serve as inputs to the model, in terms of harmonic or random distributions, as a function of the frequency or wavelength of the disturbance. In addition, the compliance of the track also contributes track disturbances which excite dynamic response of vehicles.

Current frequency response models predict decoupled lateral or vertical response to rail irregularities. For certain applications, such as a detailed investigation of a vehicle's ride vibration or component wear characteristics, a more elaborate model would be required for predicting the coupled response to lateral and vertical irregularities. Frequency response models also simulate car body flexibilities and effects of suspended masses. Excessive displacements of large suspended masses have caused operating problems on the Metroliner and SOAC vehicles, and can be a threat to other equipment items vital to safe vehicle operation. Computer programs are useful in optimizing the design of vibration isolation systems used in mounting large masses to the car body, and "tuning" them so that the suspended mass has a minimum effect on excitation of the car body flexible bending mode. Required clearances between the suspended mass and other equipment items mounted on the vehicle underframe may also be calculated.

## 2.4 CURVING PERFORMANCE

The prediction of vehicle motion on curved track can provide limits for vehicle design or track radius that permits guidance without flange/rail contact. Usually, these limits have been constraints or trade-offs on an optimal design for lateral stability.

For configurations where guidance requires flange contact, prediction of loads on the wheels during motion on curved track is fundamental to the evaluation, control, and understanding of the problems of derailment, wheel and rail wear, and the noise of wheel screech.

Current curving analyses consider only steady dynamic conditions in a turn, or somewhat unrealistic flange contact situations. The steady traverse analyses provide closed form expressions for minimum radius without flange contact. A simulation of rail vehicle dynamics for unsteady, flange guidance conditions on a curve would be particularly useful. In conjunction with current efforts by Dr. Law of Clemson University and Dr. Cooperider of Arizona State University, TSC is presently concentrating its effort toward this goal. Results of this activity will be reported under Contract DOT-TSC-902.

### 3. TSC FREQUENCY DOMAIN PROGRAMS FOR PREDICTION AND ANALYSIS OF RAIL VEHICLE DYNAMICS

Computer programs developed at TSC are capable of predicting the dynamics of single-rail vehicles on straight track for: (a) vertical or lateral response to sinusoidal and random representation of track irregularities or to wheel eccentricity effects; (b) the influence of track structure on vertical response, wheel/rail forces and deflections; (c) the influence of suspended masses on vertical response; and (d) the influence of car body flexibilities on vertical response.

These programs are linear, frequency domain programs which are relatively simple and inexpensive to use. They are particularly appropriate for first order analyses, such as estimating the effects of vehicle design parameter variation to illustrate performance trends or study trade-offs. Typical applications include assessing the effects of (a) vertical suspension and/or track parameters on wheel/rail forces, track deflections and vertical response; (b) car body bending and distributed mass characteristics on vertical response; and (c) lateral suspension and vehicle geometric parameters, such as c. g. height above rails, on vehicle lateral response.

Vehicle responses are decoupled lateral or vertical responses to a harmonic or random representation of track surface, alignment, or crosslevel irregularities. Response to harmonic track irregularities is provided in the form of transfer function plots (and/or printed data) for each system coordinate and normalized with respect to the amplitude of the track irregularity. For response to random track irregularities, a subroutine (designated RAILPL) has been coded for use with the frequency domain programs. Subroutine RAILPL allows random track irregularities to be described by one of the several experimental or empirical power spectral density representations, which, together with transfer functions computed in the frequency response programs, provides the required

data to compute vehicle response to random track irregularities. Results are expressed in the form of printed or plotted data over a continuous frequency range or as a bargraph representing mean square amplitudes over  $n^{\text{th}}$  octave bands ( $n=1, 3/4, 2/3, 1/2, 1/3, 1/4$ ).

In the following sections, each of the TSC programs is described in terms of the program's analytical capabilities, model description, equations of motion, solution procedure, input and output parameters, and program control logic. Volume II contains the program listings and card by card descriptions for inputting data for programs FULL, FLEX, LATERAL, and HALF.

## 4. PROGRAM "FULL"

### 4.1 APPLICATION

The primary usefulness of this model is to predict rigid car-body vertical and pitch acceleration and/or displacement frequency response to vertical sinusoidal rail surface irregularities for any specified point of the car body. The program also calculates vertical and pitch transmissibilities and acceleration spectra. (A detailed description of program output is contained in Section 4.5.)

The rigid car-body assumption used in this model provides a reasonable description of low frequency response until the car-body first bending mode frequency is approached, typically in the range of 6 to 12 Hz. If bending modes do not predominate the response, the useful range of the model would be extended to approximately 20 Hz.

### 4.2 MODEL DESCRIPTION

Program FULL is a digital computer program which calculates the dynamic response of a single-rail vehicle having a rigid body and two trucks, to vertical sinusoidal rail surface irregularities. Suspension characteristics are represented by rigid truck frames and linear spring and damper elements. Wheel/rail forces and track compliance are not considered.

The FULL car model shown in Figure 4-1 assumes a plane of symmetry equidistant between rails. This model represents a six-degree-of-freedom system which describes the linear and angular motion of the car in terms of the linear stiffness and damping of the suspensions, the masses and inertias of the car and trucks, and the rigid track inputs at each of the four axles. The car body is represented by a rigid body of mass  $m_2$ , moment of inertia  $I_2$ , and radius of gyration  $\rho$ , and each truck assembly is represented by the mass  $m_1$ . The primary suspension elements connecting the wheels and truck are linear springs of stiffness  $k_1$ , the equivalent of the equalizer-spring constant. The secondary suspension has a



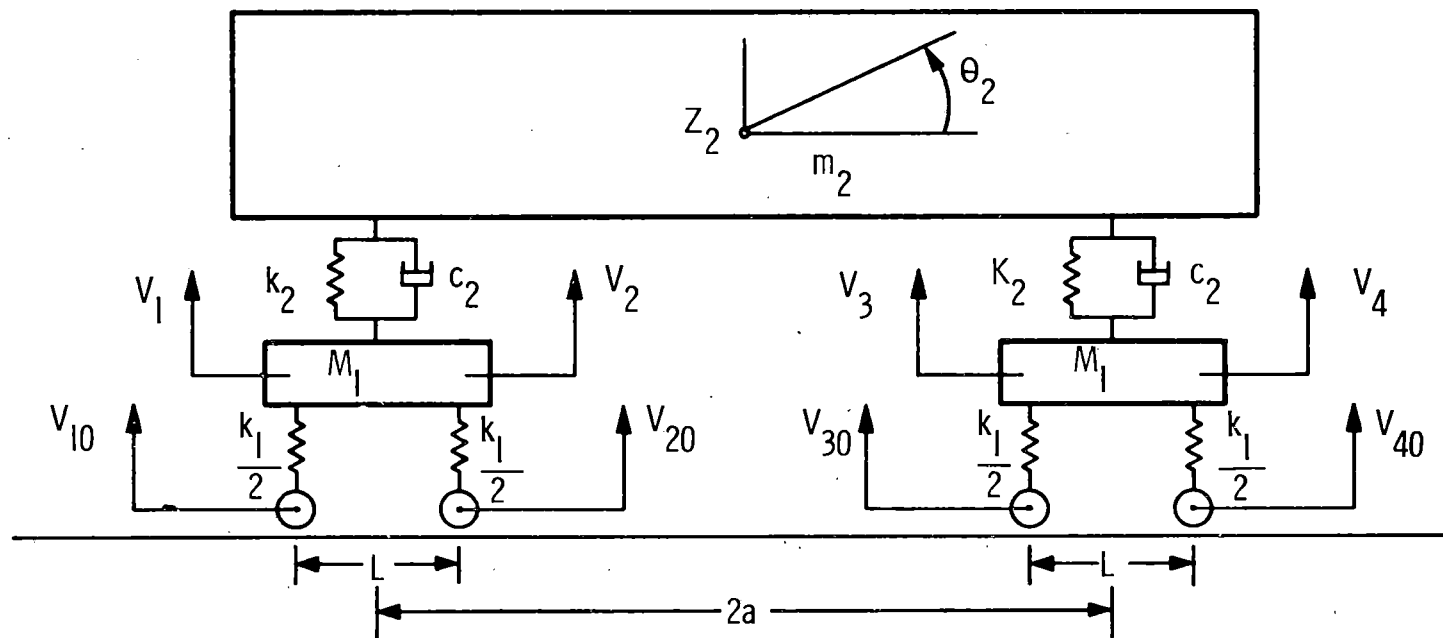


Figure 4-1. FULL Program Car Model

damping element,  $C_2$ , and bolster spring,  $k_2$ , connecting each truck to the body. The amplitude of the sinusoidal vertical track irregularity is denoted by  $v_0$ .

Since the vertical motions of a rail vehicle are effectively decoupled from the lateral motions, the response to symmetric irregularities in the vertical track profile geometry can be determined with a particularly simple rigid body model.

As a further simplification, it is assumed that the pitch motions of the trucks do not affect car body motions (e.g., Reference 1) and can be ignored so that the model response can be interpreted as the car bounce and pitch motions of two distinct two-degree-of-freedom systems. One system responds to bounce motions of the trucks driven by in-phase inputs, while the second responds to out-of-phase inputs at the wheels. For representations of track irregularities as sinusoidal inputs, the linear and angular displacements and accelerations of the carbody mass center can be expressed as independent functions of the driving frequency. The motion of any location in the car then can be calculated in terms of the motion of the mass center and its distance from the mass center.

Response of the vehicle to the track, characterized as a random input in terms of a power spectral density, can be computed from the acceleration responses by a simple extension of this analysis. (Refer to Section 3 for additional information.)

#### 4.3 EQUATIONS OF MOTION

The model used for this analysis is shown in Figure 4-1. Symmetry of the car with respect to a vertical plane in the longitudinal direction decouples vertical motions of the vehicle from lateral and roll motions. Therefore, the vertical motion of the vehicle can be conveniently and completely described in terms of a set of six coordinates,  $z_2$  and  $\phi$ , the vertical and angular displacements of the car body mass center, and  $v_1, v_2, v_3, v_4$ , the displacements of the trucks at their connections to the equalizer springs. For small amplitudes of the displacements, the linear equations of motion are:

$$m_2 \ddot{z}_2 + 2c_2 \left[ \dot{z}_2 - \frac{1}{4}(\dot{v}_1 + \dot{v}_2 + \dot{v}_3 + \dot{v}_4) \right] + 2k_2 \left[ z_2 - \frac{1}{4}(v_1 + v_2 + v_3 + v_4) \right] = 0 \quad (4-1a)$$

$$I_2 \ddot{\phi} + 2ac_2 \left[ a\dot{\phi} + \frac{1}{4}(\dot{v}_1 + \dot{v}_2 - \dot{v}_3 - \dot{v}_4) \right] + 2ak_2 \left[ a\phi + \frac{1}{4}(v_1 + v_2 - v_3 - v_4) \right] = 0 \quad (4-1b)$$

$$\begin{aligned} \frac{m_1}{4}(\ddot{v}_1 + \ddot{v}_2) + \frac{I_1}{L^2}(\ddot{v}_1 - \ddot{v}_2) + \frac{c_2}{2} \left[ a\dot{\phi} - \dot{z}_2 + \frac{1}{2}(\dot{v}_1 + \dot{v}_2) \right] + \frac{k_1}{2} v_1 \\ + \frac{k_2}{2} \left[ a\phi - z_2 + \frac{1}{2}(v_1 + v_2) \right] = \frac{1}{2} k_1 v_{10} \end{aligned} \quad (4-1c)$$

$$\begin{aligned} \frac{m_1}{4}(\ddot{v}_1 + \ddot{v}_2) - \frac{I_1}{L^2}(\ddot{v}_1 - \ddot{v}_2) + \frac{c_2}{2} \left[ a\dot{\phi} - \dot{z}_2 + \frac{1}{2}(\dot{v}_1 + \dot{v}_2) \right] + \frac{k_1}{2} v_2 \\ + \frac{k_2}{2} \left[ a\phi - z_2 + \frac{1}{2}(v_1 + v_2) \right] = \frac{1}{2} k_1 v_{20} \end{aligned} \quad (4-1d)$$

$$\begin{aligned} \frac{m_1}{4}(\ddot{v}_3 + \ddot{v}_4) + \frac{I_1}{L^2}(\ddot{v}_3 - \ddot{v}_4) + \frac{c_2}{2} \left[ \frac{1}{2}(\dot{v}_3 + \dot{v}_4) - a\dot{\phi} - \dot{z}_2 \right] + \frac{k_1}{2} v_3 \\ + \frac{k_2}{2} \left[ \frac{1}{2}(v_3 + v_4) - a\phi - z_2 \right] = \frac{1}{2} k_1 v_{30} \end{aligned} \quad (4-1e)$$

$$\begin{aligned} \frac{m_1}{4}(\ddot{v}_3 + \ddot{v}_4) - \frac{I_1}{L^2}(\ddot{v}_3 - \ddot{v}_4) + \frac{c_2}{2} \left[ \frac{1}{2}(\dot{v}_3 + \dot{v}_4) - a\dot{\phi} - \dot{z}_2 \right] + \frac{k_1}{2} v_4 \\ + \frac{k_2}{2} \left[ \frac{1}{2}(v_3 + v_4) - a\phi - z_2 \right] = \frac{1}{2} k_1 v_{40} \end{aligned} \quad (4-1f)$$

The wheel inputs,  $v_{10}$ ,  $v_{20}$ ,  $v_{30}$ ,  $v_{40}$ , represent the average of the two rail profiles.

Setting  $v_1 = v_2 = v_3 = v_4 = z_1$ , the in-phase trucks translation, in the equations of motion reduces the six equations to

$$m_1 \ddot{z}_1 + c_2(\dot{z}_1 - \dot{z}_2) + k_1 z_1 + k_2(z_1 - z_2) = \frac{1}{4} k_1 (v_{10} + v_{20} + v_{30} + v_{40}) \quad (4-2a)$$

$$m_2 \ddot{z}_2 + 2c_2(\dot{z}_2 - \dot{z}_1) + 2k_2(z_2 - z_1) = 0 \quad (4-2b)$$

Thus, the vertical translation of the car body can be viewed as the motion of mass  $m_2$  in the equivalent system of Figure 4-2(a) which is also described by the preceding two equations. Similarly, substitution of  $v_1 = v_2 = z_3 = -v_3 = -v_4$ , representing unsymmetric truck translation, in equations 4-1 (a-f) yields:

$$m_1 \ddot{z}_3 + c_2(\dot{z}_3 + a\dot{\phi}) + k_1 z_3 + k_2(z_3 + a\phi) = \frac{1}{4} k_1 (v_{10} + v_{20} - v_{30} - v_{40}) \quad (4-3a)$$

$$I_2 \ddot{\phi} + 2a c_2(\dot{z}_3 + a\dot{\phi}) + 2a k_2(z_3 + a\phi) = 0 \quad (4-3b)$$

which are also the equations of motion for the system in Figure 4-2(b).

These equivalent systems provide an interpretation of the car body motion as responses to simple base motions  $\bar{v}$  and  $v_\Delta$  shown in in Figure 4-2. Vertical translation of the car is excited by the average of the wheel motions,  $\bar{v} = \frac{1}{4}(v_{10} + v_{20} + v_{30} + v_{40})$ , while angular car body motion is driven by  $v_\Delta = 1/4(v_{10} + v_{20} - v_{30} - v_{40})$ .

The transfer functions for the linear and angular responses to these equivalent wheel inputs are

$$\frac{\bar{z}_2}{\bar{v}} = H_1(s) = \frac{\left(1 + \frac{2\beta s}{\omega_2}\right)}{\left[1 + \frac{2\beta s}{\omega_2} + \frac{\omega_1^2 + (2+\mu)\omega_2^2}{2\omega_1^2\omega_2^2} s^2 + \frac{(2+\mu)\beta}{\omega_2\omega_1^2} s^3 + \frac{s^4}{2\omega_1^2\omega_2^2}\right]} \quad (4-4)$$

$$\frac{a\bar{\phi}}{v_\Delta} = H_2(s) = \frac{-\left(1 + \frac{2G\beta s}{\omega_2}\right)}{\left[1 + \frac{2G\beta s}{\omega_2} + \frac{\omega_1^2 + (2G+\mu)\omega_2^2}{2G\omega_1^2\omega_2^2} s^2 + \frac{(2G^2+\mu)\beta}{G\omega_2\omega_1^2} s^3 + \frac{s^4}{2G\omega_1^2\omega_2^2}\right]} \quad (4-5)$$

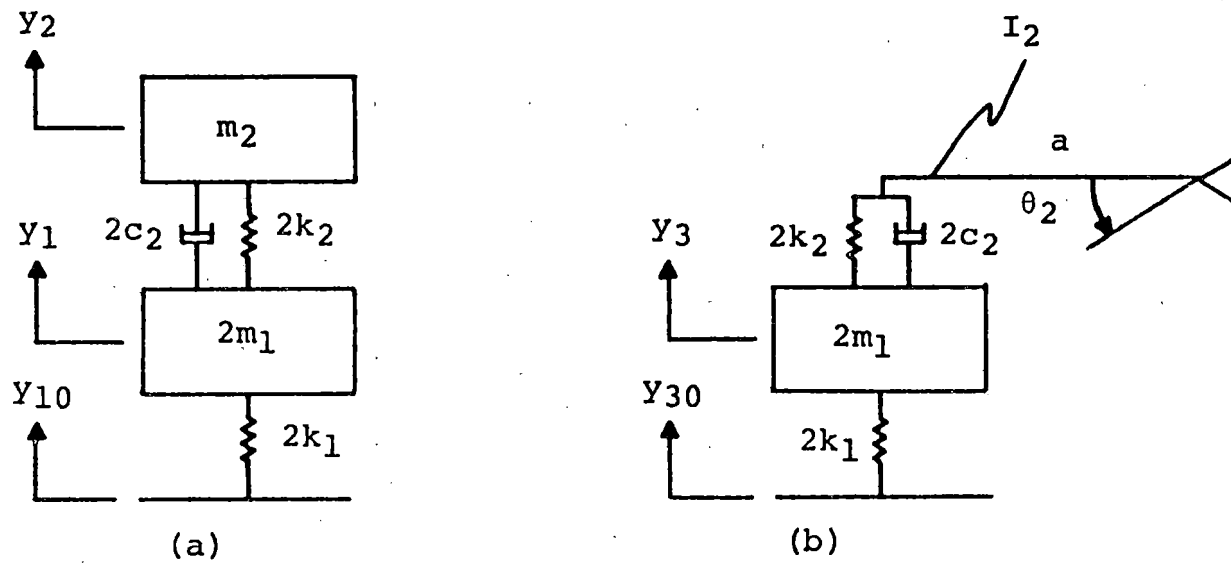


Figure 4-2. Simplified Models for Vertical Dynamics

where

$$\omega_1^2 = \frac{k_1}{m_1}, \quad \omega_2^2 = \frac{k_2}{m_2}, \quad \mu = \frac{m_2}{m_1}, \quad G = \frac{a^2}{\rho^2}, \quad \beta = \frac{c_2}{2m_2\omega_2}$$

Using these responses, the vertical motion of any other point in the car body,  $z$ , located at a distance,  $x_1$ , measured positively to the right of the mass center, can be computed as

$$z = z_2 + x_1\phi \quad (4-6)$$

The equivalent car model represented by the above system of equation is shown in Figure 4-3.

#### 4.4 SOLUTION PROCEDURE AND PROGRAM FLOW

After the data are read in, the values of the symmetrical and antisymmetric transfer functions are computed for each specified value of  $\beta$ , the damping ratio, and  $f$ , the perturbation frequency. Frequencies are generated over the desired response bandwidth by specifying a constant vehicle velocity ( $V$ ) or track wavelength ( $\lambda$ ) and adjusting the unspecified parameter according to  $f = V/\lambda$  to compute desired frequency values. The quantities

$$\left| \frac{z_2}{\bar{v}} \right| \quad \text{and} \quad \left| \frac{a\phi}{v_\Delta} \right| \quad (4-7)$$

are computed from Equations 4-4 and 4-5. From these responses,

$$\left| \frac{z_2}{v_o} \right| \quad \text{and} \quad \left| \frac{a\phi}{v_o} \right| \quad (4-8)$$

and obtained from multiplication by

$$\left| \frac{\bar{v}}{v_o} \right| \quad \text{and} \quad \left| \frac{v_\Delta}{v_o} \right| \quad (4-9)$$

To obtain the latter expressions, it is necessary to express the equivalent wheel inputs ( $\bar{v}$  and  $v_\Delta$ ) in terms of the harmonic driving function as follows. The vertical displacement of the lead wheelset is  $v_1 = v_o \sin \frac{2\pi x}{\lambda}$  where  $v_o$  is the amplitude of vertical track irregularity, and  $x$  is the position of the wheelset along the track irregularity wavelength  $\lambda$ . The vertical position of the remaining

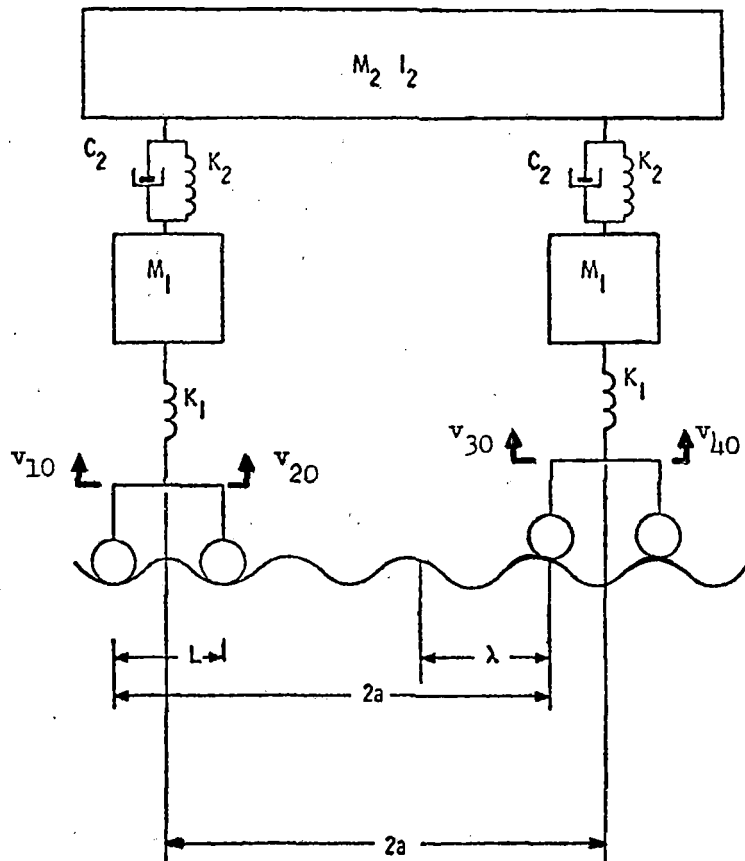


Figure 4-3. FULL Car Model Moving Over a Sinusoidal Rail Irregularity

wheelsets can be described in terms of the lead wheelset displacement plus a phase angle. Referring to Figure 4-3:

$$v_{10} = v_o \sin \frac{2\pi}{\lambda} (x) \quad (4-10a)$$

$$v_{20} = v_o \sin \frac{2\pi}{\lambda} (x+L) \quad (4-10b)$$

$$v_{30} = v_o \sin \frac{2\pi}{\lambda} (x+2a) \quad (4-10c)$$

$$v_{40} = v_o \sin \frac{2\pi}{\lambda} (x+2a+L) \quad (4-10d)$$

Substituting for the equivalent wheel inputs:

$$\bar{v} = \frac{1}{4}(v_{10}+v_{20}+v_{30}+v_{40}) \text{ and } v_{\Delta} = \frac{1}{4}(v_{10}+v_{20}-v_{30}-v_{40})$$

results in

$$\bar{v} = \frac{v_o}{4} \left[ \sin \bar{f}x + \sin \bar{f}(x+L) + \sin \bar{f}(x+2a) + \sin \bar{f}(x+L+2a) \right] \quad (4-11)$$

where  $\bar{f} = \frac{2\pi}{\lambda}$ . This can be written as

$$\frac{\bar{v}}{v_o} = \frac{1}{4} (A_1 \sin \bar{f}x + B_1 \cos \bar{f}x) \quad (4-12)$$

with

$$A_1 = 1 + \cos \bar{f}L + \cos 2\bar{f}a + \cos \bar{f}L \cos 2\bar{f}a - \sin \bar{f}L \sin 2\bar{f}a \quad (4-13a)$$

$$B_1 = \sin \bar{f}L + \sin 2\bar{f}a + \sin \bar{f}L \cos 2\bar{f}a + \cos \bar{f}L \sin 2\bar{f}a \quad (4-13b)$$

Alternatively, since  $x = Vt$ , this input in terms of the temporal frequency,  $f = \frac{2\pi v}{\lambda}$ , is

$$\frac{\bar{v}}{v_o} = \frac{1}{4} (A_1 \sin ft + B_1 \cos ft) \quad (4-14)$$

so that

$$\left| \frac{\bar{v}}{v_o} \right| = \frac{1}{4} \left[ A_1^2 + B_1^2 \right]^{1/2} \quad (4-15)$$



Similarly,

$$\frac{v_{\Delta}}{v_0} = \frac{1}{4}(A_2 \sin ft + B_2 \cos ft) \quad (4-16)$$

$$A_2 = 1 + \cos \bar{f}L - \cos 2\bar{f}a - \cos \bar{f}L \cos 2\bar{f}a + \sin \bar{f}L \sin 2\bar{f}a \quad (4-17a)$$

$$B_2 = \sin \bar{f}L - \sin 2\bar{f}a - \sin \bar{f}L \cos 2\bar{f}a - \cos \bar{f}L \sin 2\bar{f}a \quad (4-17b)$$

and

$$\left| \frac{v_{\Delta}}{v_0} \right| = \frac{1}{4} \left[ A_2^2 + B_2^2 \right]^{1/2} \quad (4-18)$$

The vertical response at one truck location is also computed as a sum of these responses, viz,

$$\left| \frac{z_2 + a\phi}{v_0} \right| \quad (4-19)$$

The acceleration responses

$$\left| \frac{\ddot{z}_2}{v_0} \right|, \left| \frac{a\ddot{\phi}}{v_0} \right|, \left| \frac{\ddot{z}_2 + a\ddot{\phi}}{v_0} \right| \quad (4-20)$$

can then be computed by a multiplication of their corresponding displacement responses by  $(2\pi f)^2/g$ . Similarly, the acceleration spectra (acceleration response to track surface irregularities of the form  $v_0 = a_1 \lambda$ )

$$\left| \frac{\ddot{z}_2}{a_1} \right|, \left| \frac{a\ddot{\phi}}{a_1} \right|, \left| \frac{\ddot{z}_2 + a\ddot{\phi}}{a_1} \right| \quad (4-21)$$

are the acceleration responses multiplied by the wavelength.

These computations are made inside two major loops, with the frequency loop nested just inside the damping ratio loop. Real variables are used throughout. Double indexed ( $\beta$  and  $f$ ) values of the responses are stored until required by the plotting routines. Sufficient time must be requested to complete all of the printing and plotting routines at the end of the program, or the output will be lost. (See Section 4.5.3).

## 4.5 PROGRAM "FULL" INPUT/OUTPUT AND CONTROL

### 4.5.1 Input

The physical input variables are given in Table 4-1, and the data control variables in Table 4-2. The program accepts up to nine frequency intervals with a different increment in each interval. The frequencies must lie in the range between 0.1 and 100 Hz, and the resulting number of frequencies is limited to 198.

### 4.5.2 Output

Up to 11 responses are plotted or printed vs. frequency, per user request. Log-log scales are used for all plotting. Program output variables are summarized in Table 4-3, and sample outputs are shown in Figures 4-4 through 4-10.

### 4.5.3 Program Control

The deck developed at TSC has been run on the Digital Equipment Corporation PDP-10 computer. Using the object desk, 10 specified frequency values require 5 seconds of CPU time and 10 minutes of Calcomp Plotter time. Approximately 5K words of memory are required. With the exception of plotting routines written at TSC, subroutines are not used. Appendix A in Volume II contains a listing of Program FULL.

TABLE 4-1. PROGRAM "FULL" REQUIRED PHYSICAL INPUT DATA

Variable (See Figure 4-1)	Description	Units	Card Format
V	Vehicle Velocity	MPH	F7.3
L	Truck Wheelbase	FEET	F7.3
2a	Vehicle Wheelbase	FEET	F7.3
W <sub>1</sub>	Truck Weight (less wheelsets)	LBS	F12.4
W <sub>2</sub>	Car Body Weight	LBS	F12.4
k <sub>1</sub>	Equalizer Spring Constant	LBS/IN	F12.4
k <sub>2</sub>	Bolster Spring Constant	LBS/IN	F12.4
β	Damping Ratio (Secondary suspension)	NONE	7F10.4
ρ	Centroidal Radius of Gyration of Car Body	FEET	F12.4

TABLE 4-2. PROGRAM "FULL" DATA CONTROL VARIABLES

INPUT Designation	Purpose	Possible Values	Card Format
NDF	Specifies number of frequency ranges considered	1 to 7	I2
IFREQ	Controls frequency ranges over which response is computed	1 or 2	I2
DF(I)	Specifies the number of points computed in a particular frequency range	I=1 to NDF	7I4
FL(I)	Specifies lower and upper frequency limits of each frequency range	I=1 to NDF+1	8F10.4
N1	Specifies number of damping ratios considered	1 to 7	I2
OPT1 OPT2 OPT3	Controls plotting options: Displacement, acceleration and acceleration spectra	TRUE (Plot) FALSE (Do not plot)	3L6
PRINT	Controls printouts	TRUE (Print) FALSE (Do not print)	L6

TABLE 4-3. PROGRAM "FULL" PLOTTED OUTPUT DATA

Figure No.	Title	Dependent Variable
4-4	Vertical Transmissibility	$z_2/\bar{v}$
4-5	Rotational (Pitching) Transmissibility	$a\phi/v_\Delta$
4-6	Vertical Car-Center Displacement due to Sinusoidal Track Irregularity	$z_2/v_o$
4-7	Pitching Response to Sinusoidal Track Irregularity	$a\phi/v_o$
4-8	Vertical Car Displacement (Over Truck) due to Sinusoidal Track Irregularity	$(z_2+a\phi)/v_o$
4-9	Vertical Car-Center Acceleration due to Sinusoidal Track Irregularity	$\ddot{z}_2/v_o$
N/A	Angular (Pitching) Acceleration due to Sinusoidal Track Irregularity	$a\ddot{\phi}/v_o$

TABLE 4-3. PROGRAM "FULL" PLOTTED OUTPUT DATA (Continued)

Figure No.	Title	Dependent Variable
N/A	Vertical Car Acceleration (Over Truck) due to Sinusoidal Track Irregularity	$(\ddot{z}_2 + a\ddot{\phi})/v_o$
4-10	Vertical Car-Center Acceleration Spectra due to Track Irregularity of the Form $v_o = a_1\lambda$	$\ddot{z}_2/a_1$
N/A	Angular (Pitching) Acceleration Spectra due to Track Irregularity of the Form $v_o = a_1\lambda$	$a\ddot{\phi}/a_1$
N/A	Vertical Car Acceleration Spectra (Over Truck) due to Track Irregularity of the Form $v_o = a_1\lambda$	$(\ddot{z}_2 + a\ddot{\phi})/a_1$

V = 60.00 MPH  
 V = 88.00 FT/SEC  
 LL = 6.83 FT  
 2R = 44.58 FT  
 RHO = 14.86 FT

M1 = 17900.00 LB  
 M2 = 74400.00 LB  
 K1 = 39850.00 LB/INCH  
 K2 = 14200.00 LB/INCH

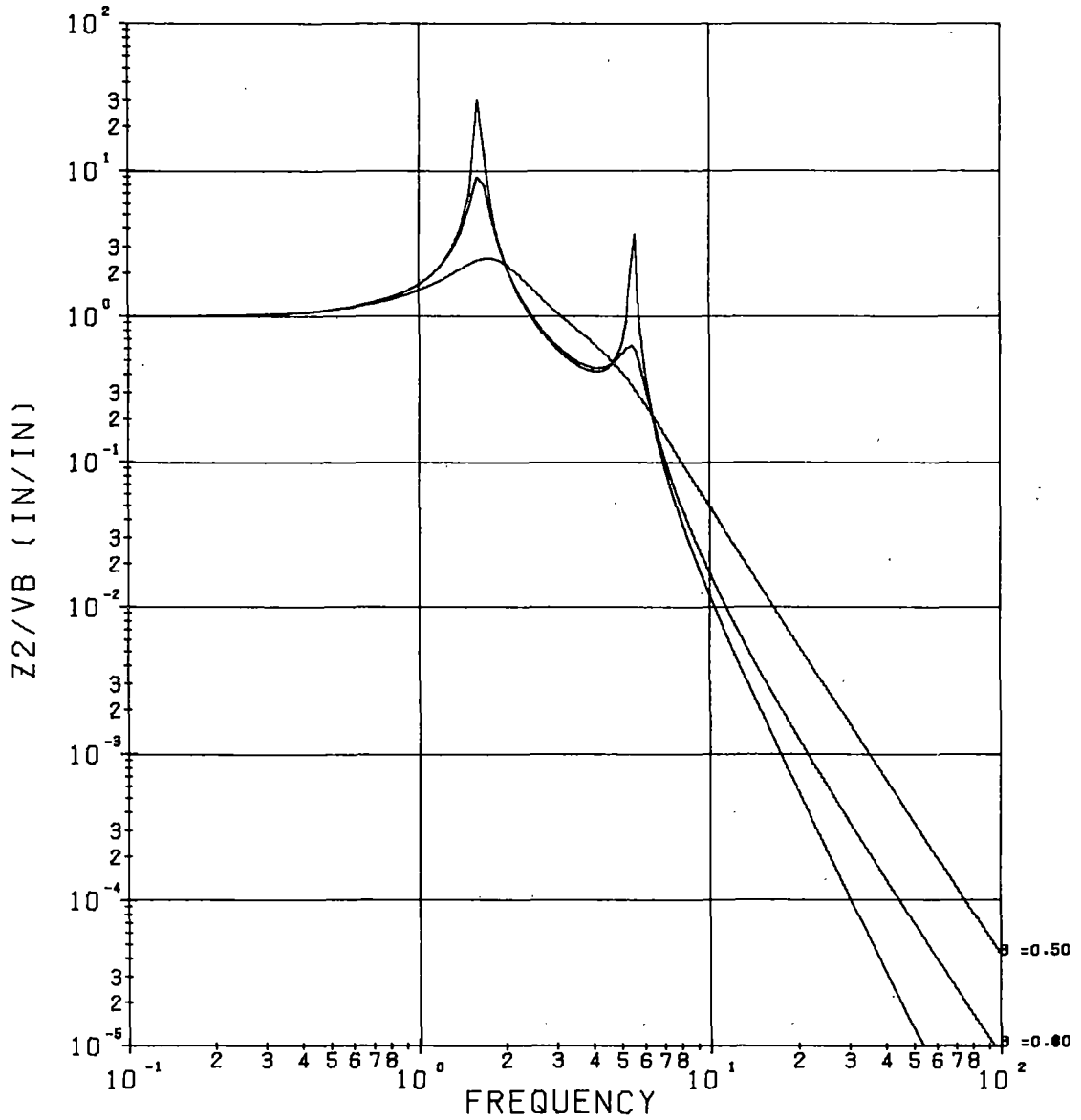


Figure 4-4. Vertical Transmissibility - Graphic Diagram

V =	60.00 MPH	H1 =	17900.00 LB
V =	88.00 FT/SEC	H2 =	74400.00 LB
LL =	6.83 FT	K1 =	39850.00 LB/INCH
2R =	44.58 FT	K2 =	14200.00 LB/INCH
RHD =	14.86 FT		

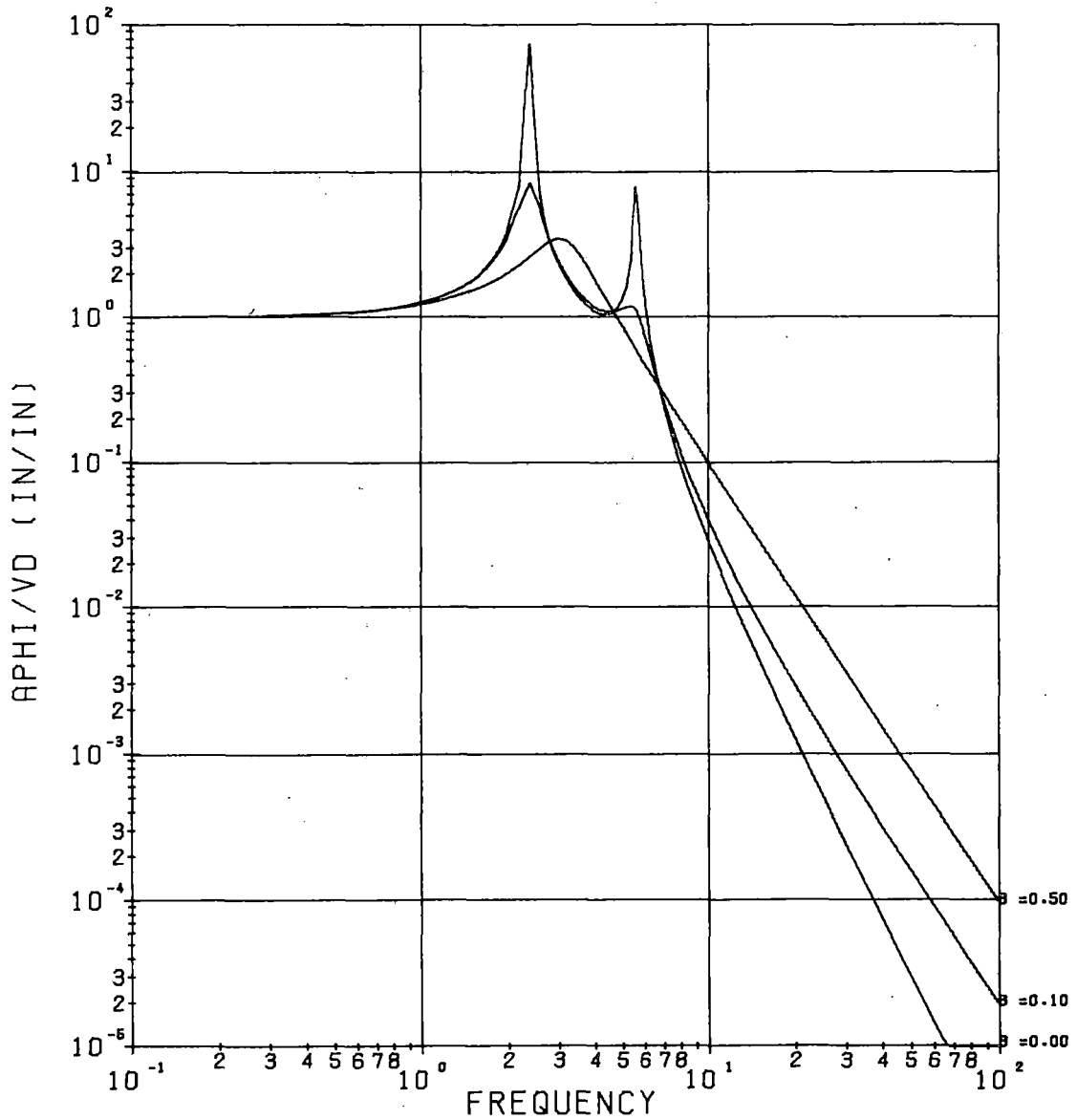


Figure 4-5. Rotational (Pitching) Transmissibility - Graphic Diagram



V =	60.00 MPH	M1 =	17900.00 LB
V =	88.00 FT/SEC	M2 =	74400.00 LB
LL =	6.83 FT	K1 =	39850.00 LB/INCH
2R =	44.58 FT	K2 =	14200.00 LB/INCH
RHO =	14.86 FT		

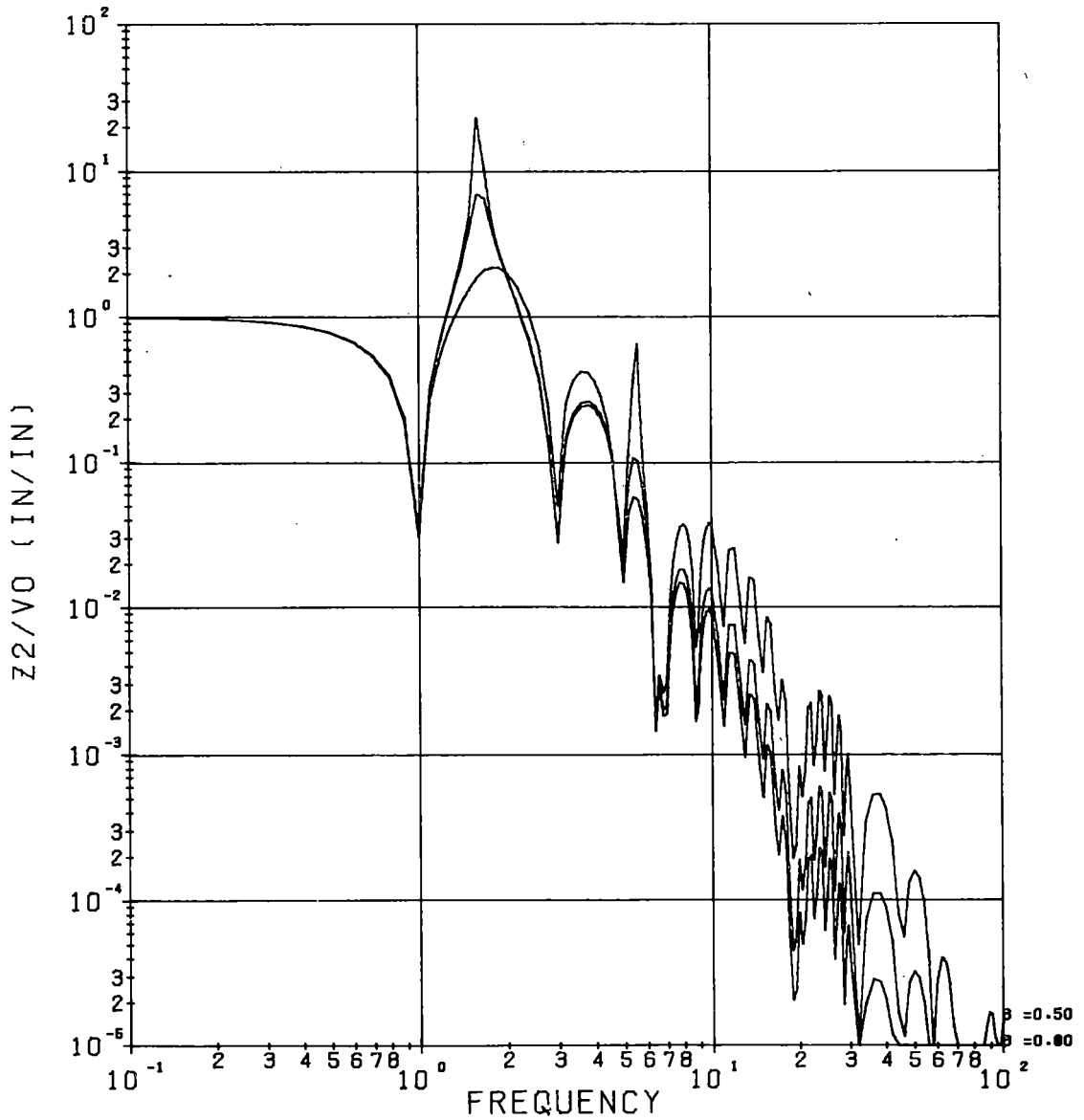


Figure 4-6. Vertical Car-Center Displacement Due to Sinusoidal Track Irregularity - Graphic Diagram

V =	60.00 MPH	W1 =	17900.00 LB
V =	88.00 FT/SEC	W2 =	74400.00 LB
LL =	6.83 FT	K1 =	39850.00 LB/INCH
2R =	44.58 FT	K2 =	14200.00 LB/INCH
RHO =	14.86 FT		

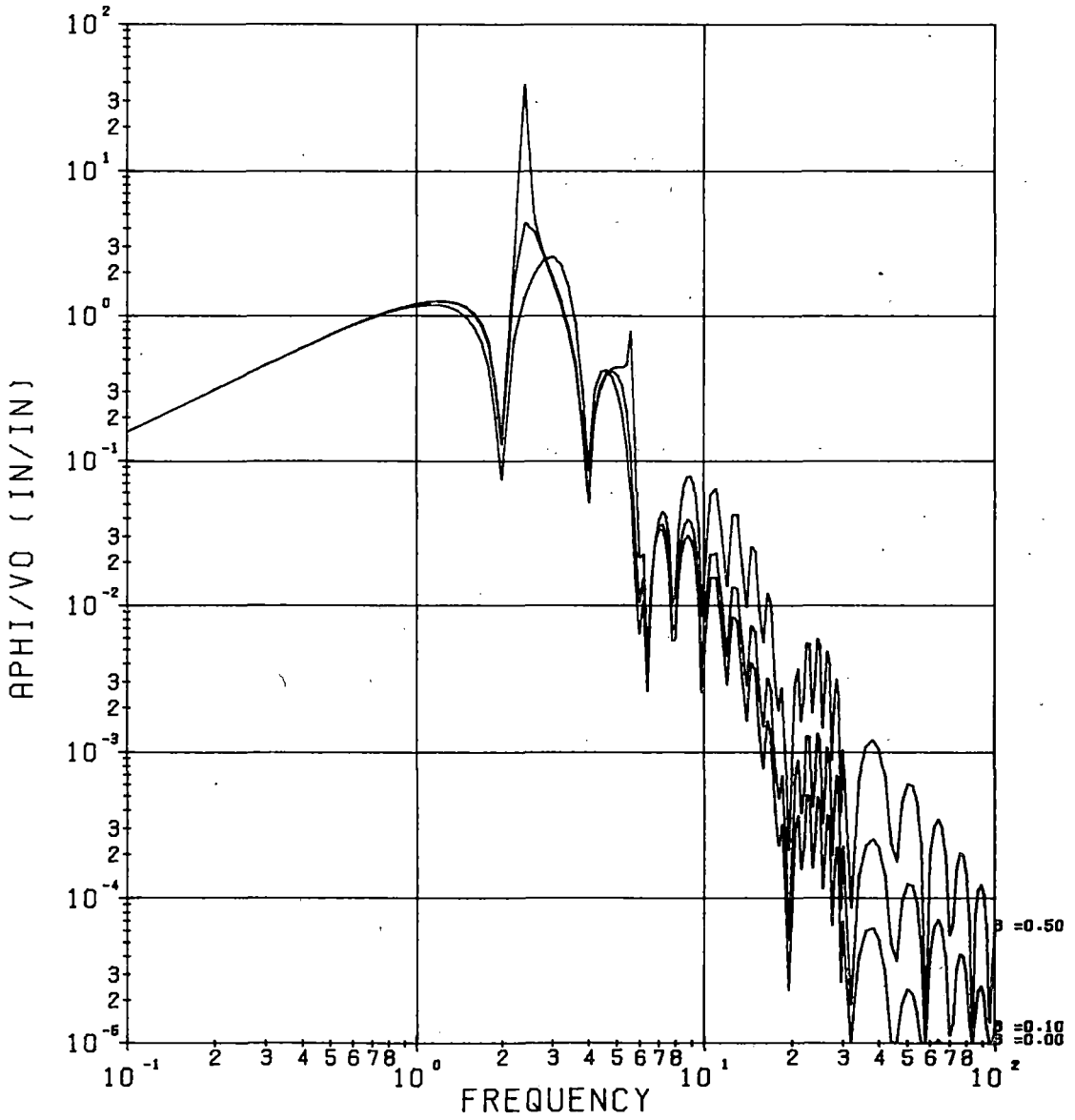


Figure 4-7. Pitching Response to Sinusoidal Track Irregularity - Graphic Diagram

V =	60.00 MPH	W1 =	17900.00 LB
V =	88.00 FT/SEC	W2 =	74400.00 LB
LL =	6.83 FT	K1 =	39850.00 LB/INCH
2R =	44.58 FT	K2 =	14200.00 LB/INCH
RHO =	14.86 FT		

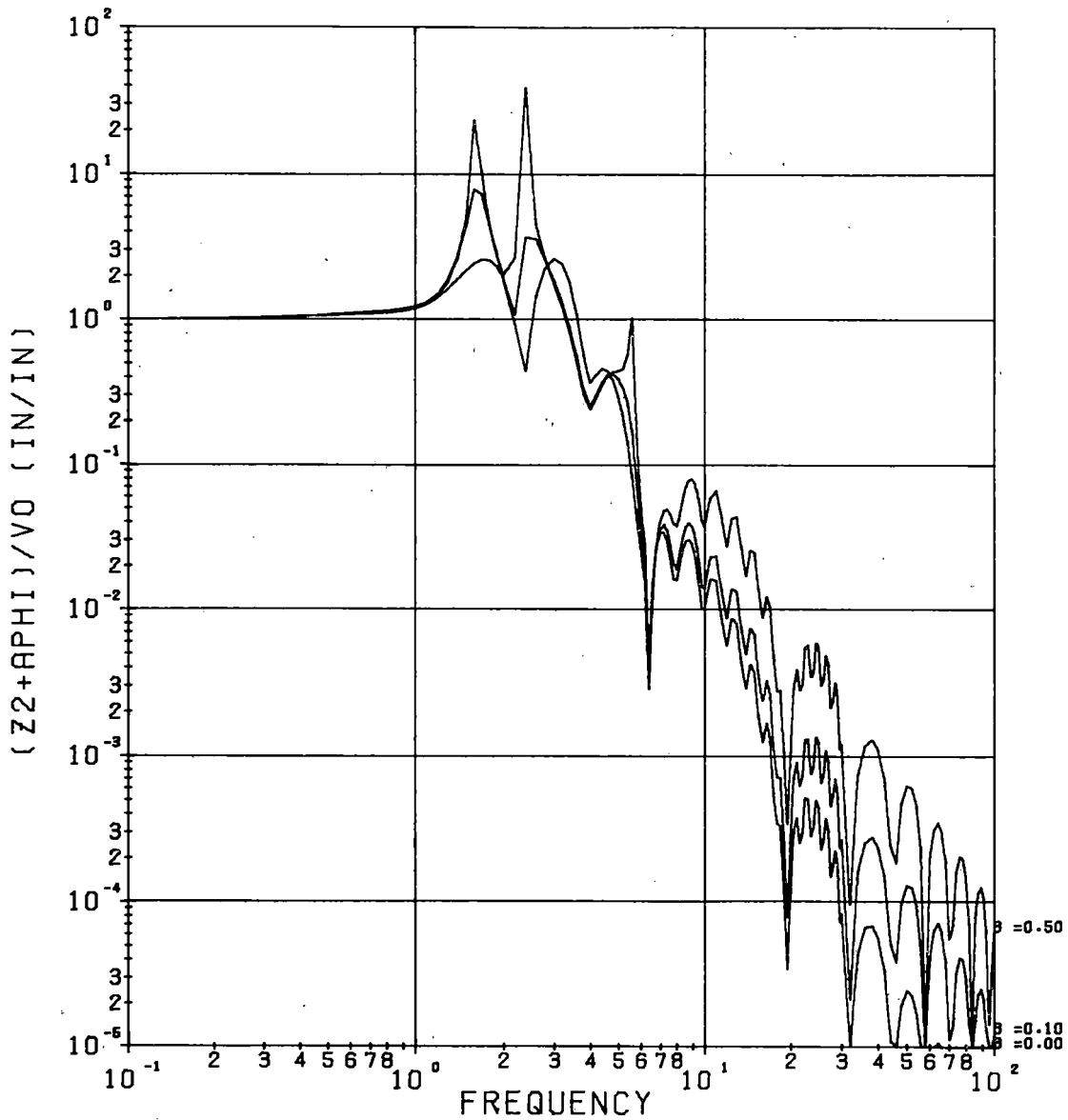


Figure 4-8. Vertical Car Displacement (Over Truck) Due to Sinusoidal Track Irregularity - Graphic Diagram

V = 60.00 MPH  
 V = 88.00 FT/SEC  
 LL = 6.83 FT  
 2R = 44.58 FT  
 RHO = 14.86 FT

W1 = 17900.00 LB  
 W2 = 74400.00 LB  
 K1 = 39850.00 LB/INCH  
 K2 = 14200.00 LB/INCH

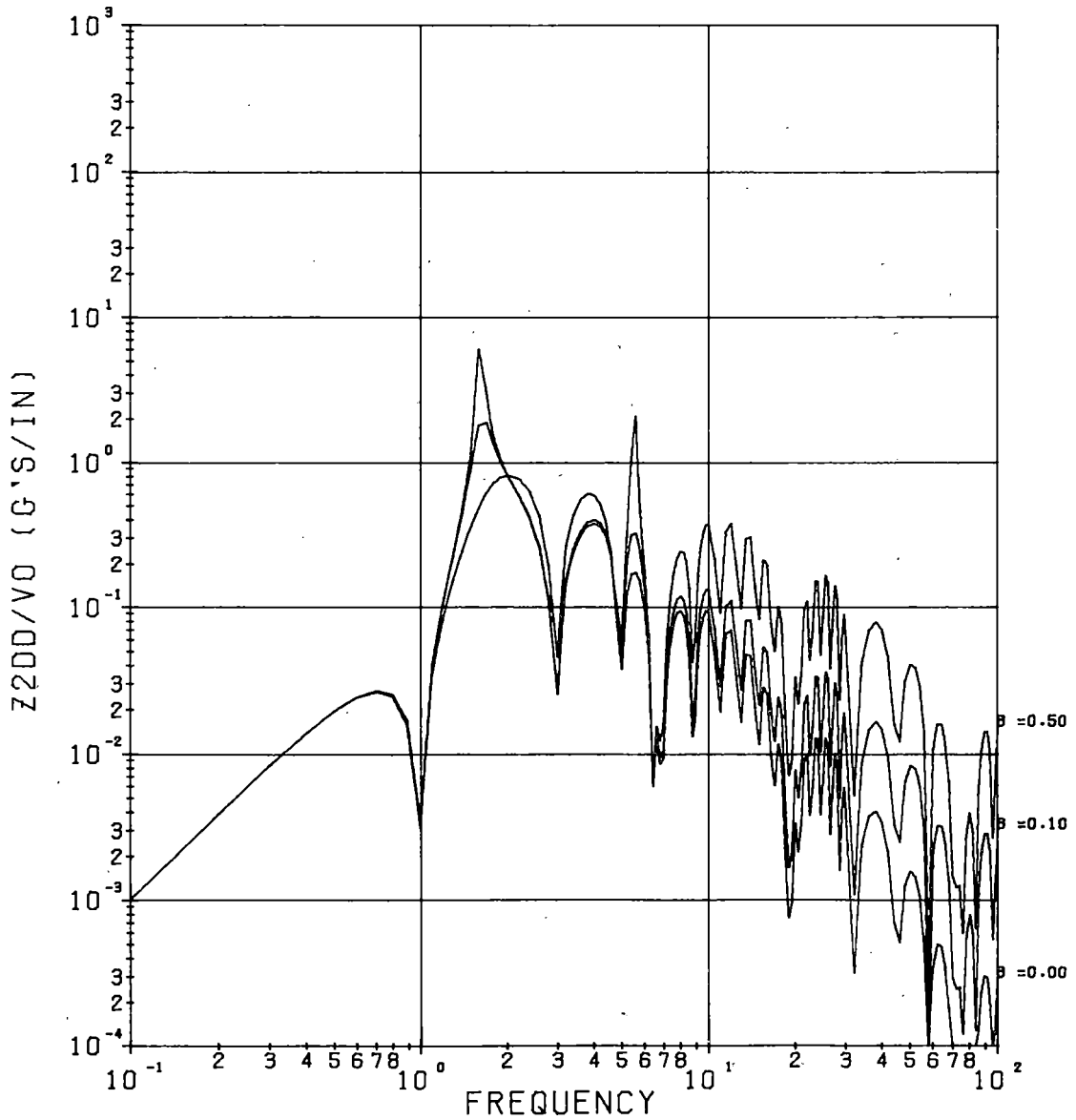


Figure 4-9. Vertical Car-Center Acceleration Due to Sinusoidal Track Irregularity - Graphic Diagram

V = 60.00 MPH  
 V = 88.00 FT/SEC  
 LL = 6.83 FT  
 2R = 44.58 FT  
 RHO = 14.86 FT

W1 = 17900.00 LB  
 W2 = 74400.00 LB  
 K1 = 39850.00 LB/INCH  
 K2 = 14200.00 LB/INCH

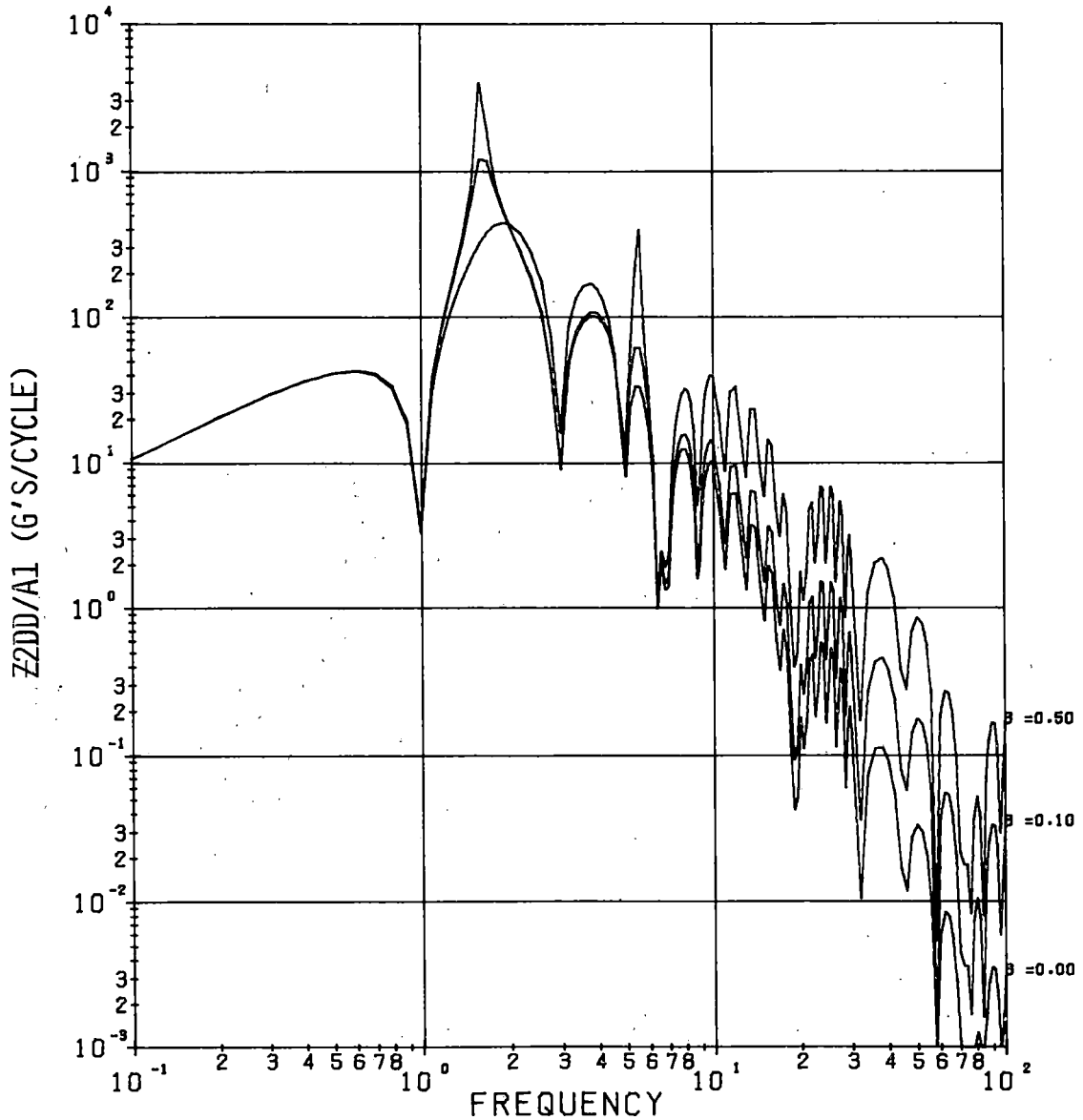


Figure 4-10. Vertical Car-Center Acceleration Due to Track Irregularity of the Form  $v_0 = A_1 \times \text{Wavelength}$  - Graphic Diagram

## 5. PROGRAM "FLEX"

### 5.1 APPLICATION

Program FLEX is used to calculate the frequency response at particular locations of a linear rail vehicle model with flexible car body to vertical sinusoidal track surface irregularities. FLEX is primarily useful for predicting effects of car-body flexibilities including the influence of mass and structural rigidity distributions, suspended masses, and wheel eccentricities on such parameters as vehicle ride roughness and relative displacements between components (which may be interpreted as an indicator of component wear).

Program FLEX is particularly suited to problems where the car-body first bending mode lies near or within the range of desired frequency response. Typically, the first bending mode has significant influence on vertical response in the 6 to 15 Hz bandwidth.

Car-body flexibility is modeled using techniques which can vary from a uniform beam approximation to the detailed prescription of distributions of mass, structure, and mode shape determined from test results. Normal program output includes plots of displacement and acceleration frequency response and acceleration spectra over the truck, at the car body center of mass, and at hanging mass positions.

### 5.2 MODEL DESCRIPTION

FLEX is a digital computer program which calculates the vertical dynamic response of a single rail vehicle having two (one-degree-of-freedom) trucks, a flexible car-body and a suspended mass, to vertical sinusoidal rail surface irregularities. Suspension characteristics are represented by linear spring elements for primary and secondary suspension and linear secondary-suspension damper elements. Wheel/rail forces and track compliance are not modeled.

It is assumed that vertical motions are decoupled from lateral motions and that the pitching motions of the trucks do not affect

car body response and can be ignored. The FLEX car model shown in Figure 5-1 assumes a plane of symmetry equidistant between rails. Model parameter descriptions (required input data) are provided in Table 5-1.

The effect of car body flexibility is considered by including the first bending mode of the car body,  $w(x,t)$ , as one of the six degrees of freedom of the vehicle. The other degrees of freedom are: the vertical position of the car-body center of mass,  $z_2$ ; the rotation of the car-body neutral axis about its center of mass,  $\phi$ ; the truck vertical displacements,  $y_1$  and  $y_2$ ; and the hanging mass (e.g., transformer) vertical displacement,  $z_1$ .

The bending mode and its natural frequency can be specified either by experimental data or a simple analytical description of the car body. A description of car body flexibility modeling options is contained in Section 5.4. In either case, it is assumed in the motion that the bending response of the car body is dominated by the first mode for excitations (loads), over a frequency band from zero to the first bending frequency. This approximation improves as the separation of the first and second natural frequencies increases and as the ratio of energy stored in the car body to the energy stored in the suspension springs increases. The program accepts four different descriptions of the fundamental bending mode of the car body. Input data required by each of the four options are listed in Table 5-2. The particular option requested is specified by the input variable INCODE. The program contains no internal check on the accuracy of the bending mode description.

The track is assumed to be rigid with sinusoidal perturbations of amplitude,  $v_0$ , in the vertical plane. An additional perturbation may also be specified in the form of wheel eccentricities. In this case, the wheel geometry is described by a circular profile having a (wheelset) center of rotation which is offset from the true center by a distance  $C_i$  ( $i = 1$  to  $4$ ) for the four wheels, normalized with respect to the track vertical perturbation,  $v_0$ . (See Figure 5-2.) A phase angle is specified for each wheel which relates the position of the maximum radius of the eccentric wheel to the horizontal track baseline.

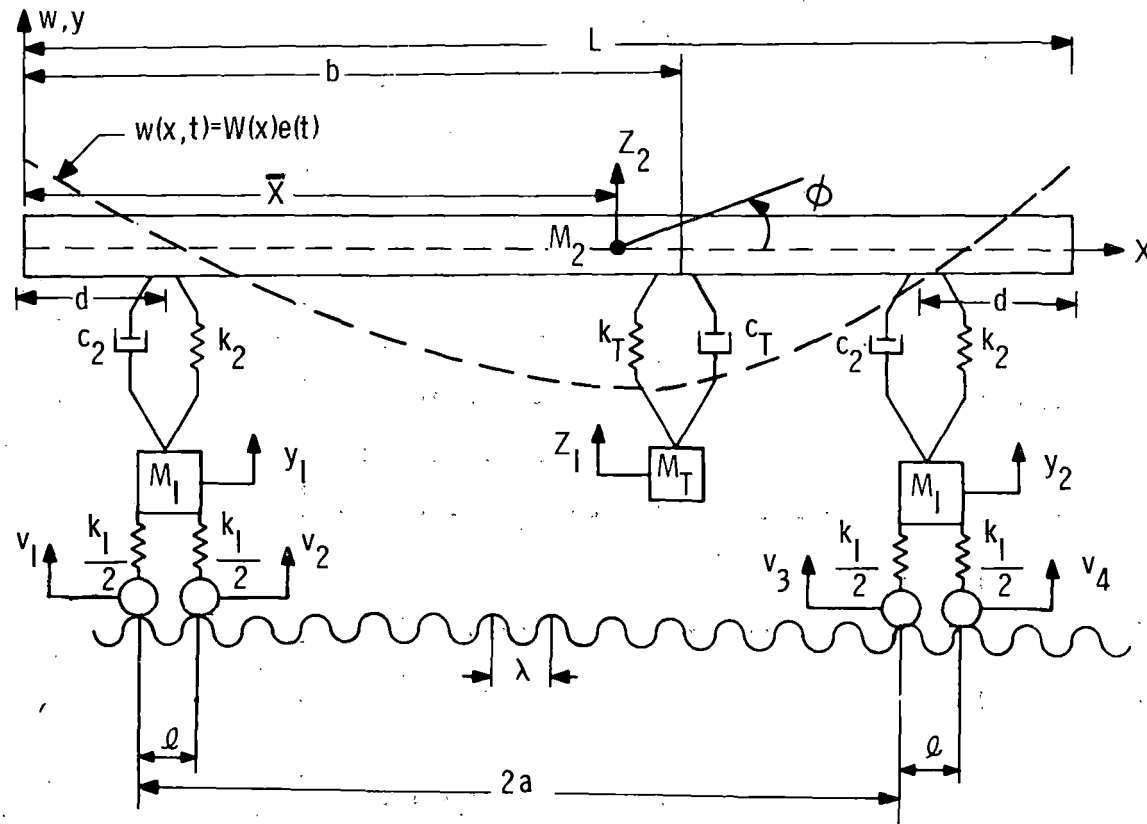


Figure 5-1. Flexible Car Model



TABLE 5-1. PROGRAM "FLEX" REQUIRED PHYSICAL INPUT DATA

Parameter (See Figure 5-1)	Designation	Description	Units	Card Format
b	B	Position of hanging mass	FEET	F10.5
$\bar{x}$	XBAR	Position of car center of gravity	FEET	F10.5
L	LL	Length of car	FEET	F10.5
d	D	Inset of truck center from each end	FEET	F10.5
$\ell$	L	Truck wheel base	FEET	F10.5
a	A	One-half of vehicle wheelbase	FEET	F10.5
$M_1$	W1	Weight of truck	LBS	F10.5
$M_2$	W2	Weight of car	LBS	F10.5
$M_T$	WT	Weight of hanging mass	LBS	F10.5
$K_1$	K1	Primary suspension spring constant	LBS/IN	F10.5
$K_2$	K2	Secondary suspension spring constant	LBS/IN	F10.5
$\beta_2$	BETA2	Secondary suspension damping ratio	NONE	F10.5
$K_T$	KT	Hanging mass spring constant	LBS/IN	F10.5
$\beta_T$	BETAT	Hanging mass damping ratio	NONE	F10.5
V <u>OR</u> $\lambda$	VORLAM	Velocity of vehicle <u>OR</u> wavelength of track irregularity	MPH <u>OR</u> INCHES/ CYCLE	F9.2
g	G	Acceleration of gravity	FT/SEC <sup>2</sup>	F10.5

WHEELSET 1  
ROUND

$$EPS1 = \frac{C_1}{v_0} = 0$$

THETA1 = 0°

WHEELSET 2  
ECCENTRIC

$$EPS2 = \frac{C_2}{v_0}$$

THETA2 = 45°

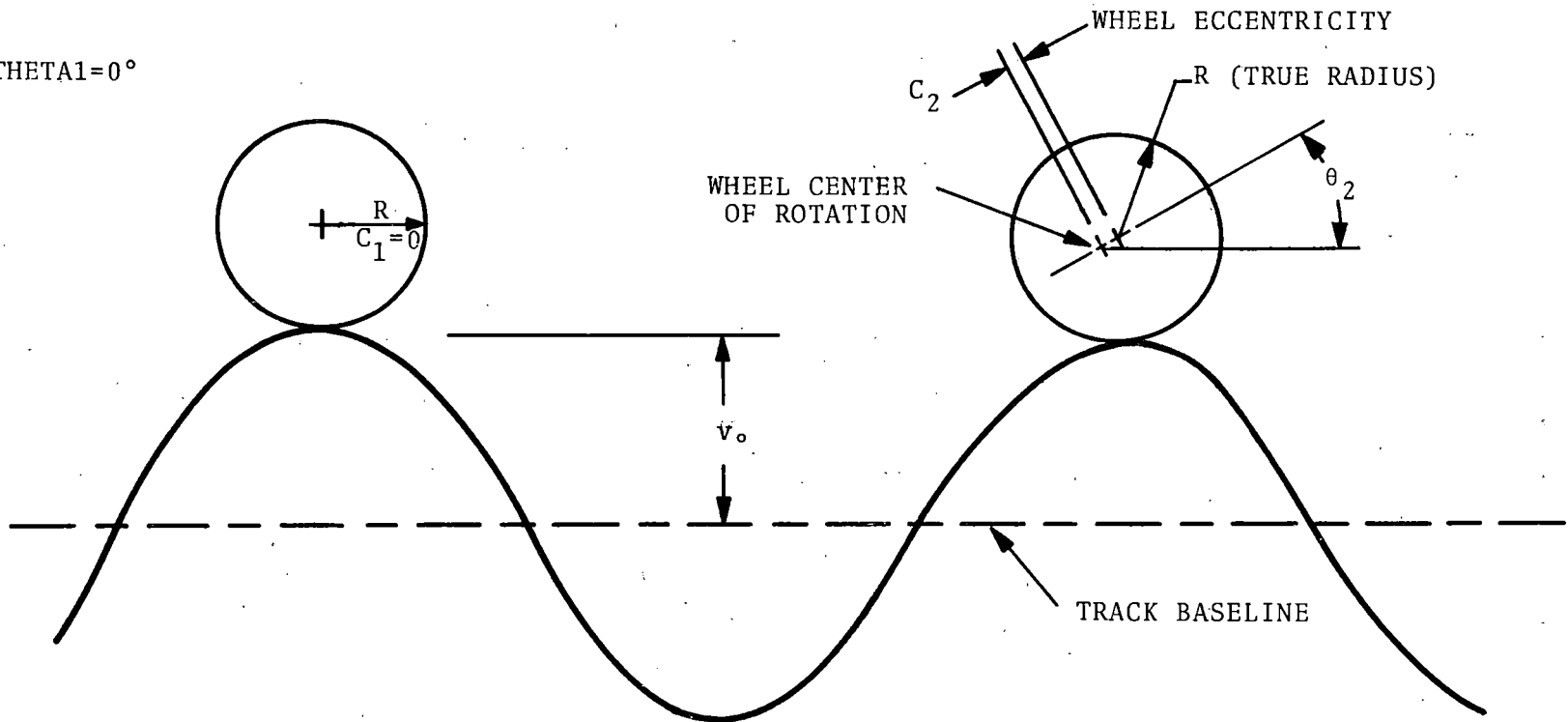


Figure 5-2. Geometry of Wheel Eccentricity

TABLE 5-2. INPUTS REQUIRED TO FORM A COMPLETE SET OF CAR BODY DATA

Value of Input Variable INCODE	Input Designation	Description	Units	Card Format
1	FB	Fundamental bending frequency of car	Hz	F10.5
2	FB	Fundamental bending frequency of car	Hz	F10.5
	INTDIM	Number of points in tabular functions	NONE	I10
	INTDEL	X-axis increments for tabular functions	FEET	F10.2
	M(I), I=1 to INTDIM	Tabular function of car weight per foot in each interval	LBS/FT	F10.5
	W(I), I=1 to INTDIM	Tabular function of the car bending mode	NONE	F10.5
3	E	Modulus of elasticity of car body material	LBS/IN <sup>2</sup>	F10.5
	ICONST	Area moment of inertia of car body	IN <sup>4</sup>	F10.5
	INTDIM	Number of points in tabular functions	NONE	I10
	INTDEL	X-axis increments for tabular functions	FEET	F10.2
	M(I), I=1 to INTDIM	Tabular function of car weight per foot in each interval	LBS/FT	F10.5

TABLE 5-2. INPUTS REQUIRED TO FORM A COMPLETE SET OF CAR BODY DATA (Continued)

Value of Input Variable INCODE	Input Designation	Description	Units	Card Format
3	W(I), I=1 to INTDIM	Tabular function of the car bending mode	NONE	F10.5
4	E	Modulus of elasticity of car body material	LBS/IN <sup>2</sup>	F10.5
	I(J), J=1 to INTDIM	Tabular function of the car body cross-sectional moment of inertia	IN <sup>4</sup>	F10.5
	INTDIM	Number of points on tabular functions	NONE	I10
	INTDEL	X-axis increments for tabular functions	FEET	F10.5
	M(I), I=1 to INTDIM	Tabular function of car weight per foot in each interval	LBS/FT	F10.5
	W(I), I=1 to INTDIM	Tabular function of the car bending mode	NONE	F10.5

### 5.3 EQUATIONS OF MOTION

The equations of motion for the flexible car model were originally developed for the model shown in Figure 5-3, which considers pitching motions of the truck.

As discussed in Section 4.2, it is assumed that the pitching motions of the trucks do not effect car body motions and can be ignored, allowing a reduction in the number of degrees of freedom required to describe car body response and accordingly, reducing the required computations. Table 5-3 contains a list of the generalized and constraint coordinates for the system shown in Figure 5-3 which will be reduced to the "equivalent" simplified model shown in Figure 5-1. The flexibility of the beam representing the car body is assumed to be described by:

$$w(x,t) = W_1(x) e_1(t) \quad (5-1)$$

where  $w(x,t)$  is the displacement from the unstrained axis of the beam, and  $e_1(t)$  and  $W_1(x)$  are the temporal and spacial components of  $w(x,t)$ . The mass distribution along the beam is given by  $m(x)$  and the mass and area moments of inertia by  $I(x)$  and  $Ia(x)$ . Expressions for system potential and kinetic energies are now written for use in developing the Lagrangian equations for the independent coordinates. (see Figure 5-3). The kinetic energy of the system is expressed as follows:

$$\begin{aligned} T = & \frac{1}{2} \int_0^L m(x) \left\{ \dot{z}_2^2 + \left[ (x-\bar{x})\dot{\phi} + \dot{w}(x,t) \right]^2 \right\} dx \\ & + \frac{1}{2} M_1 \left( \frac{\dot{v}_1 + \dot{v}_2}{2} \right)^2 + \frac{1}{2} M_1 \left( \frac{\dot{v}_3 + \dot{v}_4}{2} \right)^2 \\ & + \frac{1}{2} I_1 \left( \frac{\dot{v}_2 - \dot{v}_1}{L} \right)^2 + \frac{1}{2} I_1 \left( \frac{\dot{v}_4 - \dot{v}_3}{L} \right)^2 \\ & + \frac{1}{2} M_T \dot{z}_1^2 \end{aligned} \quad (5-2)$$

The wheel masses are constrained to move along the track and are therefore not included in the kinetic energy. The requirement

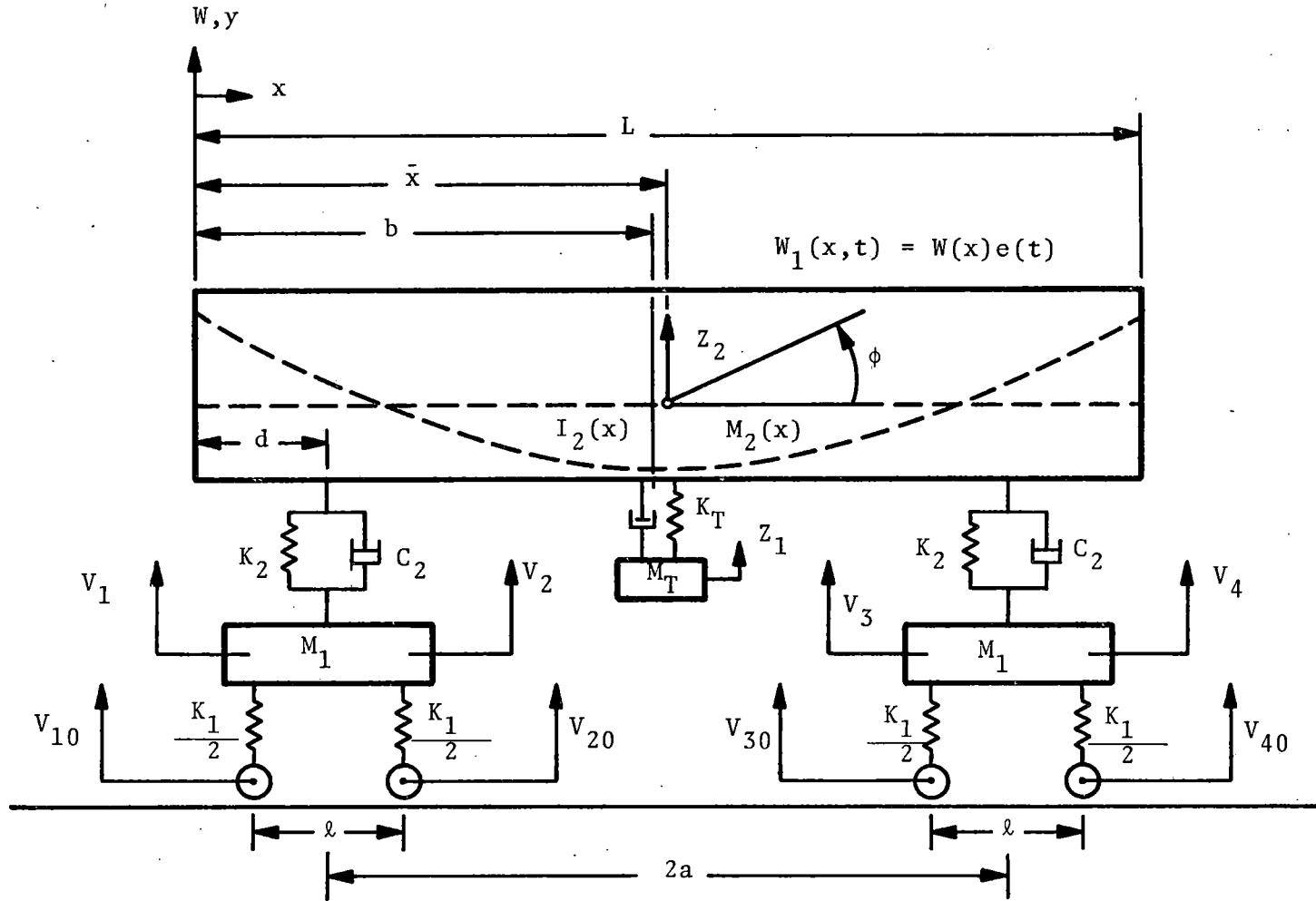


Figure 5-3. Full Car Dynamic Model with Flexible Car Body

TABLE 5-3. GENERALIZED AND CONSTRAINED COORDINATES

Coordinate	Coordinate Description	Type
$v_{10}$	Wheel displacement	Constrained
$v_{20}$	Wheel displacement	Constrained
$v_{30}$	Wheel displacement	Constrained
$v_{40}$	Wheel displacement	Constrained
$v_1$	Lead end, lead truck vertical displacement	Independent
$v_2$	Trailing end, lead truck vertical displacement	Independent
$v_3$	Lead end, trailing truck vertical displacement	Independent
$v_4$	Trailing end, trailing truck vertical displacement	Independent
$z_2$	Car body center of mass vertical displacement	Independent
$\phi$	Car body angular displacement	Independent
$z_1$	Transformer vertical displacement	Independent
$e_1$	Time dependent factor of first bending mode	Independent

that the wheels do not lift off the rails is met as long as the inertia force associated with the axle mass and the acceleration resulting from track irregularity is less than the static axle load.

The potential energy of the system is given by:

$$\begin{aligned}
 V = & \frac{1}{2} \int_0^L EI_a(x) \left[ \frac{\partial^2 w(x,t)}{\partial x^2} \right]^2 dx \\
 & + \frac{1}{2} \left\{ \frac{k_1}{2} (v_1 - v_{10})^2 + \frac{k_1}{2} (v_2 - v_{20})^2 \right. \\
 & + \left. \frac{k_1}{2} (v_3 - v_{30})^2 + \frac{k_1}{2} (v_4 - v_{40})^2 \right\} \\
 & + \frac{1}{2} \left\{ k_2 \left( z_2 - (\bar{x} - d)\phi + w(d,t) - \frac{v_2 + v_1}{2} \right)^2 \right. \\
 & + \left. k_2 \left( z_2 + (L - d - \bar{x})\phi + w(L-d,t) - \frac{v_3 + v_4}{2} \right)^2 \right\} \\
 & + \frac{1}{2} k_T \left\{ z_2 + (b - \bar{x})\phi + w(b,t) - z_1 \right\}^2
 \end{aligned} \tag{5-3}$$

where

E = Young's modulus

$I_a(x)$  = moment of inertia of the section area at x

$\bar{x}$  = beam center of mass coordinate

b = transformer attachment coordinate

L = length of the beam

d = separation distances of truck attachment points and the ends of the beam

The Rayleigh dissipation function for the system is given by:



$$\begin{aligned}
F = & \frac{c_2}{2} \left\{ \frac{\dot{v}_1 + \dot{v}_2}{2} - \left( \dot{z}_2 - (\bar{x}-d)\dot{\phi} + w(d,t) \right) \right\}^2 \\
& + \frac{c_2}{2} \left\{ \frac{\dot{v}_3 + \dot{v}_4}{2} - \left( \dot{z}_2 + (L-d-\bar{x})\dot{\phi} + \dot{w}(L-d,t) \right) \right\}^2 \\
& + \frac{c_T}{2} \left\{ \dot{z}_2 + (b-\bar{x})\dot{\phi} + \dot{w}(b,t) - \dot{z}_1 \right\}^2
\end{aligned} \tag{5-4}$$

Several of the terms in the expressions for the kinetic and potential energies can be expressed more conveniently. The first term of Equation 5-2 may be written:

$$\begin{aligned}
J = & \frac{1}{2} \int_0^L m(x) \left\{ \dot{z}_2^2 + \left[ (x-\bar{x})\dot{\phi} + \dot{e}_1 W_1(x) \right]^2 \right\} dx \\
& + \frac{1}{2} \dot{z}_2^2 \int_0^L m(x) dx + \frac{\dot{\phi}^2}{2} \int_0^L m(x) (x-\bar{x})^2 dx + \frac{\dot{e}_1^2}{2} \int_0^L m(x) W_1^2(x) dx \\
& + \dot{\phi} \dot{e}_1 \int_0^L m(x) (x-\bar{x}) W_1(x) dx
\end{aligned} \tag{5-5}$$

The last term is set equal to zero, because the angular momentum of an unconstrained beam is constant, and the bending mode  $W_1(x)$  of the car body is assumed to be that of an unconstrained beam. The angular momentum about the center of mass of an unconstrained beam is given by

$$L_0 + \dot{e}_1 \int_0^L m(x) (x-\bar{x}) W_1(x) dx = \text{Constant} \tag{5-6}$$

where  $L_0$  is angular momentum resulting from the rotation of the neutral axis,  $\phi$ . Assuming  $W_1(x)$  to be independent of both the suspended loads and  $\phi$ , we have

$$\dot{e}_1 \int_0^L m(x) (x-\bar{x}) W_1(x) dx = \text{Constant} \tag{5-7}$$

and since  $e_1 = e_1(t)$ , it is necessary that

$$\int_0^L m(x) (x-\bar{x}) W_1(x) dx = 0 \quad (5-8)$$

The preceding argument has been made only for the case when the first bending mode is present. Its extension to higher bending modes requires consideration of the orthogonality of the higher modes and the functions

$$W_{ot}(x) = \text{Constant} \quad (5-9a)$$

$$W_{or}(x) = (x-\bar{x}) \quad (5-9b)$$

which are associated with the translational and rotational motion of the neutral axis of the beam. Returning to the expression for J,

$$J = \frac{1}{2} M_2 \dot{z}_2^2 + \frac{1}{2} I_2 \dot{\phi}^2 + \dot{e}_1^2 M_{11} \quad (5-10)$$

where

$$M_2 = \int_0^L m(x) dx,$$

$$I_2 = \int_0^L m(x) (x-\bar{x})^2 dx, \text{ and}$$

$$M_{11} = \int_0^L m(x) W_1^2(x) dx.$$

Then the kinetic energy is expressed as

$$\begin{aligned} T = & \frac{1}{2} M_2 \dot{z}_2^2 + \frac{1}{2} I_2 \dot{\phi}^2 + \dot{e}_1^2 M_{11} \\ & + \frac{1}{2} M_1 \left( \frac{\dot{v}_1 + \dot{v}_2}{2} \right)^2 + \frac{1}{2} M_1 \left( \frac{\dot{v}_3 + \dot{v}_4}{2} \right)^2 \\ & + \frac{1}{2} I_1 \left( \frac{\dot{v}_2 - \dot{v}_1}{L} \right)^2 + \frac{1}{2} I_1 \left( \frac{\dot{v}_4 - \dot{v}_3}{L} \right)^2 \\ & + \frac{1}{2} M_T \dot{z}_1^2 \end{aligned} \quad (5-11)$$

Finally, the integral in the potential energy expression can be written as:

$$\frac{1}{2} \int_0^L EI_a(x) \left( \frac{\partial^2 w(x,t)}{\partial x^2} \right)^2 dx = k_{11} e_1(t), \quad (5-12a)$$

where

$$k_{11} = \frac{1}{2} \int_0^L EI_a(x) \left( \frac{\partial^2 W_1(x)}{\partial x^2} \right)^2 dx \quad (5-12b)$$

$$w(x,t) = W_1(x) e_1(t)$$

The equations of motion are derived from Lagrange's equation:

$$\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{q}_j} \right) - \frac{\partial L}{\partial q_j} + \frac{\partial F}{\partial \dot{q}_j} = 0 \quad (5-13)$$

where

$$L = T - V \text{ (the Lagrangian)}$$

$$q_j = \text{any generalized coordinate}$$

The equations of motion are derived from Lagrange's equation:

$$\underline{v_{10}} \quad v_{10} = v_0 \exp(i\omega t) \quad (5-14a)$$

$$\underline{v_{20}} \quad v_{20} = v_0 \exp\left(i2\pi\frac{\ell}{\lambda}\right) \exp(i\omega t) \quad (5-14b)$$

$$\underline{v_{30}} \quad v_{30} = v_0 \exp\left(i2\pi\frac{2a}{\lambda}\right) \exp(i\omega t) \quad (5-14c)$$

$$\underline{v_{40}} \quad v_{40} = v_0 \exp\left(i2\pi\frac{\ell+2a}{\lambda}\right) \exp(i\omega t) \quad (5-14d)$$

The equations of motion which correspond to each generalized coordinate are:

$$\begin{aligned} \underline{v_1} \quad & \frac{M_1}{4} (\ddot{v}_1 + \ddot{v}_2) + \frac{I_1}{L^2} (\ddot{v}_1 - \ddot{v}_2) + \frac{C_2}{2} \left[ \frac{\dot{v}_1 + \dot{v}_2}{2} - \left( \dot{z}_2 - (\bar{x}-d)\dot{\phi} + W_1(d) \dot{e}_1 \right) \right] \\ & + \frac{k_1}{2} (v_1 - v_{10}) - \frac{k_2}{2} \left( z_2 - (\bar{x}-d)\phi + W_1(d) e_1 - \frac{v_2 + v_1}{2} \right) = 0 \end{aligned} \quad (5-15)$$

$$\begin{aligned}
\underline{v_2} \quad & \frac{M_1}{4} (\ddot{v}_1 + \ddot{v}_2) - \frac{I_1}{L^2} (\ddot{v}_1 - \ddot{v}_2) + \frac{C_2}{2} \left[ \frac{\dot{v}_1 + \dot{v}_2}{2} - \left( \dot{z}_2 - (\bar{x}-d)\dot{\phi} + W_1(d) \dot{e}_1 \right) \right] \\
& + \frac{k_1}{2} (v_2 - v_{20}) - \frac{k_2}{2} \left[ z_2 - (\bar{x}-d)\phi + W_1(d) e_1 - \frac{v_2 + v_1}{2} \right] = 0
\end{aligned} \tag{5-16}$$

$$\begin{aligned}
\underline{v_3} \quad & \frac{M_1}{4} (\ddot{v}_3 + \ddot{v}_4) + \frac{I_1}{L^2} (\ddot{v}_3 - \ddot{v}_4) + \frac{C_2}{2} \left[ \frac{\dot{v}_3 + \dot{v}_4}{2} - \left( \dot{z}_2 + (L-d-\bar{x})\dot{\phi} + W_1(L-d)\dot{e}_1 \right) \right] \\
& + \frac{k_1}{2} (v_3 - v_{30}) - \frac{k_2}{2} \left[ z_2 - (L-d-\bar{x})W_1(L-d)e_1 - \frac{v_3 + v_4}{2} \right] = 0
\end{aligned} \tag{5-17}$$

$$\begin{aligned}
\underline{v_4} \quad & \frac{M_1}{4} (\ddot{v}_3 + \ddot{v}_4) - \frac{I_1}{L^2} (\ddot{v}_3 - \ddot{v}_4) + \frac{C_2}{2} \left[ \frac{\dot{v}_3 + \dot{v}_4}{2} - \left( z_2 + (L-d-\bar{x})\dot{\phi} + W_1(L-d)\dot{e}_1 \right) \right] \\
& + \frac{k_1}{2} (v_4 - v_{40}) - \frac{k_2}{2} \left[ z_2 + (L-d-\bar{x})W_1(L-d)e_1 - \frac{v_3 + v_4}{2} \right] = 0
\end{aligned} \tag{5-18}$$

$$\begin{aligned}
\underline{z_2} \quad & M_2 \ddot{z}_2 + 2C_2 \left[ \dot{z}_2 + \dot{e}_1 (W_1(d) + W_2(L-d)) + \dot{\phi} (L-2\bar{x}) - \frac{\dot{v}_1 + \dot{v}_2 + \dot{v}_3 + \dot{v}_4}{4} \right] \\
& + C_T (\dot{z}_2 + (b-\bar{x})\dot{\phi} + W_1(b)\dot{e}_1 - \dot{z}_1) \\
& + 2k_2 \left[ z_2 + e_1 (W_1(d) + W_1(L-d)) + \phi (L-2\bar{x}) - \frac{v_1 + v_2 + v_3 + v_4}{4} \right] \\
& + k_T (z_2 + (b-\bar{x})\phi + W_1(b)e_1 - z_1) = 0
\end{aligned} \tag{5-19}$$

$$\begin{aligned}
\underline{\phi} \quad & I_2 \ddot{\phi} + C_2 \left\{ \dot{z}_2 (L-2\bar{x}) + \dot{\phi} \left[ (\bar{x}-d)^2 + (L-d-\bar{x})^2 \right] - W_1(d) (\bar{x}-d) \dot{e}_1 \right. \\
& + W_1(L-d) (L-d-\bar{x}) \dot{e}_1 + \frac{\dot{v}_1 + \dot{v}_2}{2} (\bar{x}-d) - \frac{\dot{v}_3 + \dot{v}_4}{2} (L-d-\bar{x}) \left. \right\} \\
& + C_T (\dot{z}_2 + (b-\bar{x})\dot{\phi} + W_1(b)\dot{e}_1 - \dot{z}_1) (b-\bar{x}) \\
& + k_2 \left\{ z_2 (L-2\bar{x}) + \phi \left[ (\bar{x}-d)^2 + (L-d-\bar{x})^2 \right] - W_1(d) (\bar{x}-d) e_1 \right. \\
& + W_1(L-d) (L-d-\bar{x}) e_1 + \frac{v_1 + v_2}{2} (\bar{x}-d) - \frac{v_3 + v_4}{2} (L-d-\bar{x}) \left. \right\} \\
& + k_T \left\{ z_2 + (b-\bar{x})\phi + W_1(b)e_1 - z_1 \right\} (b-\bar{x}) = 0
\end{aligned} \tag{5-20}$$

$$\begin{aligned} \underline{z_1} \quad M_T \ddot{z}_1 - C_T \left( \dot{z}_2 + (b-x)\dot{\phi} + W_1(b)\dot{e}_1 - \dot{z}_1 \right) \\ - k_T \left( z_2 + (b-\bar{x})\phi + W(b)e_1 - z_1 \right) = 0 \end{aligned} \quad (5-21)$$

$$\begin{aligned} \underline{e_1} \quad \ddot{e}_1 M_{11} + C_2 \left( \dot{z}_2 - (\bar{x}-d)\dot{\phi} + W_1(d)\dot{e}_1 - \frac{\dot{v}_1 + \dot{v}_2}{2} \right) W_1(d) \\ + C_2 \left( \dot{z}_2 + (L-d-\bar{x})\dot{\phi} + W_1(L-d)\dot{e}_1 - \frac{\dot{v}_3 + \dot{v}_4}{2} \right) W_1(L-d) \\ + C_T \left( \dot{z}_2 + (b-\bar{x})\dot{\phi} + W_1(b)\dot{e}_1 - \dot{z}_1 \right) W_1(b) \\ + k_2 \left( z_2 - (\bar{x}-d)\phi + W_1(d)e_1 - \frac{v_1 + v_2}{2} \right) W_1(d) \\ + k_2 \left( z_2 + (L-d-\bar{x})\phi + W_1(L-d)e_1 + \frac{v_3 + v_4}{2} \right) W_1(L-d) \\ + k_T \left( z_2 + (b-\bar{x})\phi + W_1(b)e_1 - z_1 \right) W_1(b) \\ + k_{11} e_1 = 0 \end{aligned} \quad (5-22)$$

The preceding equations (5-14 to 5-22) can be reduced to six equations containing six unknowns by using equations (5-14a, b, c, d) to eliminate  $v_{10}$ ,  $v_{20}$ ,  $v_{30}$ ,  $v_{40}$  from equations (5-15 to 5-18) by adding (5-15) to (5-16) and (5-17) to (5-18), and setting

$$y_1 = \frac{v_1 + v_2}{2} \quad (5-23a)$$

$$y_2 = \frac{v_3 + v_4}{2} \quad (5-23b)$$

The resulting six equations are then combined in a matrix form to obtain:

$$[M] \{\ddot{q}\} + [C] \{\dot{q}\} + [K] \{q\} = \{Q\} \quad (5-24)$$

where

$[M]$  = mass matrix

$[C]$  = damping matrix

$[K]$  = stiffness matrix

$[Q]$  = forcing function vector

and

$$\{q(t)\} = \begin{Bmatrix} e(t) \\ z_2(t) \\ \phi(t) \\ y_1(t) \\ y_2(t) \\ z_1(t) \end{Bmatrix} \quad (5-25)$$

$$\{Q(t)\} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ \frac{k_1}{2}(v_1(t)+v_2(t)) \\ \frac{k_1}{2}(v_3(t)+v_4(t)) \\ 0 \end{Bmatrix} \quad (5-26)$$

where, again, as shown in Figure 5-1, wheelset displacements are defined in terms of the unit rotating vector representation of track surface irregularity frequency, with respect to the lead wheelset together with an appropriate phase lag associated with trailing wheelsets:

$$\begin{Bmatrix} v_1(t) \\ v_2(t) \\ v_3(t) \\ v_4(t) \end{Bmatrix} = v_0 \begin{Bmatrix} \exp(i\omega t) \\ \exp\left(i\left[\frac{2\pi\ell}{\lambda} + \omega t\right]\right) \\ \exp\left(i\left[\frac{2\pi(2a)}{\lambda} + \omega t\right]\right) \\ \exp\left(i\left[\frac{2\pi(\ell+2a)}{\lambda} + \omega t\right]\right) \end{Bmatrix} \quad (5-27)$$

If wheel eccentricity ratios and phase angles are specified as described in Section 5.2 and Figure 5-2, the forcing functions  $Q(t)$  are augmented at one (and only one) of the discrete frequencies. This frequency is given by  $\omega_\epsilon = \frac{V}{r}$ , where  $V$  is the vehicle velocity and  $r$  is the radius of the wheels. At this frequency, the forcing functions are augmented as follows:

$$Q(t) = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ \frac{k_1}{2}(v_1^i + v_2^i) \\ \frac{k_1}{2}(v_3^i + v_4^i) \\ 0 \end{Bmatrix} \quad (5-28)$$

where

$$\begin{Bmatrix} v_1^i \\ v_2^i \\ v_3^i \\ v_4^i \end{Bmatrix} = \begin{Bmatrix} v_1 (1 + \epsilon_1 \exp(i[\omega_\epsilon t + \theta_1])) \\ v_2 (1 + \epsilon_2 \exp(i[\omega_\epsilon t + \theta_2])) \\ v_3 (1 + \epsilon_3 \exp(i[\omega_\epsilon t + \theta_3])) \\ v_4 (1 + \epsilon_4 \exp(i[\omega_\epsilon t + \theta_4])) \end{Bmatrix} \quad (5-29)$$

Required input data for specifying wheel eccentricities is shown in Table 5-4.

#### 5.4 CAR BODY FLEXIBILITY - MODELING OPTIONS

##### 5.4.1 Description of the Fundamental Bending Mode

Various options for modeling car body flexibility have been incorporated into Program FLEX to allow either analytical expressions or tabulated engineering data to be used for computing the maximum strain energy associated with the first bending mode (represented by  $K_{11}$  of Equation 5-12a). Since the car body mass motion of inertia is a function of the mass distribution, an appropriate expression for computing this parameter must also be specified.

Car body flexible modes may be described using Rayleigh's Equation which equates the maximum kinetic and potential energy functions in a conservative system and results in an expression relating mode shape and natural frequency. For a slender beam, this relation is:

TABLE 5-4. OPTIONAL INPUTS DESCRIBING WHEEL ECCENTRICITY

Input Designation	Description	Units	Card Format
R	Wheel radius	FEET	F10.5
EPS1	First wheel eccentricity ratio	NONE	F10.5
EPS2	Second wheel eccentricity ratio	NONE	F10.5
EPS3	Third wheel eccentricity ratio	NONE	F10.5
EPS4	Fourth wheel eccentricity ratio	NONE	F10.5
THETA1	First wheel eccentricity phase angle	DEGREES	F10.5
THETA2	Second wheel eccentricity phase angle	DEGREES	F10.5
THETA3	Third wheel eccentricity phase angle	DEGREES	F10.5
THETA4	Fourth wheel eccentricity phase angle	DEGREES	F10.5



$$\omega = \sqrt{\frac{E \int_0^L I_a(x) (d^2W(x)/dx^2) dx}{\int_0^L m(x) (W(x))^2 dx}} \quad (5-30)$$

where

$W(x)$  = beam (car-body) deflection along its length (i.e., mode shape)

$I_a(x)$  = area cross-sectional moments of inertia of car body along its length

$m(x)$  = mass distribution of car body per unit length

The numerator in equation (5-30) is identical to  $k_{11}$  defined in equation 5-12b. This allows equation (5-12b) to be evaluated, if the first bending mode frequency (FB) is specified, viz:

$$k_{11} = E \int_0^L I(x) (d^2W(x) / dx^2) dx = (2\pi FB)^2 \int_0^L m(x) (W(x))^2 dx, \quad (5-31)$$

where

$$\omega = 2\pi(FB)$$

#### 5.4.2 Modeling Options

The program control parameter "INCODE" is used to select the desired method for modeling car body flexibility. Input required for each option is contained in Table 5-2. The four options represent various levels of bending mode and structural detail, as follows.

INCODE = 1. This option is the most basic of the options provided. The car body is represented by a uniform unconstrained beam. A solution to the differential equation of motion for a free-free beam in bending is used to approximate the mode shape. The classical beam equation is:

$$\frac{\partial^2}{\partial x^2} \left[ EI \frac{\partial^2 W(x)}{\partial x^2} \right] + m \frac{\partial^2 W(x)}{\partial x^2} = 0 \quad (5-32)$$

The solution, for a free-free uniform beam is:

$$W(x) = \cosh \beta x + \cos \beta x - \alpha (\sinh \beta x + \sin \beta x) \quad (5-33)$$

For the first bending mode,  $\beta = \frac{4.73}{L}$  and  $\alpha = 0.9825$ . In equation (5-31),  $m(x)$ , the mass distribution per unit length, is a constant. Also in equation (5-33):

$$\int_0^L (W(x))^2 dx = L \quad (5-34)$$

and,  $mL = M_2 = \text{car body mass}$ . The expression for  $k_{11}$  in equation (5-31) reduces to:

$$k_{11} = (2\pi \text{ FB})^2 M_2 \quad (5-35)$$

It is not necessary to input values of the car-body elastic modulus (E) and area moment of inertia (Ia) in this option. The mass moment of inertia for the slender rod representation of the car body is:

$$I_2 = \frac{M_2 L^2}{12} \quad (5-36)$$

INCODE = 2. In this option, the car body is represented by a distributed mass,  $M(x_i)$ , and the bending mode shape,  $W(x_i)$  specified in tabular form. The number of points and the longitudinal spacing of the data are defined by the control parameters INTDIM and INTDEL, respectively. The bending mode frequency (FB) is also specified. These data are used to evaluate the right-hand side of equation (5-31) and, again, the term  $k_{11}$  may be computed from:

$$k_{11} = (2\pi \text{ FB})^2 \sum_i M(x_i) W(x_i)^2 \quad (5-37)$$

(where  $i=1$  to INTDIM)

Again, car body material and area moment of inertia properties are not needed. The mass moment of inertia is computed relative to the car body CG as:

$$I_2 = \sum_i M(x_i - \bar{x})^2 \quad (5-38)$$

where

$$\bar{x} = \frac{\sum_i M(x_i) (x_i)}{\sum_i M(x_i)} \quad (5-39)$$

(for,  $i=1$  to INTDIM)

INCODE = 3. In this option, the car-body structure is modeled as a uniform beam having a constant area moment of inertia (ICONST) and elastic modulus E. The distributed mass  $M(x_i)$  and the bending mode  $W(x_i)$  shapes are specified in tabular form at a number of points (INTDIM) and a specified longitudinal spacing (INTDEL). The bending mode frequency is not required for this option. The term  $k_{11}$  is evaluated using equation (5-31), as

$$k_{11} = (E)(ICONST) \sum_i \left[ \frac{W(x_i) - 2W(x_{i+1}) + W(x_{i+2}))}{INTDEL} \right]^2 \quad (5-40)$$

( $i=1$  to INTDIM)

The mass moment of inertia for this option is again computed from equations (5-38) and (5-39).

INCODE = 4. This option is the most general, and it is applicable to car body configurations which have a variable load-carrying cross section and non-uniform mass distribution. The area moment of inertia, along the car length,  $I_a(x_i)$ , is input in tabular form, along with tabulated descriptions of car body mass distribution  $M(x_i)$ , and the (first) bending mode shape  $W(x_i)$ . Other parameters specified are the car body elastic modulus (E) and the number of points, and longitudinal spacing (INTDIM and INTDEL) associated with the tabulated data. The bending mode frequency is not required for this option. The term  $k_{11}$  is evaluated as follows:

$$k_{11} = E \sum_i I_a(x_i) \left[ \frac{W(x_i) - 2W(x_{i+1}) + W(x_{i+2}))}{INTDEL} \right]^2 \quad (5-41)$$

(i=1 to INTDIM)

The mass moment of inertia is evaluated according to equations (5-38) and (5-39).

### 5.5 SOLUTION PROCEDURE AND PROGRAM FLOW

After the data are read in (refer to Tables 5-1 and 5-2) the [M], [C], and [K] coefficient matrices of equation (5-24) are computed.

The damping constants  $C_2$  and  $C_T$  (see Figure 5-1) are obtained from the damping ratios  $\beta_2$  and  $\beta_T$  according to:

$$C_2 = 2 M_2 \sqrt{\frac{k_2}{M_2}} \beta_2 \quad (5-42a)$$

$$C_T = 2 M_T \sqrt{\frac{k_T}{M_T}} \beta_T \quad (5-42b)$$

where

$M_2$  = mass of the car body

$M_T$  = mass of the suspended mass

$k_2$  = secondary suspension stiffness constant

$k_T$  = suspended mass stiffness constant

The equations of motion are linear differential equations with constant coefficients. The solutions are of the form

$$q_j(t) = \bar{q}_j(\omega) \exp(i\omega t), \quad (5-43)$$

where  $\bar{q}_j(\omega)$  gives the complex amplitude of the oscillation for each coordinate  $q_j$ , at the angular frequency  $\omega$ . Similarly, the complex amplitude of the forcing function  $Q_j(t)$  (defined by equation (5-26) without wheel eccentricities and by equation (5-28) with wheel eccentricities) is given by  $\bar{Q}_j(\omega)$  where

$$\bar{Q}_j(t) = Q_j(\omega) \exp(i\omega t) \quad (5-44)$$

The frequency response  $\bar{q}_j(\omega)$  is obtained by solving the following complex equations

$$(-\omega^2) [M]\{\bar{q}(\omega)\} + i\omega[C]\{\bar{q}(\omega)\} + [K]\{\bar{q}(\omega)\} = \{\bar{Q}(\omega)\} \quad (5-45)$$

at discrete frequencies over the range of interest. A complex coefficient matrix  $[A(\omega)]$  is formed and solved by premultiplying by its inverse, for each value of  $\omega$  considered. Frequencies are generated by specifying a vehicle velocity or track irregularity wavelength and adjusting the unspecified parameter to generate  $\omega$ , according to  $\omega = 2\pi V/\lambda$ . Normalized values of the coordinate vector  $q(\omega)$  are obtained by matrix inversion and stored:

$$\{q(\omega)\} = [MAT(\omega)]^{-1} \{Q(\omega)\}. \quad (5-46)$$

Computational variables are normalized with respect to the track perturbation amplitude,  $v_0$ . If required, acceleration and acceleration spectra responses are also computed and stored. Acceleration responses are computed by a multiplication of the corresponding displacement response by  $(2\pi f)^2/g$ , and the acceleration spectra are the acceleration responses multiplied by the track wavelength. Sufficient time must be requested to complete all of the printing and plotting routines at the end of the program, or the output will be lost. (See Section 5.6.3.)

## 5.6 INPUT/OUTPUT PARAMETERS AND CONTROL VARIABLES

### 5.6.1 Input

The physical input variables are given in Tables 5-1, 5-2 and 5-4. All of the variables listed in Table 5-1 are required. Input parameters to describe the fundamental bending mode of the car body are required in accordance with Table 5-2. The variables listed in Table 5-4 pertain to wheel eccentricity and are optional. Table 5-5 lists and defines the data control variables for inputting data from Tables 5-2 and 5-4. The program accepts seven frequency intervals and frequency increments within each interval for com-

TABLE 5-5. PROGRAM "FLEX" DATA CONTROL VARIABLES

Input Designation	Purpose	Possible Values	Data Format
PRINT	Controls printed output	0, Intermediate results not printed 1, Intermediate results printed	I2
DISP	Controls plotting of displacement responses	0, Displacement responses not plotted 1, Displacement responses plotted	I2
ACC	Controls calculation of acceleration responses	0, Acceleration responses not calculated 1, Acceleration responses calculated and plotted	I2
SPEC	Controls calculation of acceleration spectra	0, Acceleration spectra not calculated 1, Acceleration spectra calculated and plotted	I2
ECCEN	Includes or deletes eccentricity calculations	0, No wheel eccentricity 1, Expects wheel eccentricity data	I1
VLTEST	Prepares program to accept velocity or wavelength input	1, Velocity value accepted 2, Wavelength value accepted	I1
NDF	Specifies number of frequency ranges considered	1 to 7	I2
IFREQ	Controls frequency ranges over which response is computed	1 or 2	I2
DF(I)	Specifies the number of points computed in a particular frequency range	I=1 to NDF	7I4
FL(I)	Specifies lower and upper frequency limits of each frequency range	I=1 to NDF+1	8F10.4
INCODE	Selects data set describing car body's bending mode	1 to 4 (See Table 5-2)	I1

puting frequency response. The resulting number of frequencies is limited to two hundred.

### 5.6.2 Output

The FLEX program automatically prints the input values. In addition, two computed quantities are printed along with the input. These are FM and FW. FM is the natural frequency of the hanging mass and is calculated from the input data for the user's convenience. FW is the frequency of the wheel eccentricity contribution. It is calculated and printed when wheel eccentricity is specified and a fixed velocity is given.

If the user codes PRINT = 1 (refer to Table 5-5), some intermediate results are printed for diagnostic purposes. The values of [M], [C], and [K], the mass, damping, and spring coefficient matrices are printed. For each point in the frequency range, the normalized solution vector (q) is calculated. The real and imaginary parts of the six system variables which compose (q) are printed under the title "QVAR VALUES".

The displacement responses are always printed for three points on the car: the center of gravity of the car body, the point on the car body directly over the truck center (a distance d away from the end of the car), and the center of gravity of the hanging mass. Acceleration responses and acceleration spectra are also printed for these points, if the corresponding code variable is set to one, resulting in a total of nine possible plots (refer to Table 5-6).

Each plot is a labeled log-log graph with frequency as the independent variable. Certain input parameters are written on the plots for clarity and convenience. The displacement responses are plotted only when DISP = 1. Similarly, the acceleration response plots are produced when ACC = 1, and the acceleration spectra plots are produced when SPEC = 1. These codes are summarized in Table 5-5. Sample plots are shown in Figures 5-4 through 5-9.

TABLE 5-6. PROGRAM "FLEX" PLOTTED OUTPUT DATA

Figure No.	Title	Independent Variable
5-4	Displacement Response - Center of Gravity	$\left  \frac{\bar{z}_2(\omega) + W(\bar{x})\bar{e}(\omega)}{v_o} \right $
5-6	Displacement Response - Truck Center	$\left  \frac{\bar{z}_2(\omega) + W(L-d)\bar{e}(\omega) + (L-d-\bar{x})\bar{\phi}(\omega)}{v_o} \right $
5-8	Displacement Response - Hanging Mass	$\left  \frac{\bar{z}_1(\omega)}{v_o} \right $
5-5	Acceleration Response - Center of Gravity	$\left  \frac{\bar{z}_2(\omega) + W(\bar{x})\bar{e}(\omega)}{v_o} \right  \frac{(2\pi f)^2}{g}$
5-7	Acceleration Response - Truck Center	$\left  \frac{\bar{z}_2(\omega) + W(L-d)\bar{e}(\omega) + (L-d-\bar{x})\bar{\phi}(\omega)}{v_o} \right  \frac{(2\pi f)^2}{g}$
5-9	Acceleration Response - Hanging Mass	$\left  \frac{\bar{z}_1(\omega)}{v_o} \right  \frac{(2\pi f)^2}{g}$
5-10	Acceleration Spectra - Center of Gravity	$\left  \frac{\bar{z}_2(\omega) + W(\bar{x})\bar{e}(\omega)}{a_1} \right  \frac{(2\pi f)^2}{g} \lambda$
5-11	Acceleration Spectra - Truck Center	$\left  \frac{\bar{z}_2(\omega) + W(L-d)\bar{e}(\omega) + (L-d-\bar{x})\bar{\phi}(\omega)}{a_1} \right  \frac{(2\pi f)^2}{g} \lambda$
-	Acceleration Spectra - Hanging Mass	$\left  \frac{\bar{z}_1(\omega)}{a_1} \right  \frac{(2\pi f)^2}{g} \lambda$



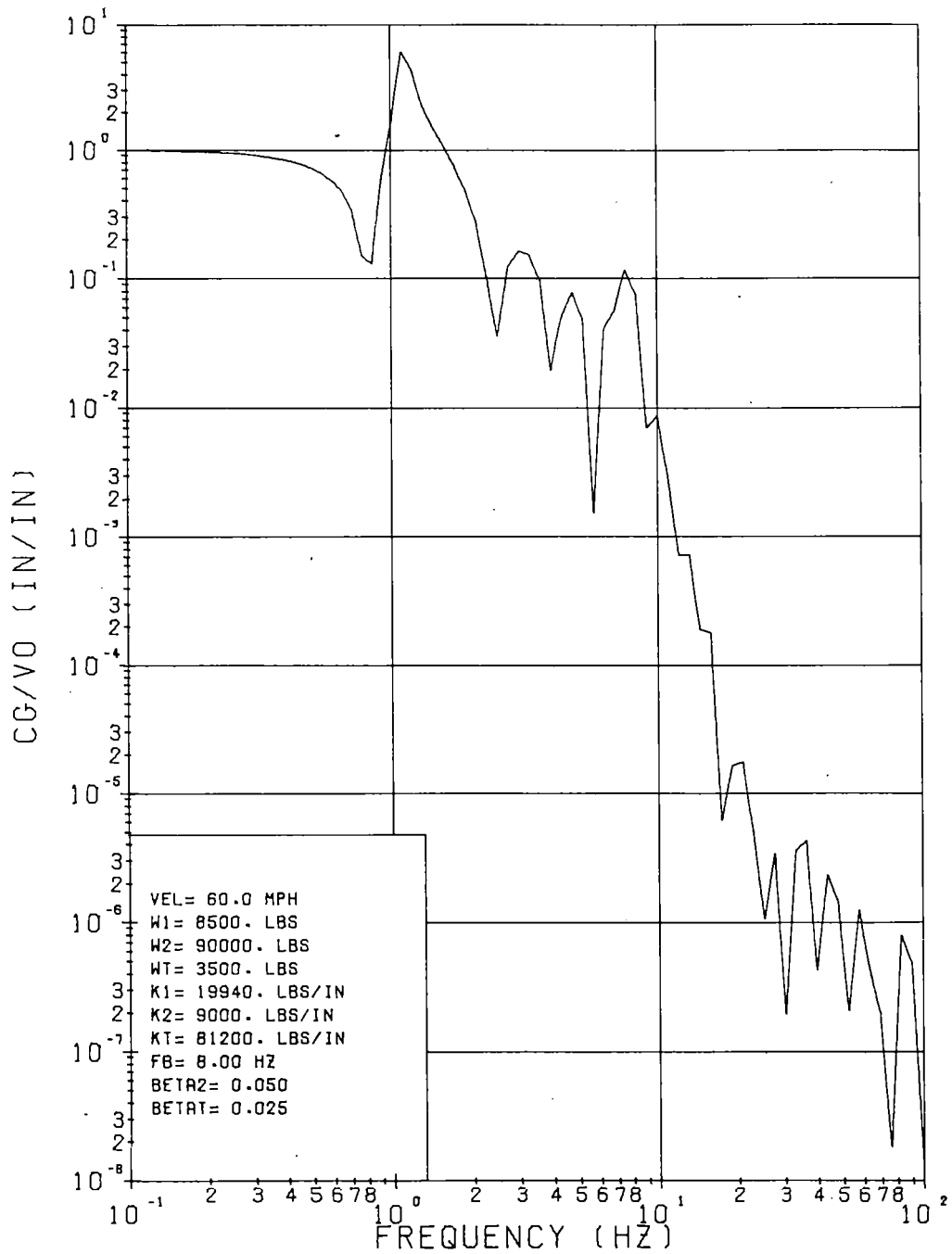


Figure 5-4. Displacement Response, Center of Gravity - Graphic Diagram

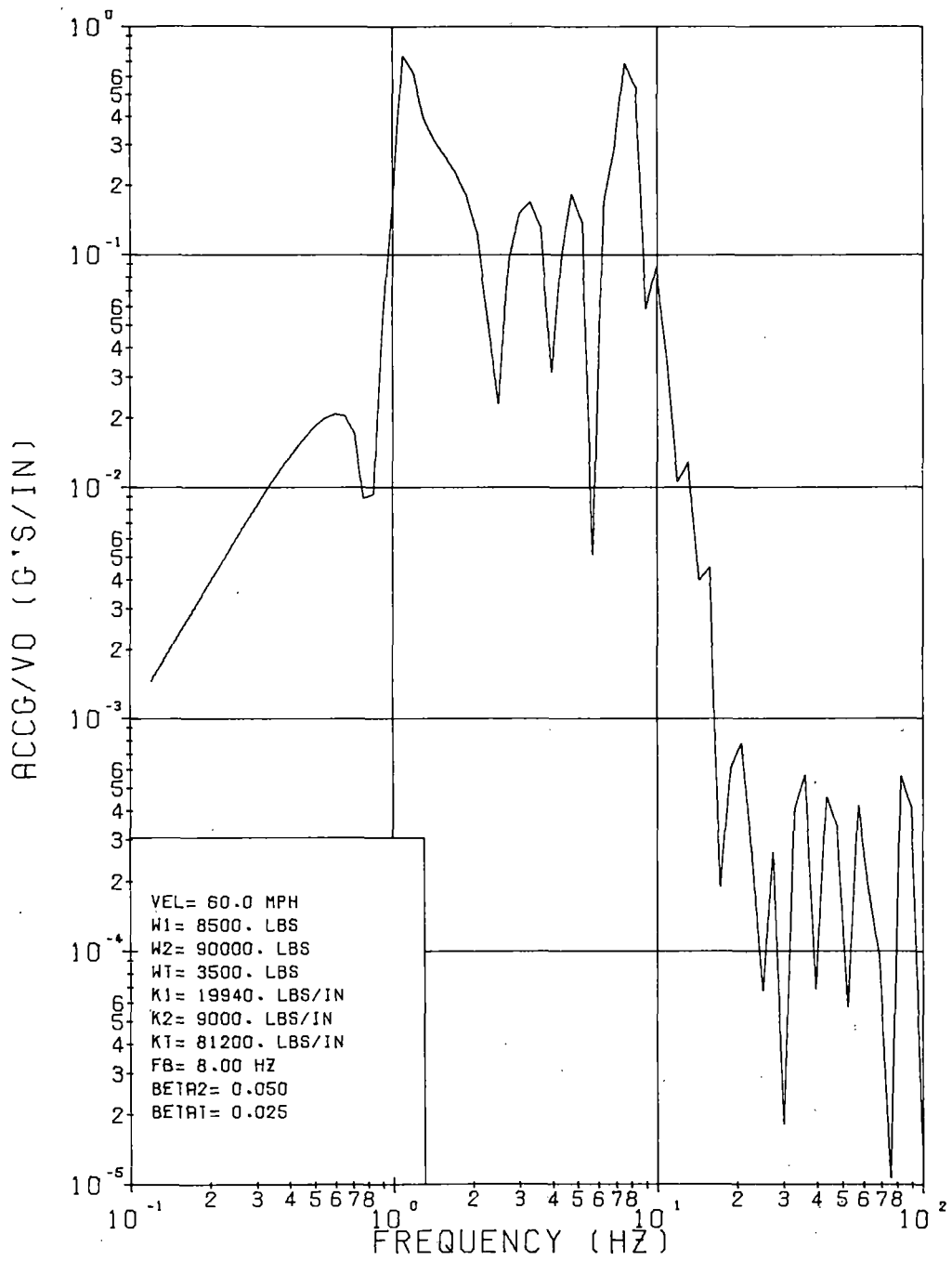


Figure 5-5. Acceleration Response, Center of Gravity - Graphic Diagram

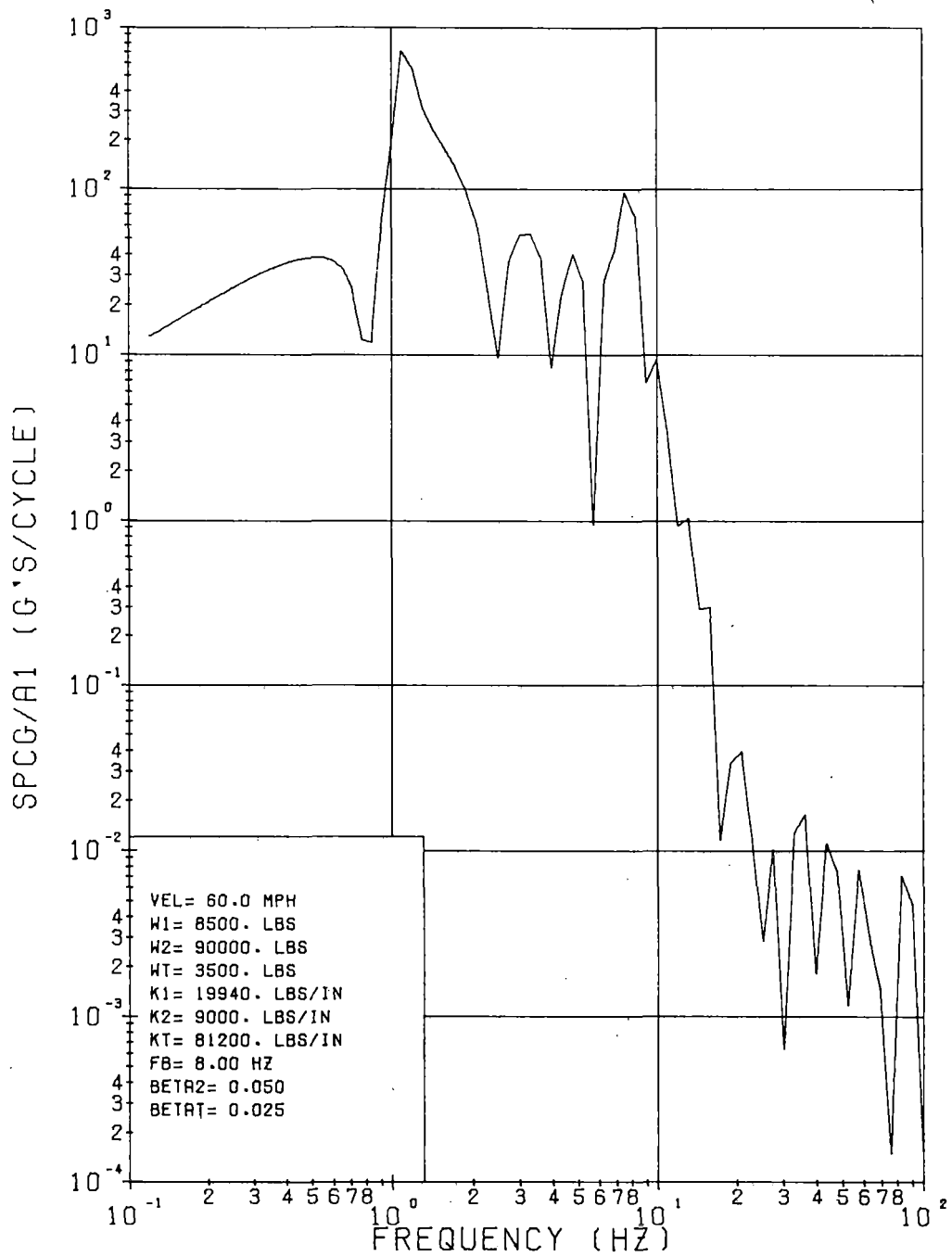


Figure 5-6. Acceleration Spectra, Center of Gravity -- Graphic Diagram

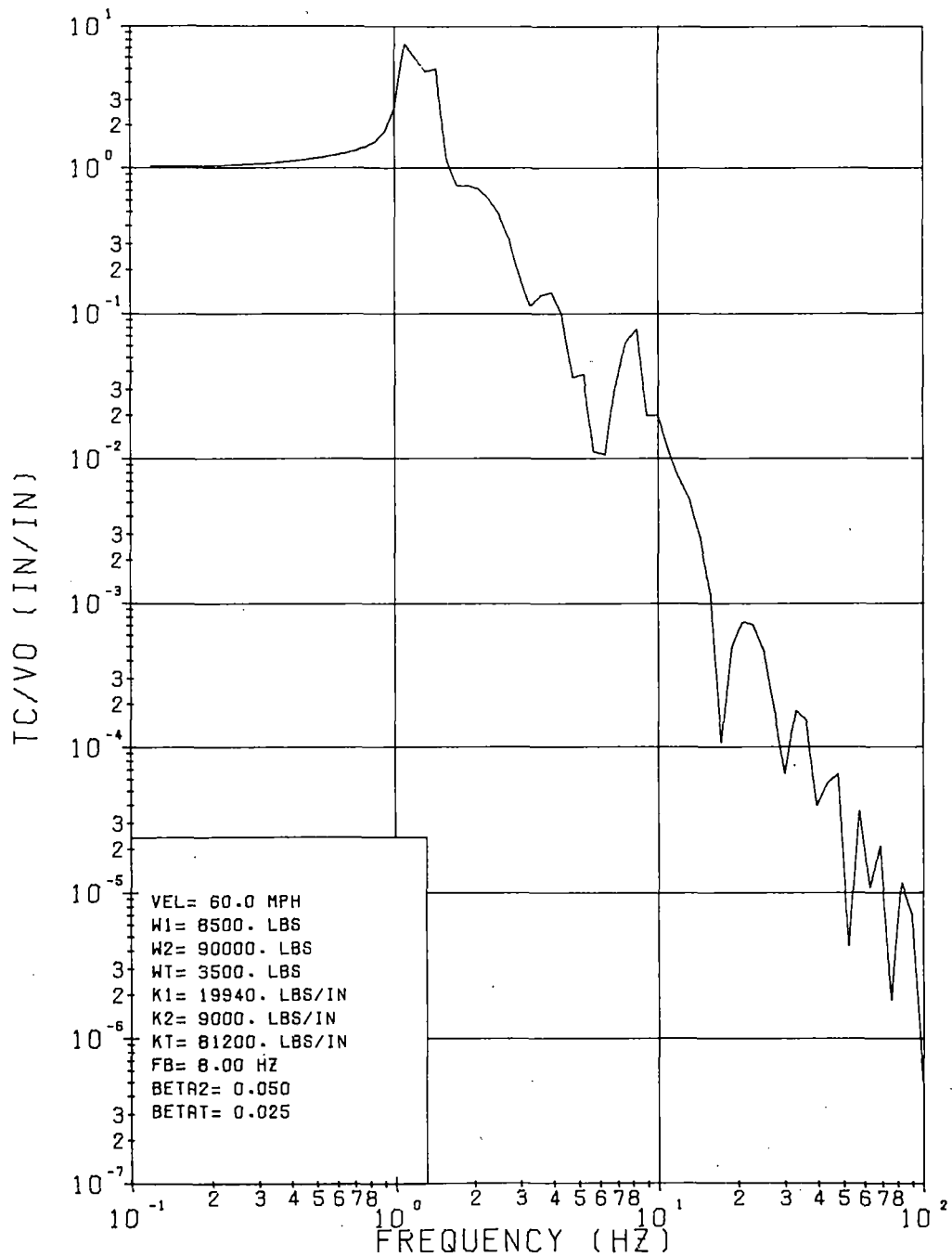


Figure 5-7. Displacement Response, Truck Center -  
Graphic Diagram

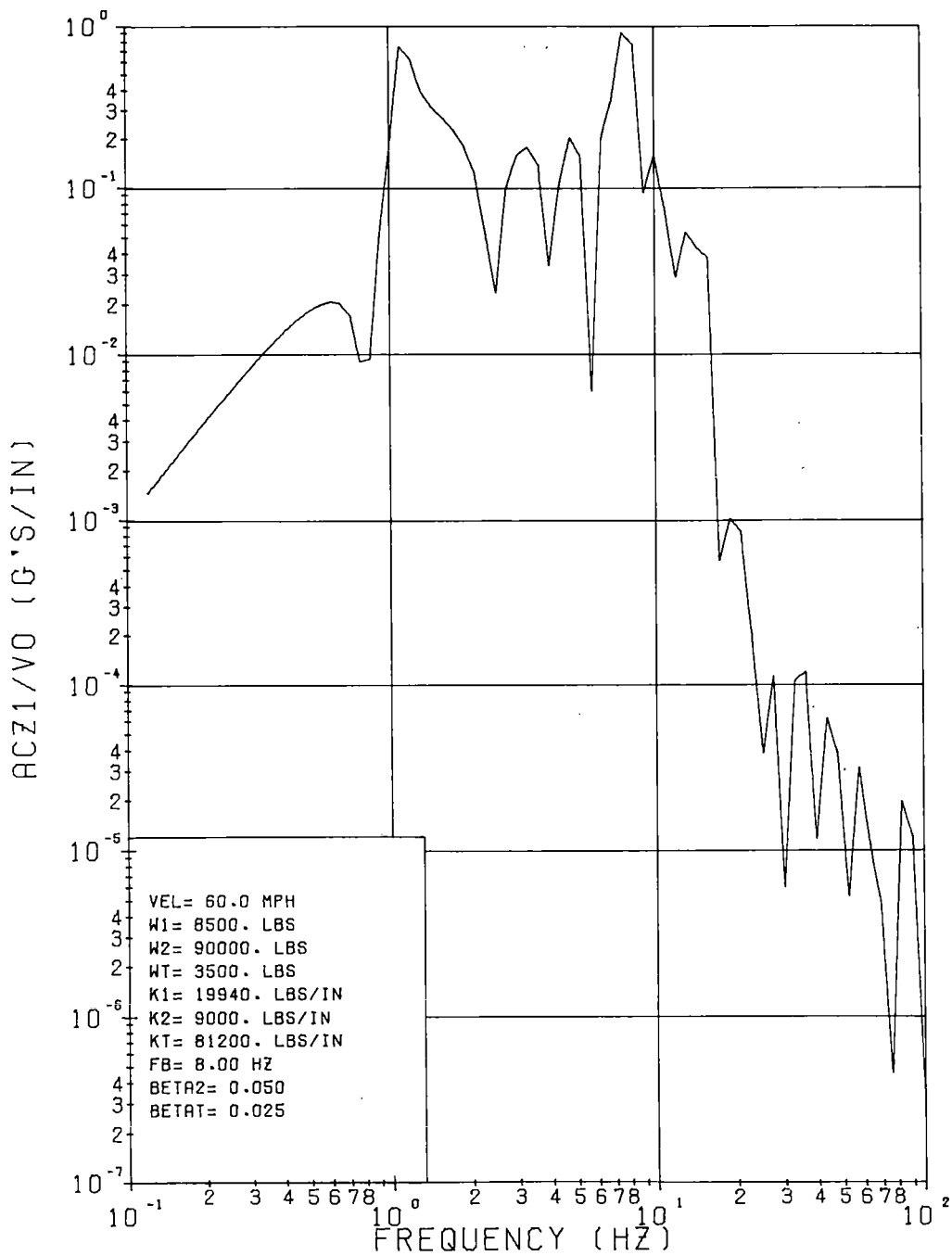


Figure 5-8. Acceleration Response, Hanging Mass - Graphic Diagram

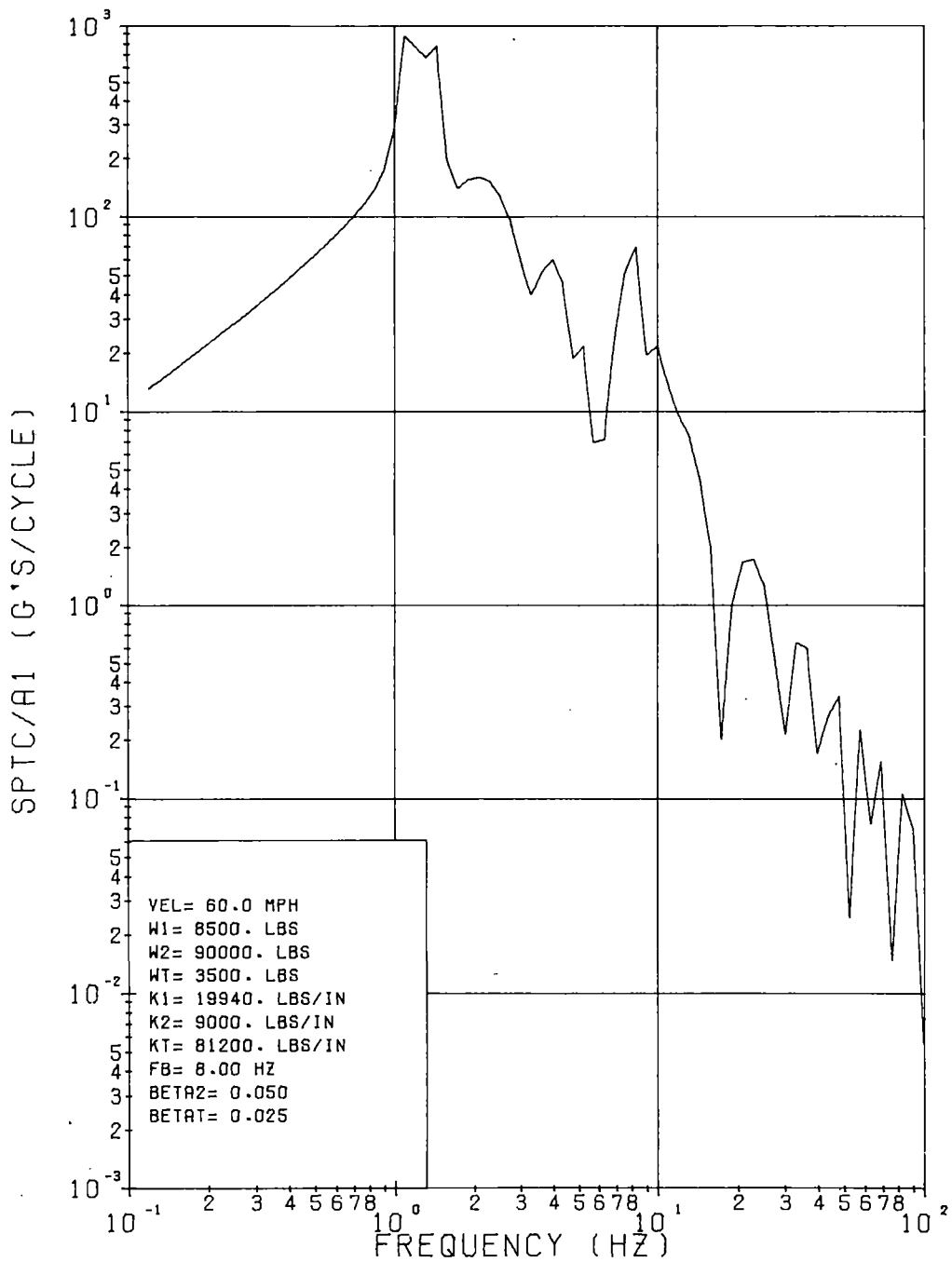


Figure 5-9. Acceleration Spectra, Truck Center - Graphic Diagram

### 5.6.3 Program Control

The deck developed at TSC has been run on the Digital Equipment Corporation PDP-10 computer. Using the object deck, 59 specified frequency values require 26 seconds of CPU time and 10 minutes of Calcomp Plotter time. Approximately 16,000 (decimal) words of memory are required. A listing and comments on running Program FLEX on the DEC System PDP-10 are contained in Volume II as Appendix B.

The only subroutine other than the plotting subroutines written at TSC is MINV which is part of the IBM scientific subroutine package. This subroutine was modified at TSC, to enable it to invert a complex matrix; as available from IBM, it was restricted to real numbers.

## 6. PROGRAM "LATERAL"

### 6.1 APPLICATION

Program LATERAL is a digital computer program which computes the lateral frequency response of a single 15-degree-of-freedom rail vehicle having a rigid car-body, to sinusoidal track irregularities. Two types of track irregularities may be specified: Option I - track centerline lateral displacement from tangent track in the horizontal plane (alignment); and Option II - crosslevel misalignment. Output consists of printed or plotted data for the acceleration or displacement (roll, yaw, and lateral) response at any of the fifteen coordinates.

These responses provide measures of passenger vibration environment, component life and reliability, and safety associated with vehicle lateral and roll displacement amplitudes. The model is also useful for studies such as design optimization of lateral suspension characteristics, and (together with related programs) evaluation of maximum speed limits for various track classifications based on simulated vehicle dynamic response to a statistical representation of track structure irregularities.

### 6.2 MODEL DESCRIPTION

The LATERAL car model shown in Figure 6-1 is a three-dimensional representation of a rigid three-degree-of-freedom car body connected through linear secondary suspension elements to two six-degree-of-freedom trucks. The two wheelsets are connected to the rigid truck frame through linear lateral and yaw stiffness elements. Lateral and longitudinal axes of symmetry exist as defined by axes A-A and B-B in Figure 6-1. The model also assumes that vertical translational and pitch rotational motions are decoupled from the lateral motions. The track is assumed to be rigid, having one of two types of sinusoidal irregularities. The first option allows the irregularity to be specified as a lateral displacement,  $\delta$ , of the track centerline in the horizontal plane; and the second option specifies a crosslevel misalignment as the



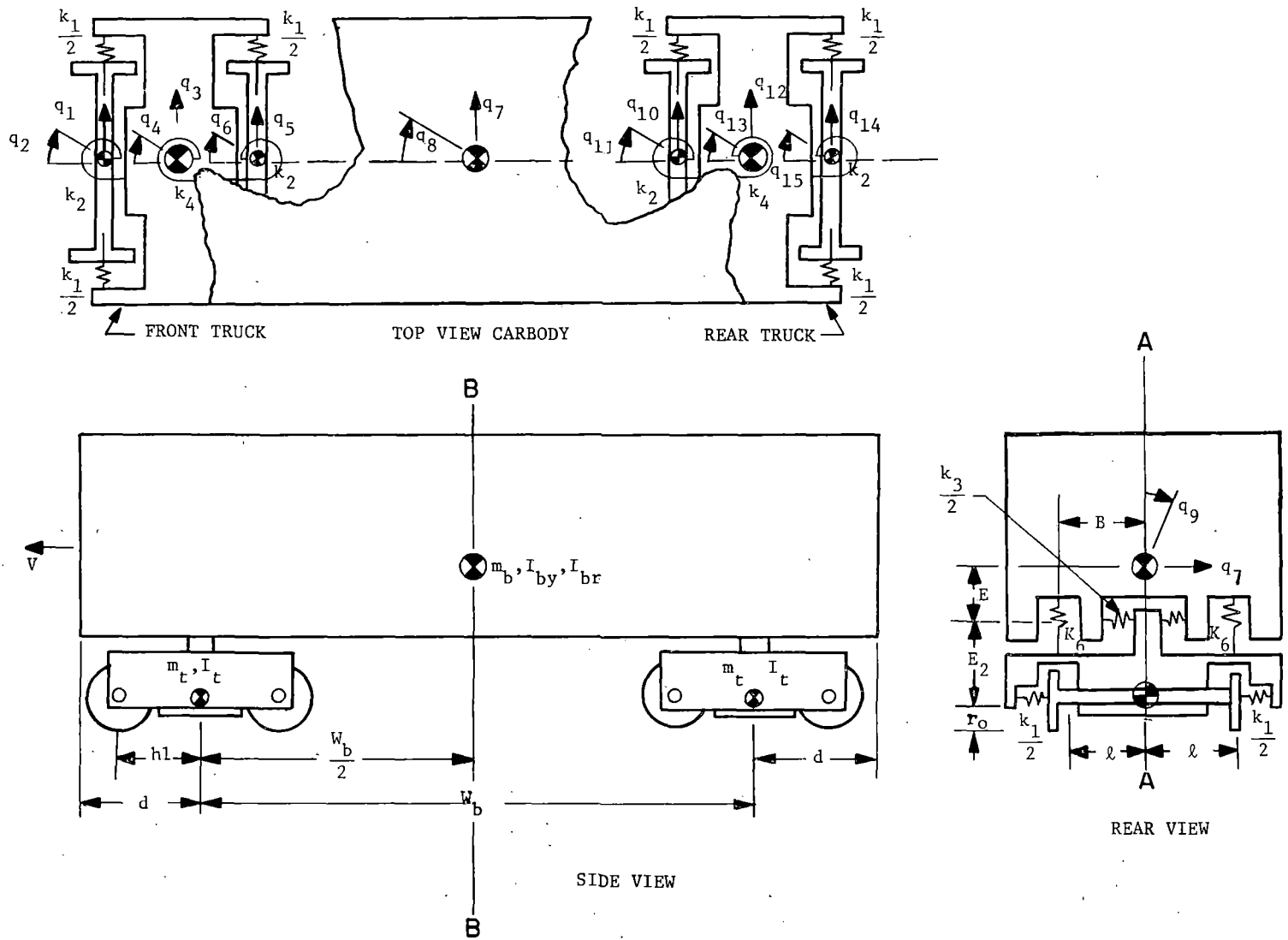


Figure 6-1. Lateral Rail Vehicle Model

ratio of difference in height of the two rails ( $z$ ), divided by the track gage,  $2\ell$ . The two options are mutually exclusive. The program does not accommodate wheel eccentricity.

Wheel rail interactions consider effects of creep forces and gravitational stiffness which is defined in Ref. 2, as the force-per-unit lateral displacement required to move a loaded wheelset laterally, in the absence of friction. A gyroscopic precession torque, caused by the cross product of the wheelset spin axis angular momentum and wheelset roll angular velocity resulting from crosslevel track irregularities, is also included as a user option. A definition of model components and the fifteen coordinate degree-of-freedom are defined in Tables 6-1 and 6-2, respectively.

### 6.3 EQUATIONS OF MOTION

The equations of motion for the lateral car model shown in Figure 6-1 are derived using Lagrange's equation for the generalized coordinates  $q_j$ ;  $j = 1-15$  (refer to Table 6-2 for coordinate descriptions). The equations are derived separately for track alignment ( $\delta$ ) or track crosslevel ( $\theta$ ) irregularities. Note that the wheelset lateral coordinates are defined as wheel displacements relative to the rail at the wheel/rail interface for crosslevel inputs and as the absolute displacement of the wheelset center of gravity for alignment inputs. In either case the two motions are easily relatable by a simple constraint equation, such as  $q_{c.g.} = q_{w/r} + \delta$  (for alignment irregularities) or  $q_{c.g.} = q_{w/r} + r_o \theta$  (for crosslevel irregularities). The motion of the wheelset c.g. is defined as the wheel/rail relative displacement plus the displacement associated with the c.g. traversing the lateral track irregularity amplitude ( $\delta$ ) for alignment inputs, or swinging through an arc determined by the distance from the center of rotation ( $r_o$ ) and the magnitude of the angular rotation ( $\theta$ ) associated with crosslevel irregularities. The remaining coordinate responses are measured with respect to an inertial coordinate system.

TABLE 6-1. COMPONENTS FOR THE LATERAL CAR MODEL

Subsystem	Component	Symbol	Units
Axle	Axle Weight	$m_w$	lbm
	Axle Lateral Damping	$C_1$	lb-sec-in <sup>-1</sup>
	Axle Lateral Stiffness	$K_1$	lb-in <sup>-1</sup>
	Axle Spacing	$2h_l$	ft
	Axle Yaw Inertia	$I_w$	lb-sec <sup>2</sup> -in
	Axle Spin Axis Inertia	$I_o$	lb-sec <sup>2</sup> -in
	Axle Torsional Yaw Stiffness	$K_2$	lb-in-rad <sup>-1</sup> ,
	Axle Torsional Yaw Damping	$C_2$	lb-sec-in-rad <sup>-1</sup>
	Wheel Radius, Conicity	$r_o, \alpha$	ft, radians
	Gage	$2l$	ft
Truck	Truck Frame Weight	$m_t$	lbm
	Truck Lateral Damping	$C_3$	lb-sec-in <sup>-1</sup>
	Truck Lateral Stiffness	$K_3$	lb-in <sup>-1</sup>
	Truck Yaw Inertia	$I_t$	lb-sec <sup>2</sup> -in
	Truck Yaw Damping	$C_4$	in-lb-sec-rad <sup>-1</sup>
	Truck Yaw Stiffness	$K_4$	in-lb-rad <sup>-1</sup>
Car Body	Body Weight	$m_b$	lbm
	Body Yaw Inertia	$I_{by}$	lb-in-sec <sup>2</sup>

TABLE 6-1. COMPONENTS FOR THE LATERAL CAR MODEL (Continued)

Subsystem	Component	Symbol	Units
Car Body - Cont.	Roll Inertia	$I_{br}$	lb-in-sec <sup>2</sup>
	Secondary Vertical Stiffness	$K_6$	lb-in <sup>-1</sup>
	Secondary Vertical Damping	$C_6$	lb-sec/in
	Body Length	L	ft
	Truck Distance from Car-End	d	ft
	Wheel Base Length	$W_b$	ft
	Body Center of Mass Height Above Secondary Lateral Suspension	E	ft
	Secondary Spring Lateral Spacing	2B	ft
	Secondary Lateral Suspension Height Above Truck Center of Mass	$E_2$	ft
Interface Coefficients	Wheel-Rail Gravitational Stiffness, lateral	$k_g$	lb-in <sup>-1</sup>
	Wheel-Rail Gravitational Stiffness, yaw	$k_a$	lb-in/rad
	Creep Coefficients	$f_L, f_T$	lb

TABLE 6-2. DEGREES OF FREEDOM IN PROGRAM "LATERAL"  
(OPTION II - CROSSLEVEL)

Coordinate Number	Coordinate Description	Symbol
1	Leading Truck Front Axle, Lateral Wheel Displacement with Respect to Rail Surface*	X1
2	Leading Truck Front Axle Yaw	PSI1
3	Leading Truck Body Lateral Displacement (Sway)	X2
4	Leading Truck Body Yaw	PSI2
5	Leading Truck Rear Axle, Lateral Wheel Displacement with Respect to Rail Surface*	X3
6	Leading Truck Rear Axle Yaw	PSI3
7	Car Body Lateral Displacement (Sway)	X4
8	Car Body Yaw	PSI4
9	Car Body Roll	THETA
10	Trailing Truck Front Axle, Lateral Wheel Displacement with Respect to Rail Surface*	X5
11	Trailing Truck Front Axle Yaw	PSI5
12	Trailing Truck Body Lateral Displacement (Sway)	X6
13	Trailing Truck Body Yaw	PSI6
14	Trailing Truck Rear Axle, Lateral Wheel Displacement with Respect to Rail Surface*	X7
15	Trailing Truck Rear Axle Yaw	PSI7

\*with respect to an inertial reference frame for alignment inputs (OPTION I)

For crosslevel track irregularities, the Lagrangian of the system ( $L=T-V$ ) is defined by the following expressions for the system kinetic ( $T$ ) and potential ( $V$ ) energies. For crosslevel inputs ( $\theta$ ), the kinetic energy function is as follows:

$$\begin{aligned}
T = & \frac{1}{2} \left[ m_b \dot{q}_7^2 + I_{by} \dot{q}_8^2 + I_{br} \dot{q}_9^2 \right] + \frac{1}{2} m_t (\dot{q}_3^2 + \dot{q}_{12}^2) \\
& + I_t (\dot{q}_4^2 + \dot{q}_{13}^2) + \frac{1}{2} m_w \left[ (\dot{q}_1 + r_o \dot{\theta}_1)^2 + (\dot{q}_5 + r_o \dot{\theta}_5)^2 + (\dot{q}_{10} + r_o \dot{\theta}_{10})^2 \right. \\
& \left. + (\dot{q}_{14} + r_o \dot{\theta}_{14})^2 \right] + \frac{1}{2} I_w \left[ \dot{q}_2^2 + \dot{q}_6^2 + \dot{q}_{11}^2 + \dot{q}_{15}^2 \right] \quad (6-1)
\end{aligned}$$

The potential energy function is written:

$$\begin{aligned}
V = & \frac{k_1}{2} \left[ (q_1 + r_o \theta_1 - q_3 - h \ell q_4)^2 + (q_5 + r_o \theta_5 - q_3 + h \ell q_4)^2 \right. \\
& \left. + (q_{10} + r_o \theta_{10} - q_{12} - h \ell q_{13})^2 + (q_{14} + r_o \theta_{14} - q_{12} + h \ell q_{13})^2 \right] \\
& + \frac{k_2}{2} \left[ (q_2 - q_4)^2 + (q_6 - q_4)^2 + (q_{11} - q_{13})^2 + (q_{15} - q_{13})^2 \right] \\
& + \frac{k_3}{2} \left[ \left( q_7 - q_3 + \frac{wb}{2} q_8 - E q_9 - E_2 \theta_3 \right)^2 + \left( q_7 - q_{12} - \frac{wb}{2} q_8 - E q_9 - E_2 \theta_{12} \right)^2 \right] \\
& + \frac{k_4}{2} \left[ (q_4 - q_8)^2 + (q_{13} - q_8)^2 \right] + k_6 B^2 \left[ (q_9 - \theta_3)^2 + (q_9 - \theta_{12})^2 \right] \quad (6-2)
\end{aligned}$$

The Rayleigh dissipation function for crosslevel inputs is as follows:

$$\begin{aligned}
D = & \frac{C_1}{2} \left[ (\dot{q}_1 + r_o \dot{\theta}_1 - \dot{q}_3 - h \ell \dot{q}_4)^2 + (\dot{q}_5 + r_o \dot{\theta}_5 - \dot{q}_3 + h \ell \dot{q}_4)^2 \right. \\
& \left. + (\dot{q}_{10} + r_o \dot{\theta}_{10} - \dot{q}_{12} - h \ell \dot{q}_{13})^2 + (\dot{q}_{14} + r_o \dot{\theta}_{14} - \dot{q}_{12} + h \ell \dot{q}_{13})^2 \right] \\
& + \frac{C_2}{2} \left[ (\dot{q}_2 - \dot{q}_4)^2 + (\dot{q}_6 - \dot{q}_4)^2 + (\dot{q}_{11} - \dot{q}_{13})^2 + (\dot{q}_{15} - \dot{q}_{13})^2 \right] \\
& + \frac{C_3}{2} \left[ \left( \dot{q}_7 - \dot{q}_3 + \frac{wb}{2} \dot{q}_8 - E \dot{q}_9 - E_2 \dot{\theta}_3 \right)^2 + \left( \dot{q}_7 - \dot{q}_{12} - \frac{wb}{2} \dot{q}_8 - E \dot{q}_9 - E_2 \dot{\theta}_{12} \right)^2 \right] \\
& + \frac{C_4}{2} \left[ (\dot{q}_4 - \dot{q}_8)^2 + (\dot{q}_{13} - \dot{q}_8)^2 \right] + C_6 B^2 \left[ (\dot{q}_9 - \dot{\theta}_3)^2 + (\dot{q}_9 - \dot{\theta}_{12})^2 \right] \quad (6-3)
\end{aligned}$$

where

$$\begin{aligned} \theta_1 = \theta_2 = \theta_o \exp(i\omega t) \exp\left(-i\frac{2\pi}{\lambda}h\ell\right); & \quad \theta_{10} = \theta_{11} = \theta_o \exp(i\omega t) \exp\left(i\frac{2\pi}{\lambda}(L-2d-h\ell)\right); \\ \theta_3 = \theta_o \exp(i\omega t); & \quad \theta_{12} = \theta_o \exp(i\omega t) \exp\left(i\frac{2\pi}{\lambda}(L-2d)\right); \\ \theta_5 = \theta_6 = \theta_o \exp(i\omega t) \exp\left(i\frac{2\pi}{\lambda}h\ell\right); & \quad \theta_{14} = \theta_{15} = \theta_o \exp(i\omega t) \exp\left(i\frac{2\pi}{\lambda}(L-2d+h\ell)\right). \end{aligned}$$

External (wheelset) forces ( $F_w$ ) are:

- (a) Lateral creep ( $f_L$ ) and gravitation stiffness ( $K_g$ ) forces of the form:

$$2f_L \left( \frac{\dot{q}_i}{V} - q_{i+1} \right) + K_g q_i \quad (6-4a)$$

for wheelset lateral coordinates,  $i=1, 5, 10, 14$

- (b) Yaw creep ( $f_t$ ) gravitational stiffness ( $K_a$ ) and precession ( $I_o \frac{V}{r_o} \dot{\theta}$ ) torques of the form:

$$2f_t \left( \dot{q}_i \frac{\ell^2}{V} + q_{i-1} \frac{\alpha \ell}{r_o} \right) - K_a q_i - I_o \frac{V}{r_o} \dot{\theta} \quad (6-4b)$$

for wheelset yaw coordinates,  $i=2, 6, 11, 15$ .

Equations of motion for crosslevel track perturbations are developed from Lagrange's equation where  $L=T-V$ :

$$\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{q}_i} \right) - \frac{\partial L}{\partial q_i} + \frac{\partial D}{\partial \dot{q}_i} + F_w = 0 \quad (6-5)$$

The system equations of motion for crosslevel inputs are given below for sinusoidal crosslevel misalignment - [zero phase angle  $\phi$  input at lead truck ( $q_3$ )]:

Front Axle - Front Truck

$$\begin{aligned} M_w \ddot{q}_1 + C_1 (\dot{q}_1 - \dot{q}_3 - h\ell \dot{q}_4) + k_1 (q_1 - q_3 - h\ell q_4) + 2f_L \dot{q}_1 / V \\ - 2f_L q_2 + k_g q_1 = Q(1) \end{aligned} \quad (6-6)$$

$$I_w \ddot{q}_2 + C_2(\dot{q}_2 - \dot{q}_4) + k_2(q_2 - q_4) + 2f_T q_1 \alpha \ell / r_o + 2f_T \dot{q}_2 \ell^2 / V - k_a q_2 = Q(2) \quad (6-7)$$

### Front Truck

$$M_t \ddot{q}_3 + C_1(2\dot{q}_3 - \dot{q}_1 - \dot{q}_5) + k_1(2q_3 - q_1 - q_5) + C_3(\dot{q}_3 - \dot{q}_7 - \frac{W_b}{2} \dot{q}_8 + E\dot{q}_9) + k_3(q_3 - q_7 - \frac{W_b}{2} q_8 + Eq_9) = Q(3) \quad (6-8)$$

$$I_t \ddot{q}_4 + C_2(2\dot{q}_4 - \dot{q}_2 - \dot{q}_6) + k_2(2q_4 - q_2 - q_6) + C_4(\dot{q}_4 - \dot{q}_8) + k_4(q_4 - q_8) + C_1(2h\ell\dot{q}_4 - \dot{q}_1 + \dot{q}_5)h\ell + k_1(2h\ell q_4 - q_1 + q_5)h\ell = Q(4) \quad (6-9)$$

### Rear Axle - Front Truck

$$M_w \ddot{q}_5 + C_1(\dot{q}_5 - \dot{q}_3 + h\ell\dot{q}_4) + k_1(q_5 - q_3 + h\ell q_4) + 2f_L \dot{q}_5 / V - 2f_L q_6 + k_g q_5 = Q(5) \quad (6-10)$$

$$I_w \ddot{q}_6 + C_2(\dot{q}_6 - \dot{q}_4) + k_2(q_6 - q_4) + 2f_T q_5 \alpha \ell / r_o + 2f_T \dot{q}_6 \ell^2 / V - k_a q_6 = Q(6) \quad (6-11)$$

### Car Body

$$M_b \ddot{q}_7 + C_3(2\dot{q}_7 - \dot{q}_3 - \dot{q}_{12}) - 2C_3 E \dot{q}_9 + k_3(2q_7 - q_3 - q_{12}) - 2k_3 E q_9 = Q(7) \quad (6-12)$$

$$I_{by} \ddot{q}_8 + C_4(2\dot{q}_8 - \dot{q}_4 - \dot{q}_{13}) + k_4(2q_8 - q_4 - q_{13}) + C_3 \frac{W_b}{2} [W_b \dot{q}_8 - \dot{q}_3 + \dot{q}_{12}] + k_3 \frac{W_b}{2} [W_b q_8 - q_3 + q_{12}] = Q(8) \quad (6-13)$$

$$I_{br} \ddot{q}_9 + 4k_6 B^2 q_9 + 4C_6 B^2 \dot{q}_9 + k_3 E (2Eq_9 + q_3 + q_{12} - 2q_7) + C_3 E (2E\dot{q}_9 + \dot{q}_3 + \dot{q}_{12} - 2\dot{q}_7) = Q(9) \quad (6-14)$$



Front Axle-Rear Truck

$$M_w \ddot{q}_{10} + C_1 (\dot{q}_{10} - \dot{q}_{12} - h\ell \dot{q}_{13}) + k_1 (q_{10} - q_{12} - h\ell q_{13}) + 2\dot{q}_{10} f_L / V - 2f_L q_{11} + k_g q_{10} = Q(10) \quad (6-15)$$

$$I_w \ddot{q}_{11} + C_2 (\dot{q}_{11} - \dot{q}_{13}) + k_2 (q_{11} - q_{13}) + 2f_T q_{10} \alpha \ell / r_o + 2f_T \dot{q}_{11} \ell^2 / V - k_a q_{11} = Q(11) \quad (6-16)$$

Rear Truck

$$M_T \ddot{q}_{12} + C_1 (2\dot{q}_{12} - \dot{q}_{10} - \dot{q}_{14}) + k_1 (2q_{12} - q_{10} - q_{14}) + C_3 \left( \dot{q}_{12} - \dot{q}_7 + \frac{W_b}{2} \dot{q}_8 + E \dot{q}_9 \right) + k_3 \left( q_{12} - q_7 + \frac{W_b}{2} q_8 + E q_9 \right) = Q(12) \quad (6-17)$$

$$I_T \ddot{q}_{13} + C_2 (2\dot{q}_{13} - \dot{q}_{11} - \dot{q}_{15}) + k_2 (2q_{13} - q_{11} - q_{15}) + C_4 (\dot{q}_{13} - \dot{q}_8) + k_4 (q_{13} - q_8) + C_1 h\ell (2h\ell \dot{q}_{13} - \dot{q}_{10} + \dot{q}_{14}) + k_1 h\ell (2h\ell q_{13} - q_{10} + q_{14}) = Q(13) \quad (6-18)$$

Rear Axle - Rear Truck

$$M_w \ddot{q}_{14} + C_1 (\dot{q}_{14} - \dot{q}_{12} + h\ell \dot{q}_{13}) + k_1 (q_{14} - q_{12} + h\ell q_{13}) + 2f_L \dot{q}_{14} / V - 2f_L q_{15} + k_g q_{14} = Q(14) \quad (6-19)$$

$$I_w \ddot{q}_{15} + C_2 (\dot{q}_{15} - \dot{q}_{13}) + k_2 (q_{15} - q_{13}) + 2f_T q_{14} \alpha \ell / r_o + 2f_T \dot{q}_{15} \ell^2 / V - k_a q_{15} = Q(15) \quad (6-20)$$

The (crosslevel) forcing function vector is as follows:

$$Q(1) = -r_o (m_w \ddot{\theta} + C_1 \dot{\theta} + k_1 \theta) \exp\left(-i \frac{2\pi h\ell}{\lambda}\right) \quad (6-21)$$

$$Q(2) = I_o V / r_o \dot{\theta} \exp\left(-i \frac{2\pi h\ell}{\lambda}\right) \quad (6-22)$$

$$Q(3) = -E_2 [k_3 \theta + C_3 \dot{\theta}] + r_o (k_1 \theta + C_1 \dot{\theta}) \left[ \exp\left(i \frac{2\pi h\ell}{\lambda}\right) + \exp\left(-i \frac{2\pi h\ell}{\lambda}\right) \right] \quad (6-23)$$

$$Q(4) = h\ell r_o (k_1 \theta + C_1 \dot{\theta}) \left[ \exp\left(-i \frac{2\pi h\ell}{\lambda}\right) - \exp\left(i \frac{2\pi h\ell}{\lambda}\right) \right] \quad (6-24)$$

$$Q(5) = - r_o (m_w \ddot{\theta} + C_1 \dot{\theta} + k_1 \theta) \exp\left(i \frac{2\pi h \ell}{\lambda}\right) \quad (6-25)$$

$$Q(6) = I_o V / r_o \dot{\theta} \exp\left(i \frac{2\pi h \ell}{\lambda}\right) \quad (6-26)$$

$$Q(7) = E_2 [k_3 \theta + C_3 \dot{\theta}] \left\{ 1 + \exp\left[i \frac{2\pi}{\lambda} (L-2d)\right] \right\} \quad (6-27)$$

$$Q(8) = \frac{W_b}{2} E_2 (k_3 \theta + C_3 \dot{\theta}) \left\{ 1 - \exp\left[i \frac{2\pi}{\lambda} (L-2d)\right] \right\} \quad (6-28)$$

$$Q(9) = (2B^2 k_6 - k_3 E E_2) \theta \left\{ 1 + \exp\left[i \frac{2\pi}{\lambda} (L-2d)\right] \right\} \\ + (2B^2 C_6 - C_3 E E_2) \dot{\theta} \left\{ 1 + \exp\left[i \frac{2\pi}{\lambda} (L-2d)\right] \right\} \quad (6-29)$$

$$Q(10) = - r_o (m_w \ddot{\theta} + C_1 \dot{\theta} + k_1 \theta) \exp\left[i \frac{2\pi}{\lambda} (L-2d-h\ell)\right] \quad (6-30)$$

$$Q(11) = I_o V / r_o \dot{\theta} \exp\left[i \frac{2\pi}{\lambda} (L-2d-h\ell)\right] \quad (6-31)$$

$$Q(12) = - E_2 [k_3 \theta + C_3 \dot{\theta}] \exp\left[i \frac{2\pi}{\lambda} (L-2d)\right] + r_o (k_1 \theta + C_1 \dot{\theta}) \\ \exp\left[i \frac{2\pi}{\lambda} (L-2d-h\ell)\right] + \exp\left[i \frac{2\pi}{\lambda} (L-2d+h\ell)\right] \quad (6-32)$$

$$Q(13) = h \ell r_o (k_1 \theta + C_1 \dot{\theta}) \left\{ \exp\left[i \frac{2\pi}{\lambda} (L-2d-h\ell)\right] - \exp\left[i \frac{2\pi}{\lambda} (L-2d+h\ell)\right] \right\} \quad (6-33)$$

$$Q(14) = - r_o (m_w \ddot{\theta} + C_1 \dot{\theta} + k_1 \theta) \exp\left[i \frac{2\pi}{\lambda} (L-2d+h\ell)\right] \quad (6-34)$$

$$Q(15) = I_o V / r_o \dot{\theta} \exp\left[i \frac{2\pi}{\lambda} (L-2d+h\ell)\right] \quad (6-35)$$

where  $\theta = \theta_o \exp(i\omega t)$

$\theta_o$  = maximum value of crosslevel input ( $z/2\ell$ )

$\lambda = V/f$

$\omega = 2\pi f$

$V$  = vehicle velocity (program input)

$f$  = cyclical frequency (program input)

The same procedure is used to derive equations of motion for track alignment perturbations. The left hand side for this

system of equations is identical with equations (6-6) through (6-20), with the non-zero terms of the (alignment) forcing function as follows:

$$Q(1) = \delta_o K_g \exp(i\omega t) \quad (6-36)$$

$$Q(2) = \delta_o 2f_T \exp(i\omega t) \alpha \ell / r_o \quad (6-37)$$

$$Q(5) = \delta_o K_g \exp\left[i\frac{2\pi}{\lambda} 2h\ell\right] \exp(i\omega t) \quad (6-38)$$

$$Q(6) = \delta_o 2f_T \exp\left[i\frac{2\pi}{\lambda} 2h\ell\right] \exp(i\omega t) \alpha \ell / r_o \quad (6-39)$$

$$Q(10) = \delta_o K_g \exp\left[i\frac{2\pi}{\lambda} (L-2d)\right] \exp(i\omega t) \quad (6-40)$$

$$Q(11) = \delta_o 2f_T \exp\left[i\frac{2\pi}{\lambda} (L-2d)\right] \exp(i\omega t) \alpha \ell / r_o \quad (6-41)$$

$$Q(14) = \delta_o K_g \exp\left[i\frac{2\pi}{\lambda} (L-2h\ell)\right] \exp(i\omega t) \quad (6-42)$$

$$Q(15) = \delta_o f_T \exp\left[i\frac{2\pi}{\lambda} (L-2d+2h\ell)\right] \exp(i\omega t) \alpha \ell / r_o \quad (6-43)$$

where  $\delta_o$  = maximum lateral displacement of the centerline. The preceding equations are combined into a matrix equation of the form

$$[M] \{\ddot{q}\} + [C] \{\dot{q}\} + [K] \{q\} = \{Q\} \quad (6-44)$$

where

[M] = mass matrix

[C] = augmented damping matrix

[K] = augmented stiffness matrix

{Q} = forcing function vector

{q} = generalized coordinate vector

The damping and stiffness matrices are assembled by grouping the coefficients of the generalized coordinates  $\dot{q}_j$  and  $q_j$  ( $J=1,15$ ), respectively, on the left hand side of each equation. Since these matrices include creep-force and gravitational stiffness terms, these matrices are not symmetric and are referred to as "augmented" matrices. The two forcing function vectors (q) given by equations (6-21) through (6-35) and (6-36) through (6-43) for options I or II, respectively, are independent of the  $q_j$ 's and their derivatives.

#### 6.4 SOLUTION PROCEDURE AND PROGRAM FLOW

The equations of motion (equations 6-6 through 6-35) are linear differential equations with constant coefficients. Solutions of the form

$$q_j(t) = \bar{q}_j(\omega) \exp(i\omega t) \quad j = 1, 15 \quad (6-45)$$

where  $\bar{q}_j(\omega)$  is the complex amplitude of the oscillation for each coordinate  $q_j$  at the angular frequency  $\omega$  in response to the complex forcing function defined by equations (6-36) through (6-43) for alignment track irregularities (Option I) or by equations (6-21) through (6-35) for crosslevel track irregularities (Option II).

After the data listed in Table 6-1 and the track perturbation option are read in, the [M], [C], and [K] coefficient matrices are calculated and combined to form the complex coefficient matrix:

$$\text{MAT}(\omega) = -\omega^2[M]^2 + i\omega[C] + [K] \quad (6-46)$$

The program then enters the main frequency loop and repeatedly solves the matrix equation for each value of  $\omega$  by first computing the forcing function coefficients and then premultiplying by the inverse of the complex coefficient matrix, to solve for the normalized values of the coordinate vector  $\{q_j(\omega)\}$ , according to

$$\{\bar{q}_j(\omega)\} = [\text{MAT}(\omega)]^{-1} \{\bar{Q}_j(\omega)\} \quad (6-47)$$

Frequencies are generated by specifying vehicle velocity (a program input) and adjusting the track irregularity wavelength (alignment or crosslevel), to produce the desired frequency bandwidth according to  $\omega = 2\pi f = 2\pi V/\lambda$ . Discrete values of frequency (limits and increments) are program inputs.

In the program coding, the generalized coordinate responses  $q_j$ ,  $j=1, 15$ , are normalized with respect to the maximum amplitude of the specified track perturbation. In the case of Option I, all the  $q_j$ 's (both the lateral displacement and angular coordinates) are normalized with respect to  $\delta_0$ , the maximum lateral displacement

of the track centerline from tangent track in the horizontal plane. In the case of Option II, the lateral displacement coordinates are normalized with respect to the maximum difference in the height of the rails at opposing points along the track; and the angular (roll and yaw) coordinates are normalized with respect to  $\theta_0$ , the maximum difference in the height of the rails divided by the gage. If required, acceleration responses are also computed and stored by multiplying the corresponding displacement response by  $(2\pi f)^2/g$ .

## 6.5 INPUT/OUTPUT PARAMETERS AND CONTROL LOGIC

### 6.5.1 Physical Input Data and Control Logic

At the beginning of the "lateral" run the program will request values for the track perturbation option (IOPT = 1 for alignment irregularities, and IOPT = 2 for crosslevel irregularities), the vehicle velocity (V) and the acceleration of gravity (g). The physical input parameters describing car-body and truck components, as listed in Table 6-1, are then input. The final inputs required are inputs necessary to define the frequency response bandwidth (refer to Table 6-3) and to specify control variables to obtain (a) the desired response coordinates (refer to Tables 6-2 and 6-4); (b) type of response (acceleration or displacement frequency responses); and (c) the form of response (plots or printed output).

### 6.5.2 Outputs

The values of the coefficient matrices [M], [C], and [K] and the frequency range control data (Table 6-3) are printed at the beginning of the output. The components of the normalized solution vector  $q(\omega)$  and the corresponding accelerations  $\omega^2 q_j(\omega)$ ,  $j=1, 15$ , are printed and plotted for each frequency, if requested as indicated in Table 6-4. The units of the normalized displacement responses were discussed in Section 6.4. Acceleration responses are normalized in the same way except that the rectilinear coordinates are expressed in units of g's per  $\delta_0$  (Option I) or g's per  $2\ell\theta_0$  (Option II), and the angular coordinates are expressed in units of radians-sec<sup>-2</sup> per  $\delta_0$  (Option I) or radians-sec<sup>-2</sup> per  $\theta_0$  (Option II). Elements of a sample output are presented in Figure

TABLE 6-3. VARIABLES DESCRIBING FREQUENCY POINTS

Input Designation	Purpose	Possible Values
NDF	Specified number of frequency ranges considered.	1 to 10
DF(I) I=1,NDF	Frequency increment within I <sup>th</sup> frequency range	up to 200 frequency points
FL(I) I=1,NDF+1	Lower bound of I <sup>th</sup> frequency range, all ranges continuous. FL(NDF+1) gives upper bound of last range.	up to 200 frequency points

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TABLE 6-4. PRINTING AND PLOTTING CONTROL VARIABLES

Input Designation	Purpose	Possible Values
DTAG(N)	Controls printing and plotting of N <sup>th</sup> coordinate (N=1,15)	0, No Output 1, Print Out 2, Print and Plot Out
ATAG(N)	Controls printing and plotting of acceleration of N <sup>th</sup> coordinate (N=1,15)	0, No Output 1, Print Output 2, Print and Plot Out

6-2 (tabulation of input values) and Figures 6-3 through 6-6 (sample plots).

A listing of Program LATERAL is included in Volume II as Appendix C.

INPUT VALUES

LATERAL PARAMETRIC RLN II-7 IOPT=2 6-14-75

IOPT = 2  
 V = 0.5200E+02 MPH  
 G = 0.3864E+03 IN/SEC\*\*2

AXLE

MNG = 0.5220E+04 LBS  
 C1 = 0.1160E+03 LBS-SEC/IN  
 C2 = 0.2200E+06 IN-LBS-SEC/RAD  
 K1 = 0.2786E+05 LBS/IN  
 K2 = 0.4400E+03 IN-LBS/RAD  
 2HL = 0.8500E+01 FT  
 IW = 0.8129E+04 IN-LBS-SEC\*\*2

TRUCK

MTG = 0.1261E+05 LBS  
 C3 = 0.1730E+03 LBS-SEC/IN  
 C4 = 0.2800E+06 IN-LBS-SEC/RAD  
 K3 = 0.3389E+04 LBS/IN  
 K4 = 0.2800E+03 IN-LBS/RAD  
 IT = 0.5500E+05 IN-LBS-SEC\*\*2

CAR BODY

MBG = 0.1276E+06 LBS  
 K6 = 0.3900E+04 LBS/IN  
 IBY = 0.2870E+09 IN-LBS-SEC\*\*2  
 IBR = 0.6552E+06 IN-LBS-SEC\*\*2  
 LL = 0.8500E+02 FT  
 2L = 0.4830E+01 FT  
 D = 0.1290E+02 FT  
 W = 0.5950E+02 FT  
 E = 0.2100E+01 FT  
 E2 = 0.1900E+01 FT  
 2B = 0.3850E+01 FT  
 C6 = 0.5750E+02 LBS-SEC/IN

WHEEL

KA = 0.5800E+05 IN-LBS/RAD  
 KG = 0.3820E+05 LBS/IN  
 FSUBL = 0.1200E+07 LBS  
 FT = 0.1200E+07 LBS  
 ALPHA = 0.5200E-01 RADIANS  
 RO = 0.1500E+01 FT  
 IZERO = 0.8200E+00 IN-LBS-SEC\*\*2

Figure 6-2. Sample Output



DIAGNOSTIC L= 0.1023E+04 ZEE= 0.3552E+03

IFREQ= 1 NDF= 1

DF = 75 0 0 0 0 0 0

FL(HZ) = 0.012 10.0000 0.0000 0.0000 0.0000 0.0000

REQUESTED OUTPUT	PRINT	PLOT
DISPLACEMENT - LEAD TRUCK FRONT AXLE LAT,DISP	X	X
DISPLACEMENT - LEADING TRUCK FRONT AXLE YAW	X	X
DISPLACEMENT - LEADING TRUCK BODY LAT,DISP,	X	X
DISPLACEMENT - CAR BODY LATERAL DISPLACEMENT	X	X
DISPLACEMENT - CAR BODY ROLL	X	X
DISPLACEMENT - TRAILING TRUCK LATERAL DISP,	X	X

Figure 6-2. Sample Output - Continued

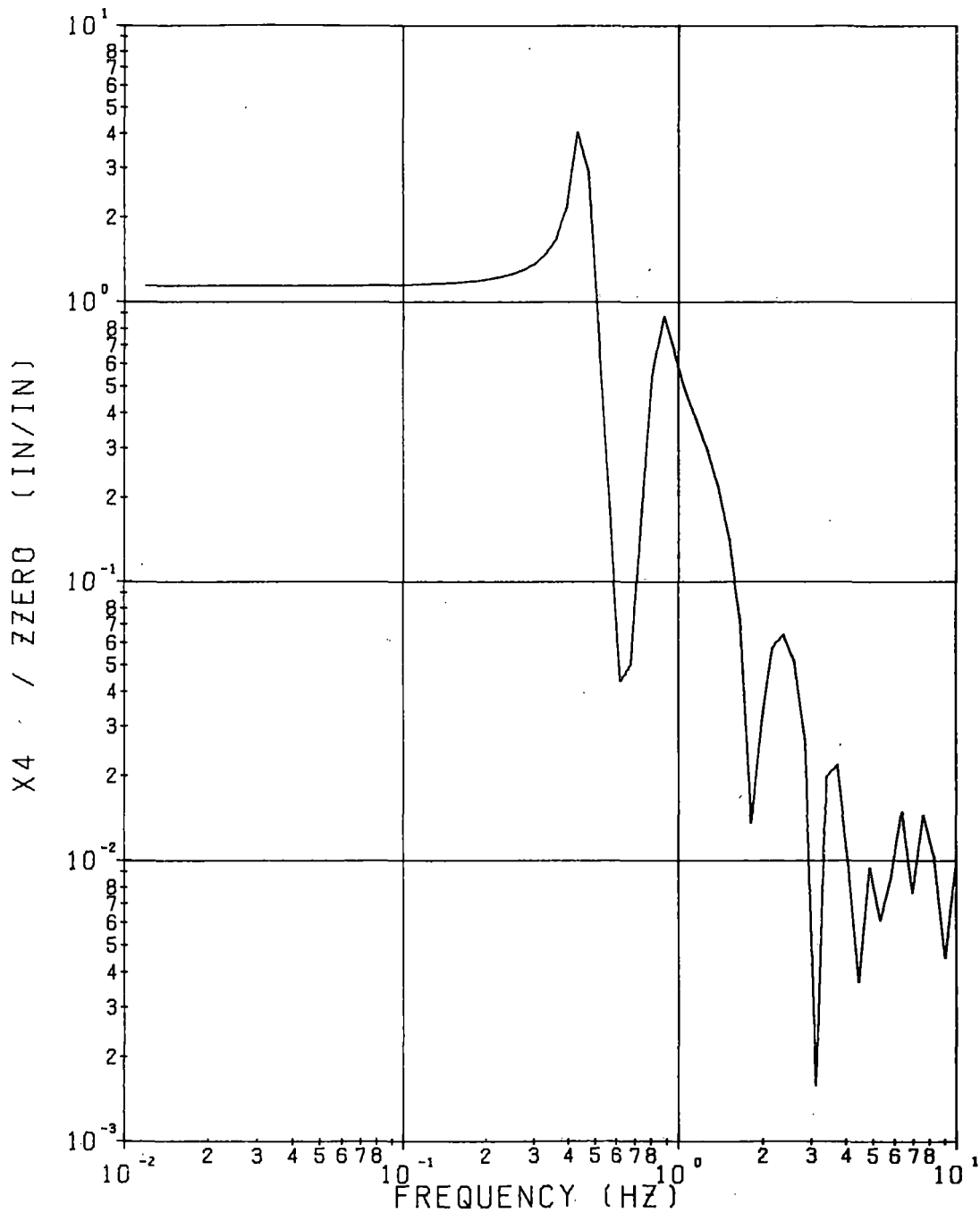


Figure 6-3. Crosslevel Perturbation Response of Car Body Lateral Displacement - Graphic Diagram

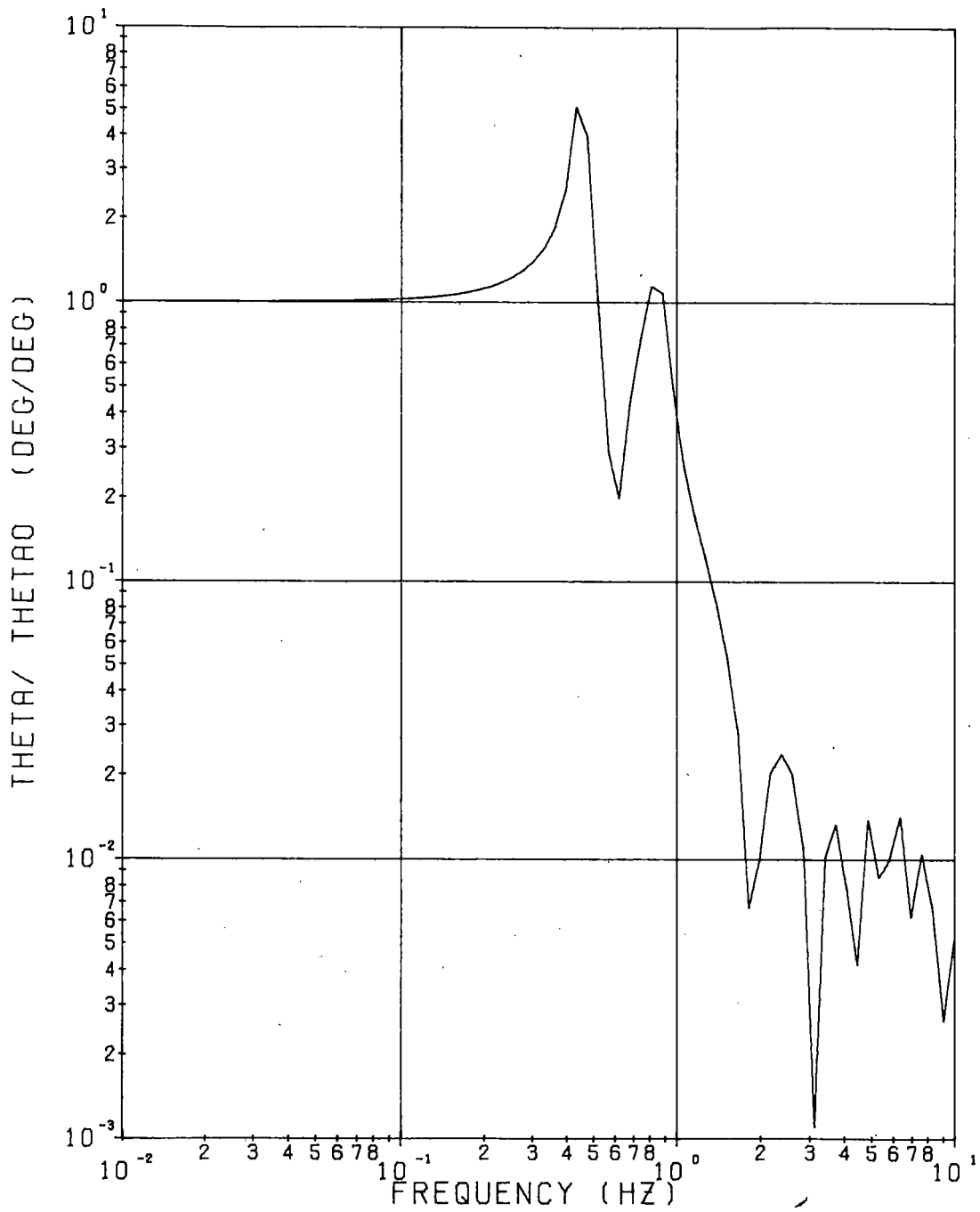


Figure 6-4. Crosslevel Perturbation Response of Car Body Roll - Graphic Diagram

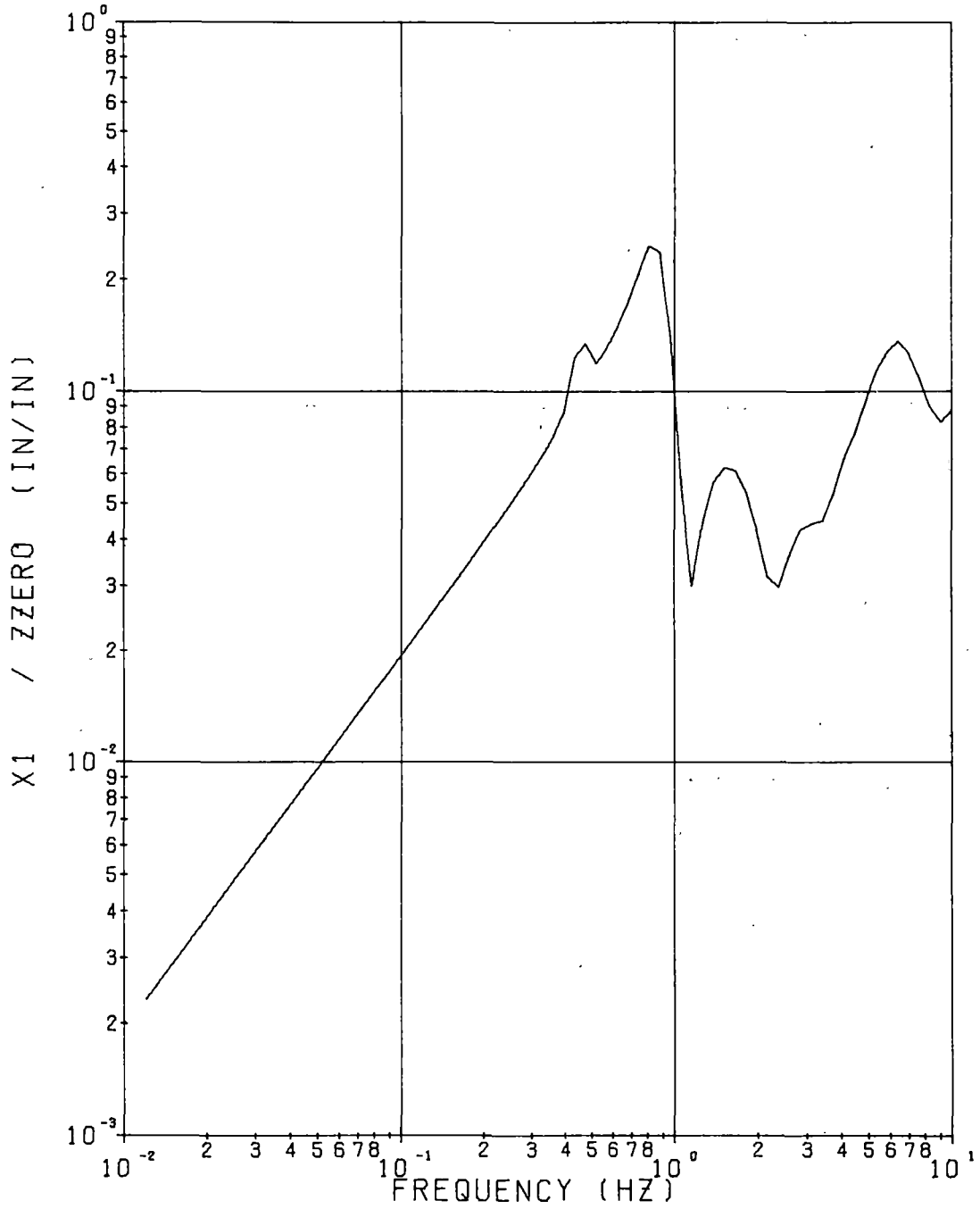


Figure 6-5. Crosslevel Perturbation Response of Lead Truck  
 Front Axle Lateral Displacement - Graphic Diagram

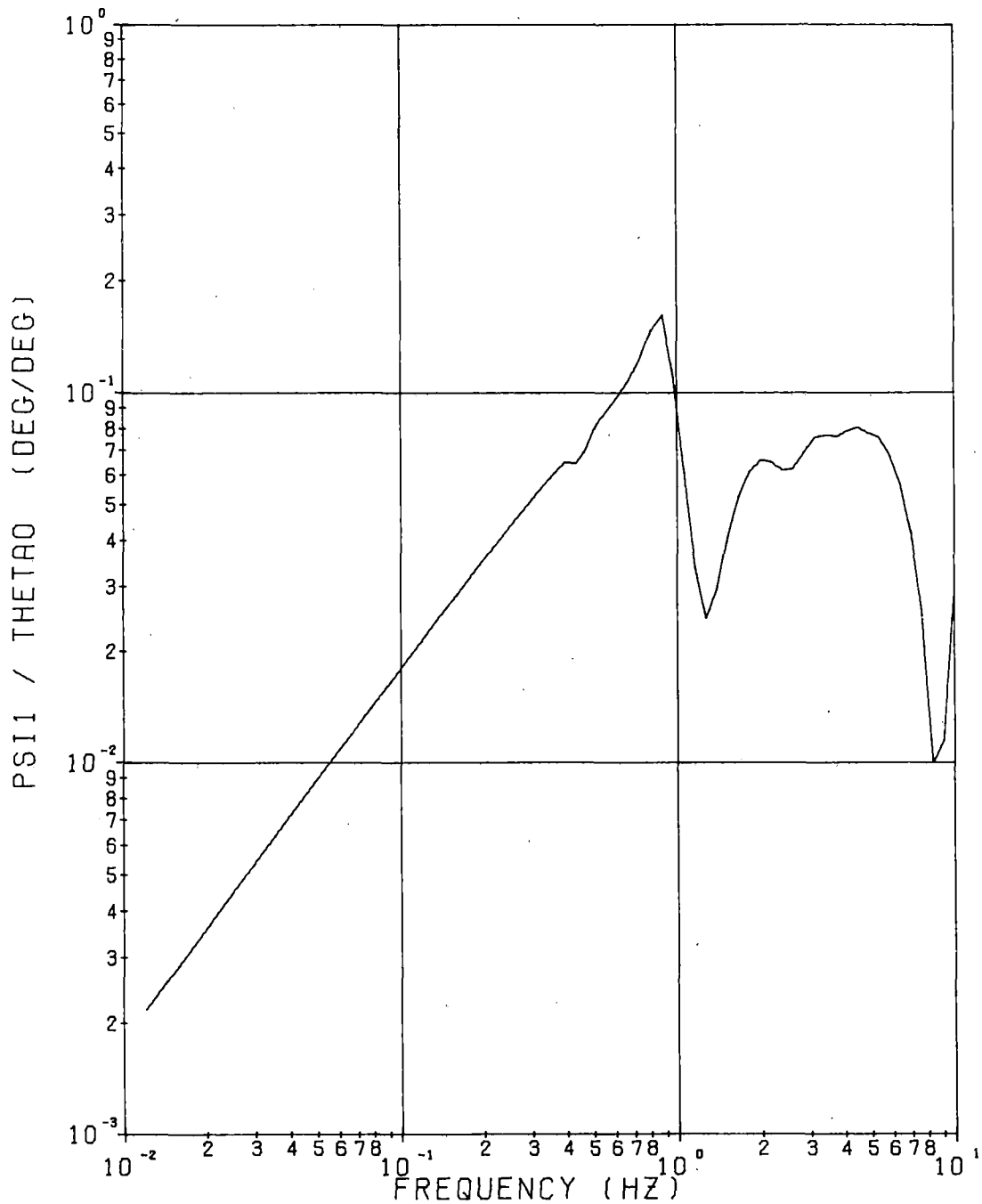


Figure 6-6. Crosslevel Perturbation Response of Lead Truck Front Axle Yaw - Graphic Diagram

## 7. PROGRAM "HALF"

### 7.1 APPLICATION

The primary usefulness of this model is to calculate vertical wheel-rail forces and track deflections in terms of vehicle suspension parameters and track structure properties, in response to vertical sinusoidal track (surface) irregularities. These parameters provide indicators of component wear and safety (in terms of maximum and minimum wheel loads and track deflection) and are useful for studies such as establishing maximum speed limits and matching vehicle vertical suspension and track structure characteristics. The model may also be used to predict the vertical frequency response of car body, truck, and wheelset components.

### 7.2 MODEL DESCRIPTION

Program HALF is a digital computer program which models one-half of a car body, one truck, and track structure impedance, represented by a beam on a visco-elastic foundation. The model shown in Figure 7-1 considers vertical forces and motions at the wheel/rail interface and neglects effects of car body pitch dynamics. Since track forces generally become significant at frequencies above the vehicle pitch natural frequency this assumption is valid. One-half of the car body mass is lumped over and connected to a truck through linear secondary suspension spring and damper elements. The truck is modeled as a rigid mass with rigid wheelsets connected to an equalizer bar, through the linear primary suspension spring element.

The vehicle moves over a track structure whose unloaded track profile ( $F_1 = F_2 = 0$ ) is specified by the vertical perturbation  $V_1$  and  $V_2$  (functions of a sinusoidal vertical track irregularity) at the left and right wheels. The deflections of the track from its unloaded positions  $V_1$  and  $V_2$  are given by the coordinates  $\delta_1$  and  $\delta_2$ . The track model is shown in Figure 7-2, along with a tabulated description of parameters which characterize the track structure.

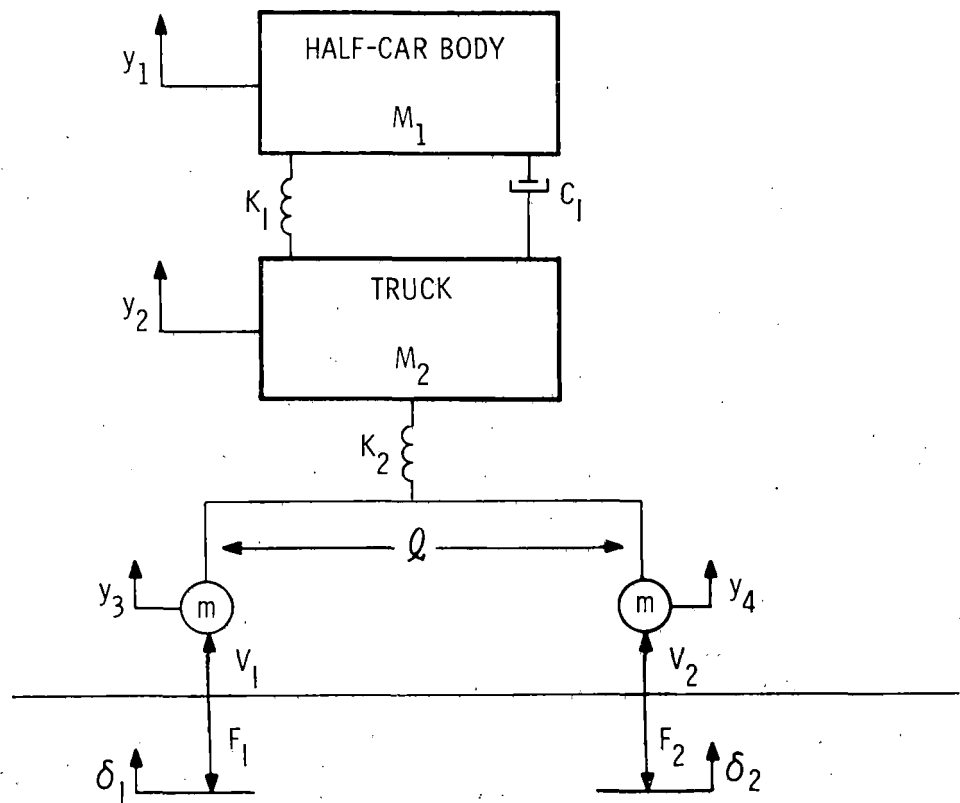
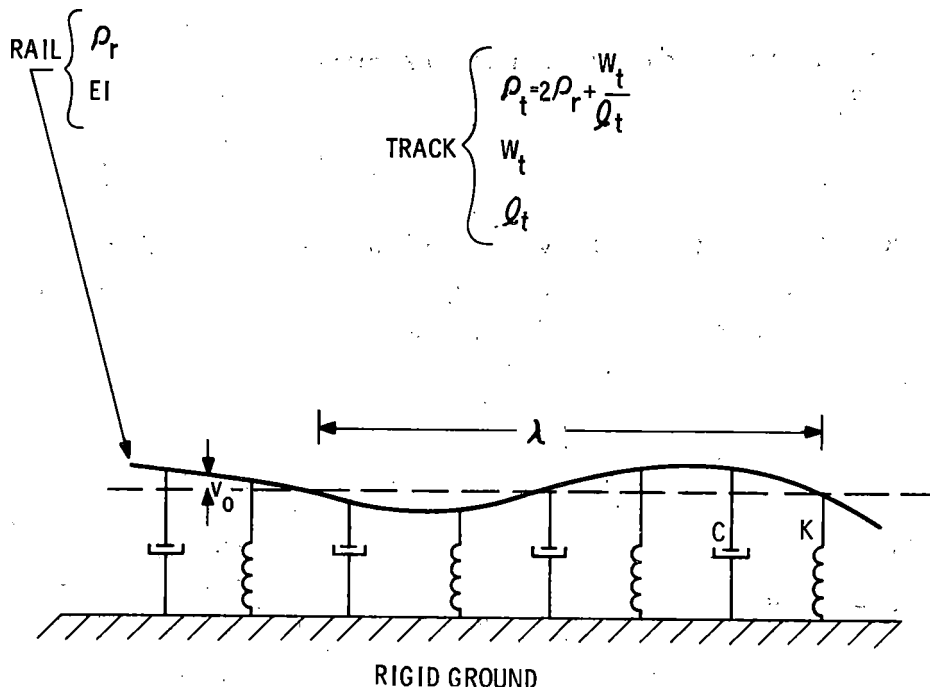


Figure 7-1. Program "HALF" Vehicle Model



VARIABLE	DEFINITION	UNITS
$\rho_r$	SINGLE RAIL DENSITY PER UNIT LENGTH	LBS/IN
$EI$	SINGLE RAIL FLEXURAL RIGIDITY	LBS-IN <sup>4</sup>
$w_t$	WEIGHT OF A TIE	LBS
$l_t$	TIE SPACING	IN
$\lambda$	WAVELENGTH OF RAIL DISTURBANCE	IN
$v_0$	AMPLITUDE OF TRACK IRREGULARITY	IN
$l$	TRUCK WHEELBASE	IN
$c$	DAMPING COEFFICIENT PER UNIT TRACK LENGTH	$\frac{\text{LBS/IN/SEC}}{\text{IN}}$
$k$	FOUNDATION STIFFNESS PER UNIT TRACK LENGTH	$\frac{\text{LBS/IN}}{\text{IN}}$

Figure 7-2. Program "HALF" Track Model



The entire model has eight unknowns:  $Y_1, Y_2, Y_3, Y_4, \delta_1, \delta_2, F_1,$  and  $F_2$ .

### 7.3 EQUATIONS

The following system of equations is solved for  $F_1/v_0, F_2/v_0, y_1/v_0, y_2/v_0, y_3/v_0, y_4/v_0, \delta_1/v_0, \delta_2/v_0$  as a function of frequency. (Refer to Figures 7-1 and 7-2.)

$$y_3 = v_1 + \delta_1 \quad (7-1)$$

$$y_4 = v_2 + \delta_2 \quad (7-2)$$

$$\delta_1 = G_{11}F_1 + G_{12}F_2 \quad (7-3)$$

$$\delta_2 = G_{11}F_2 + G_{12}F_1 \quad (7-4)$$

$$F_1 - m\ddot{y}_3 = F_2 - m\ddot{y}_4 \quad (7-5)$$

$$\frac{y_1}{y_2} = \frac{1 + 2j\beta\frac{\omega}{\omega_1}}{1 - \left(\frac{\omega}{\omega_1}\right)^2 + 2j\beta\frac{\omega}{\omega_1}} \quad (7-6)$$

$$F_1 + F_2 = m(\ddot{y}_3 + \ddot{y}_4) + M_2\ddot{y}_2 + M_1\ddot{y}_1 \quad (7-7)$$

$$K_2\left(y_2 - \frac{y_3 + y_4}{2}\right) + M_2\ddot{y}_2 + M_1\ddot{y}_1 = 0 \quad (7-8)$$

where

$v_0$  = amplitude of vertical irregularity

$$v_1 = v_0 \sin\left(\frac{2\pi x}{\lambda}\right); \quad v_2 = v_0 \sin\left(2\pi\frac{x+l}{\lambda}\right)$$

$$\ddot{y}_i = -\omega^2 y_i, \quad i = 1, 2, 3, 4$$

$x = Vt$ ,  $V$  is the vehicle's velocity

$$\omega = \frac{2\pi V}{\lambda}, \quad f = \frac{V}{\lambda} = \frac{\omega}{2\pi}, \quad \omega_1 = \sqrt{K_1/M_1}$$

$$\beta = \frac{C_1}{2} (K_1 M_1)^{-1/2}$$

The functions  $G_{11}$  and  $G_{12}$  are the dynamic compliance coefficients of a beam on a visco-elastic foundation and are given by the following expressions:

$$\omega_a = \sqrt{K/\rho_t}; \quad \omega/\omega_a = \bar{\omega} \quad (7-9)$$

$$\lambda_0 = 2\pi\sqrt{4EI/K} \quad (7-10)$$

$$\zeta = \frac{C}{2} \sqrt{1/K\rho_t} \quad (7-11)$$

for  $\bar{\omega} < 1$

$$\phi = \tan^{-1} \left\{ \frac{2\zeta\bar{\omega}}{1-\bar{\omega}^2} \right\} \quad (7-12)$$

$$\beta_1 = \frac{2\pi}{\lambda_0} \left[ (1-\bar{\omega}^2)^2 + (2\zeta\bar{\omega})^2 \right]^{1/8} \quad (7-13)$$

$$G_{11} = \frac{\pi e^{-j\left(\frac{3\phi}{4}\right)}}{K\lambda_0 \left[ (1-\bar{\omega}^2)^2 + (2\zeta\bar{\omega})^2 \right]^{3/8}} \quad (7-14)$$

$$G_{12} = G_{11} \frac{e}{\sqrt{2}} \left\{ e^{-j\left(\frac{\pi}{4}\right)} \left\{ e^{\sqrt{2}\beta_1} e^{j\left(\frac{3\pi}{4} + \frac{\phi}{4}\right)} + j e^{\sqrt{2}\beta_1} e^{j\left(\frac{5\pi}{4} + \frac{\phi}{4}\right)} \right\} \right\} \quad (7-15)$$

for  $\bar{\omega} > 1$

$$\phi = \tan^{-1} \left\{ \frac{2\zeta\bar{\omega}}{\bar{\omega}^2 - 1} \right\} \quad (7-16)$$

$$\beta_1 = \frac{2\pi}{\lambda_0} \left[ (\bar{\omega}^2 - 1)^2 + (2\zeta\bar{\omega})^2 \right]^{1/8} \quad (7-17)$$

$$G_{11} = \frac{\pi(1+i) e^{i\left(\frac{3\phi}{4}\right)}}{\sqrt{2} K \lambda_0 \left[ (\bar{\omega}^2 - 1)^2 + (2\zeta\bar{\omega})^2 \right]^{3/8}} \quad (7-18)$$

$$G_{12} = \frac{G_{11}}{(1+i)} \left\{ e^{\sqrt{2}\beta_1} e^{i(\pi-\phi/4)} + i e^{\sqrt{2}\beta_1} e^{i\left(\frac{3\pi}{2} - \frac{\phi}{4}\right)} \right\} \quad (7-19)$$

#### 7.4 SOLUTION PROCEDURE AND PROGRAM FLOW

After the input variables are read in and converted to units of inches, pounds and seconds, a complex coefficient matrix  $[A(\omega)]$  is formed. The program then enters the main frequency loop and repeatedly solves the matrix equation for each value of  $\omega$  by premultiplying by the inverse of the coefficient matrix. Frequencies are generated by specifying a vehicle velocity or track irregularity wavelength and adjusting the unspecified parameter to generate  $\omega$  according to  $\omega = 2\pi f = 2\pi V/\lambda$ :

$$[A(\omega)] \begin{Bmatrix} y_1/v_0 \\ y_2/v_0 \\ y_3/v_0 \\ y_4/v_0 \\ \delta_1/v_0 \\ \delta_1/v_0 \\ F_1/v_0 \\ F_2/v_0 \end{Bmatrix} = \begin{Bmatrix} 1 \\ \exp(j 2\pi \frac{\ell}{\lambda}) \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \quad (7-20)$$

Solutions are stored and then printed and plotted. Sufficient time must be requested to complete all of the printing and plotting routines at the end of the program, or the output will be lost (see Section 7.5.3). Optional print statements within the frequency loop can be used to obtain values of key matrix elements and to check on the accuracy of the solutions. These optional print statements are indicated by comment cards in the program listing.

#### 7.5 PROGRAM "HALF" INPUT/OUTPUT PARAMETERS AND CONTROL

##### 7.5.1 Input

The physical input variables are described in Table 7-1, while Table 7-2 lists the data control variables. The program accepts up to seven consecutive frequency intervals, and a different increment

TABLE 7-1. PROGRAM "HALF" REQUIRED PHYSICAL INPUT DATA

Variable (See Figs. 7-1 and 7-2)	Description	Units
K	Foundation Stiffness per Unit Track Length	LBS/IN <sup>2</sup>
K <sub>1</sub>	Secondary Suspension Spring Constant	LBS/INCH
K <sub>2</sub>	Primary Suspension Spring Constant	LBS/INCH
W <sub>1</sub> =M <sub>1</sub> g	Half-Car Body Weight	LBS
W <sub>2</sub> =M <sub>2</sub> g	Truck Weight	LBS
W=mg	Wheelset Weight	LBS
ρ <sub>t</sub>	Single Rail Weight per Unit Length	LBS/YD
I	Single Rail Area Moment of Inertia	INCHES <sup>4</sup>
E	Young's Modulus for Steel Rail	LBS/IN <sup>2</sup>
V	Velocity of Car	MPH
<u>OR</u>	<u>OR</u>	<u>OR</u>
λ	Wavelength of Track Irregularity	FEET/CYCLE

TABLE 7-1 PROGRAM "HALF" REQUIRED PHYSICAL INPUT DATA (Continued)

Variable (See Figs. 7-1 and 7-2)	Description	Units
$l$	Truck Wheel Base	INCHES
$\beta$	Vehicle Damping Ratio	NONE
$\zeta$	Track Damping Ratio	NONE
$W_t$	Weight of a Tie	LBS
$l_t$	Tie Spacing	INCHES

TABLE 7-2. PROGRAM "HALF" DATA CONTROL VARIABLES

Input Designation	Purpose	Possible Values
NDF	Specifies number of frequency ranges considered	1 to 7
IFREQ	Controls frequency ranges over which response is computed	1 or 2
DF(I)	Specifies the number of points computed in a particular frequency range	I=1 to NDF
FL(I)	Specifies lower and upper frequency limits of each frequency range	I=1 to NDF+1
IFV	Selects Vehicle Velocity or Wavelength of Track Irregularity	.1 for $\lambda$ 2 for V

in each interval. The required information consists of (a) the number of intervals, (b) the increment for each of the intervals in order, and (c) the boundaries of the intervals. The resulting number of frequencies is limited to one hundred.

### 7.5.2 Output

The eight dependent variables and the dynamic compliance coefficients listed in Table 7-3 are plotted on four graphs as a function of frequency. The irregularity wavelength (or the velocity, if wavelength option is specified) is written alongside the frequency. Separate curves are drawn for the front and rear wheels. Sample curves are shown in Figures 7-3 through 7-6.

The printed output consists of the input data, the magnitude and phase of each of the dependent variables ( $y_1/v_0$ ,  $y_2/v_0$ ,  $y_3/v_0$ ,  $y_4/v_0$ ,  $\delta_1/v_0$ ,  $\delta_2/v_0$ ,  $F_1/v_0$ ,  $F_2/v_0$ ) as a function of the selected frequencies, and the magnitude and phase of  $G_{11}$  and  $G_{12}$  as a function of frequency.

### 7.5.3 Program Control

The deck developed at TSC has been run on the IBM 7094 and DECsystem PDP-10. When using the object deck, a set of 80 frequencies requires approximately 60 seconds of CPU time and five minutes of Calcomp Plotter time. Approximately 20K (decimal) words of memory core are required to load.

The only subroutine other than the plotting subroutines written at TSC is MINV which is part of the IBM scientific subroutine package. This subroutine was modified at TSC, to enable it to invert a complex matrix; as available from IBM, it was restricted to real numbers. A listing of Program HALF and subroutines is included in Volume II as Appendix D.

TABLE 7-3. PROGRAM "HALF" PLOTTED OUTPUT DATA

Figure No.	Title	Dependent Variables
7-3	Displacement Amplitude Ratio	$\frac{y_1}{v_0}, \frac{y_2}{v_0}, \frac{y_3}{v_0}, \frac{y_4}{v_0}$
7-4	Wheel-Rail Forces Produced by Unit Track Irregularity (LB/IN)	$\frac{F_1}{v_0}, \frac{F_2}{v_0}$
7-5	Track Deflection Amplitude Ratio	$\frac{\delta_1}{v_0}, \frac{\delta_2}{v_0}$
7-6	Track Compliance Function	$G_{11}, G_{12}$



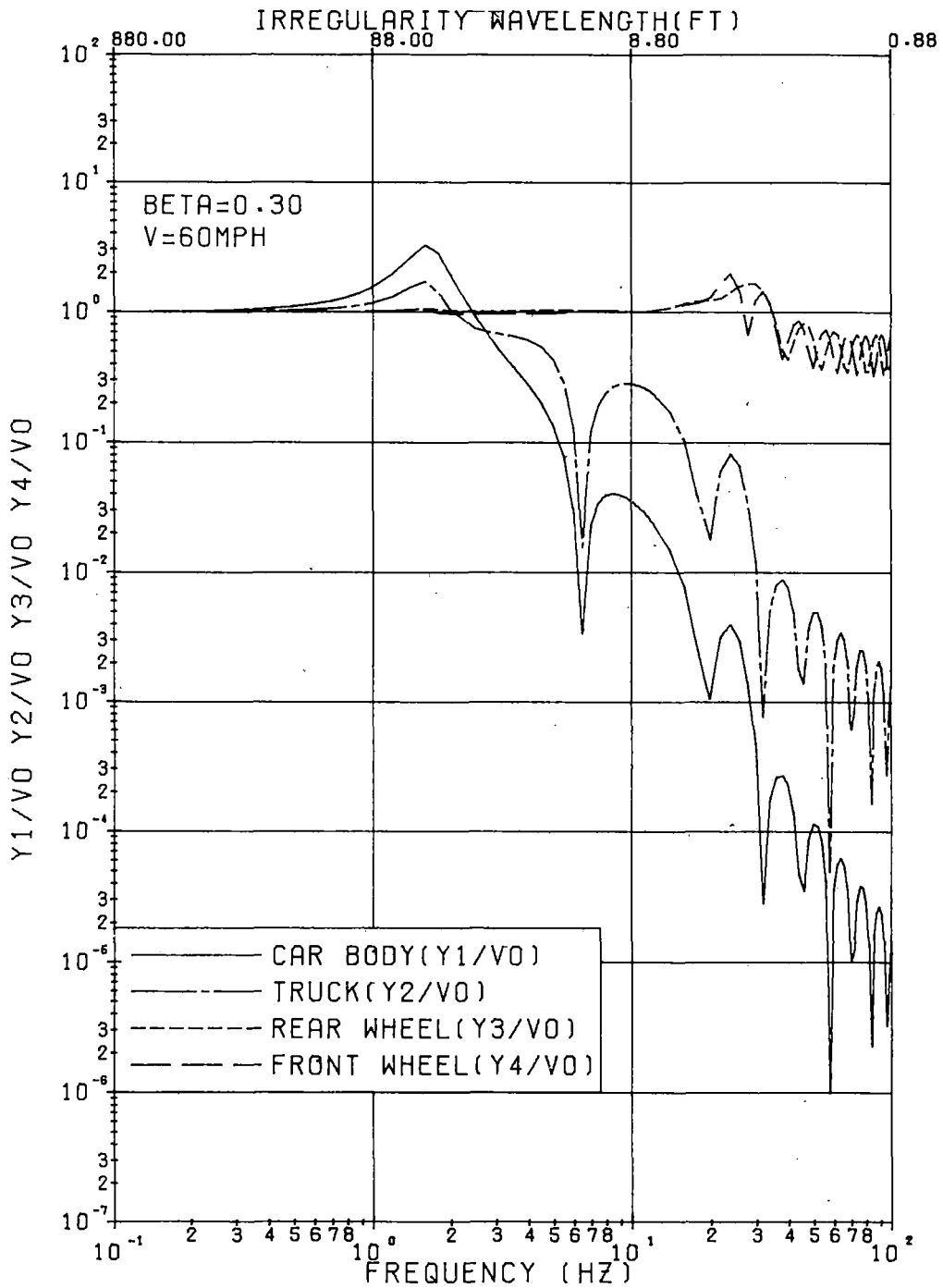


Figure 7-3. Car Body, Truck, and Wheel Displacement Amplitude Ratio - Graphic Diagram

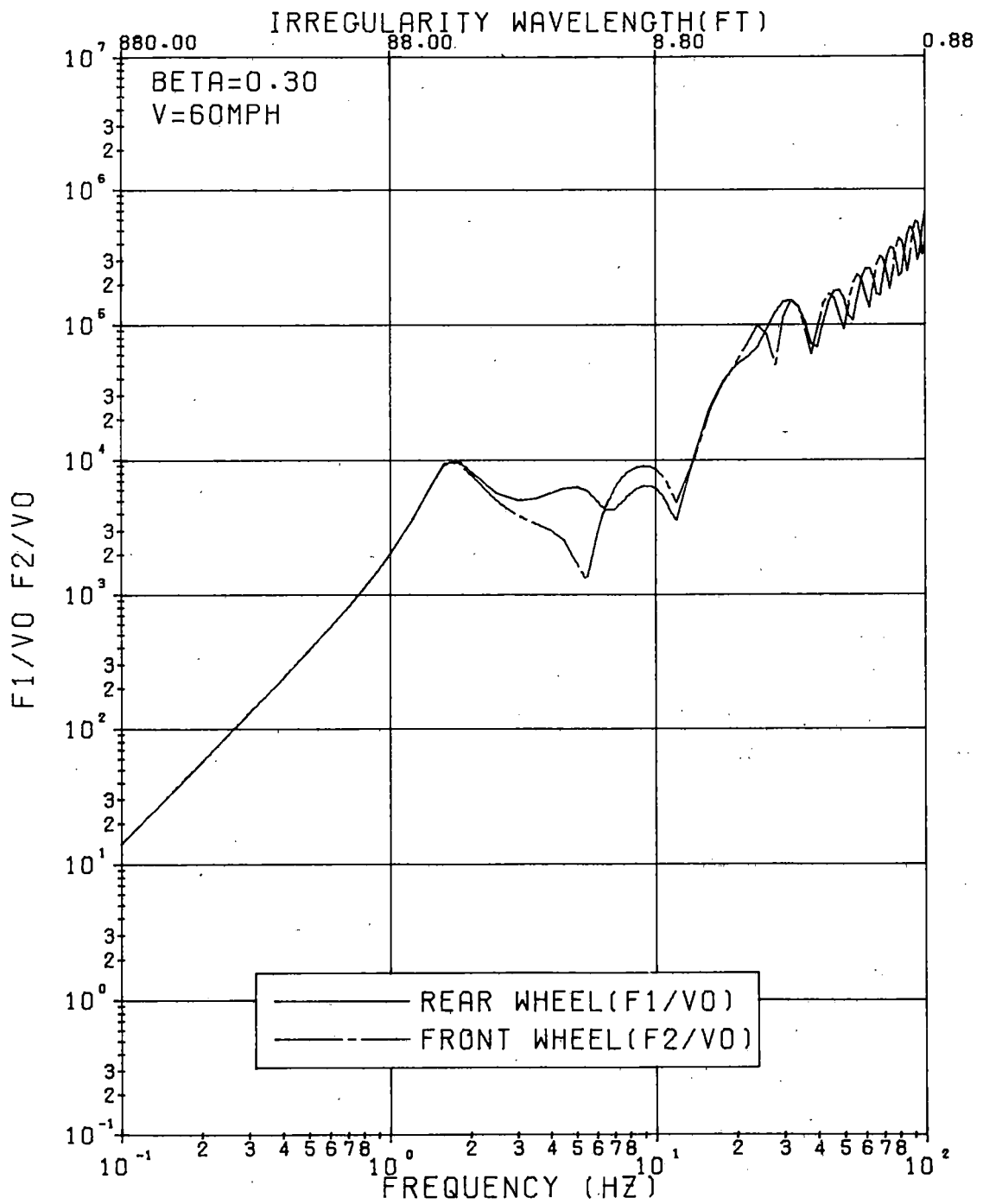


Figure 7-4. Wheel-Rail Forces Produced by Unit Track Irregularity (Lb/In) - Graphic Diagram

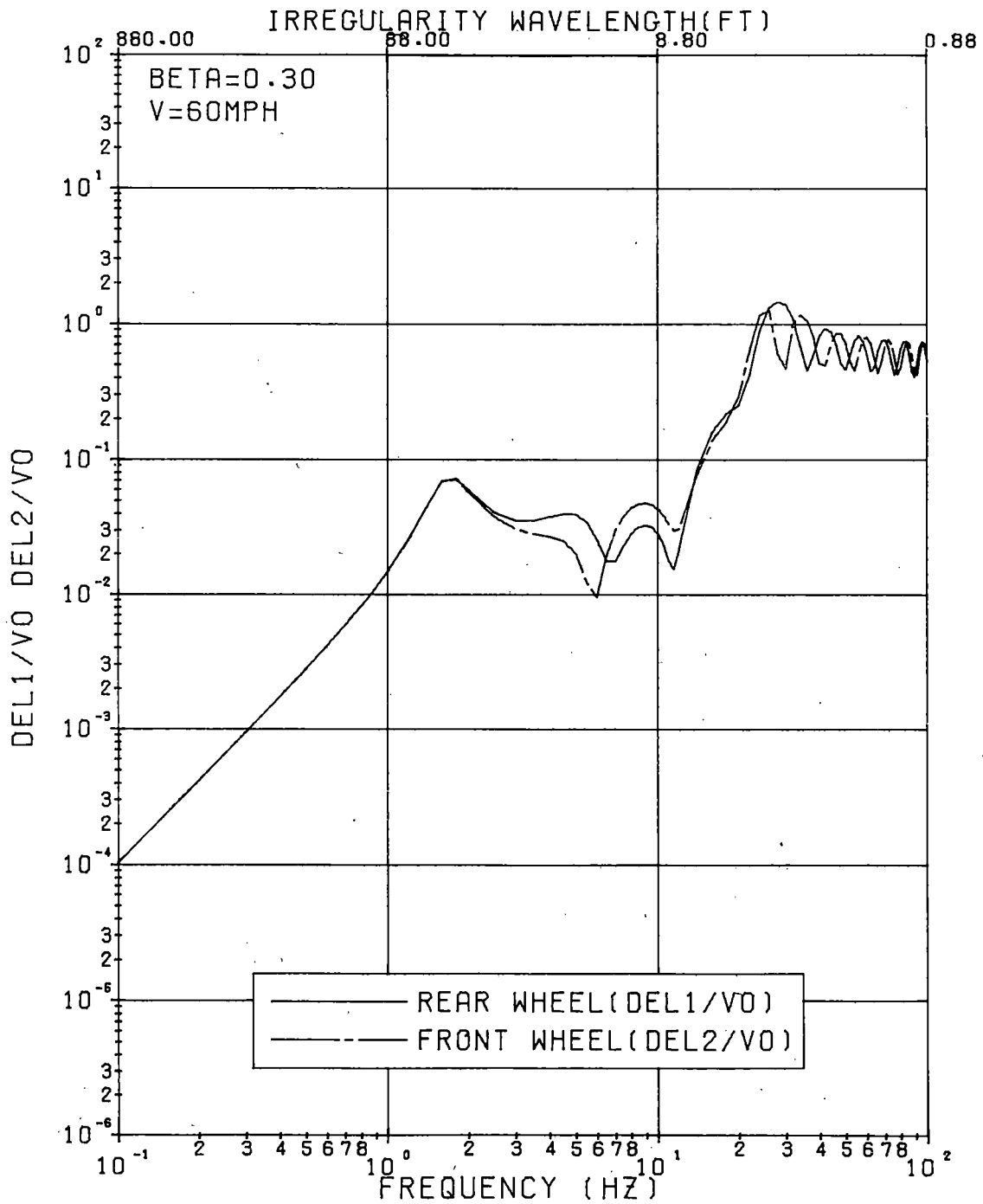


Figure 7-5. Track Deflection Amplitude Ratio - Graphic Diagram

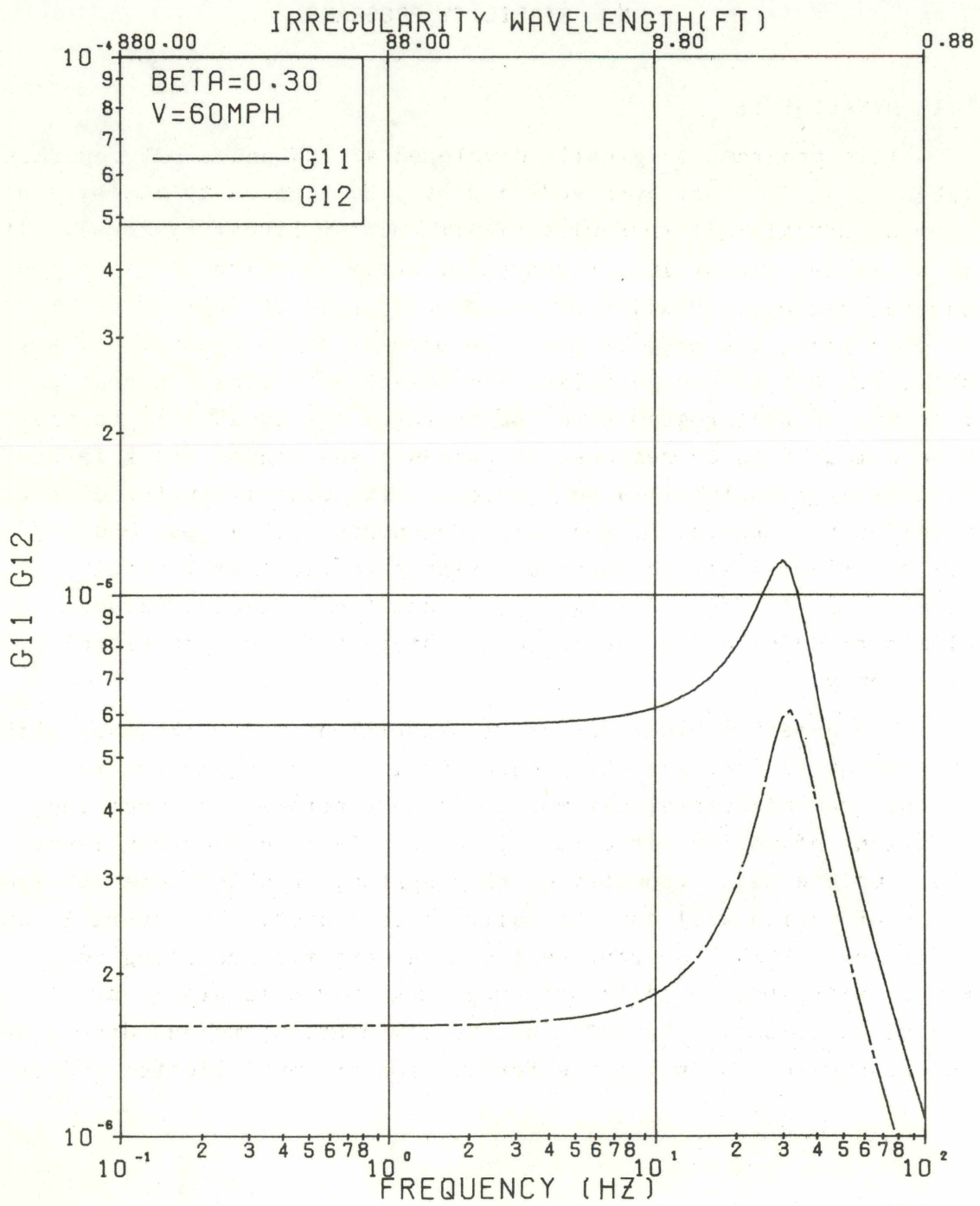


Figure 7-6. Track Compliance Function - Graphic Diagram

## 8. STABILITY PROGRAMS

### 8.1 DYNALIST II

This program, originally developed at TRW under DOT contract (Refs. 3 and 4), has been modified by J.H. Wiggins to its present form as DYNALIST II (Dynamics of Articulated Linear Systems). It is a complex eigenvalue, eigenvector analysis which predicts the lateral dynamic stability of systems of up to 50 degrees of freedom. At this size, the program has been used to model dynamics of wheelsets, trucks, single vehicles, and three-car trains. Recent extensions of the program have incorporated the capability to provide computation of response to harmonic and random track irregularities. DYNALIST II also provides: (a) capability for direct modal representation of flexible components such as car bodies; (b) the flexibility to construct either vertical or lateral models; and (c) for editing (i.e. truncating) of eigenvalues, to eliminate modes above the range of interest for computational efficiency.

The namelist block format of DYNALIST provides the user with the option to exercise the program by inputting parameters to a pre-programmed lateral car model, or to generate and input the equations of motion himself. Alternatively, the user can input the complete modal geometry of this system, in which case the equations of motion will automatically be generated. The overall capabilities of DYNALIST represent a more versatile modeling tool which, with some sophisticated modeling, may simulate a variety of dynamic problems. The modeling and computation options described below, however, simplify the formulation of sophisticated models.

## 8.2 DYNALIST II OPTIONS

### 8.2.1 Direct System Method

Using this option, the system is modeled as a single component (i.e., a vehicle, several connected vehicles, or a single vehicle component) having up to 50 degrees of freedom. Complex modes are then evaluated for the system, and may be truncated at the user's discretion for frequency response computations.

### 8.2.2 Direct Subsystem Method

With this option, the system is modeled as an assembly of subsystems (e.g., a truck, a car body, a hanging mass, etc.), each having a maximum of 25 independent coordinates with connections between components defined by coordinate constraint equations. The total system is limited to 50 independent coordinates. Subsystem modes are not computed with this option. System modes are computed directly, and again, may be truncated by the user for frequency response computations.

This option differs from the first in that it permits rapid assembly of a system from a list of subsystems on file and is ideal for evaluating effects that changes in major components (e.g., trucks) have on such parameters as critical speed and frequency response.

### 8.2.3 Modal Synthesis Method

This option is similar to the previous option in that the system is modeled as an assembly of subsystems. The difference is that subsystem modes are computed and may be truncated prior to assembling the system. System modes are then generated and may again be truncated prior to initiating frequency response computations.

As an operational program at TSC, however DYNALIST II has been successfully exercised in only the modal synthesis option, while the other two options are in the checkout stage. A check case has been run with DYNALIST II and Program LATERAL (described in Section 6), to compare the lateral frequency response to track alignment

irregularities predicted by each program. Results indicated good agreement at all coordinate responses computed.

### 8.3 TRKHNT AND CARHNT

Battelle Columbus Laboratories (BCL) has developed computer program (Ref. 5) for evaluating the lateral hunting stability of a single 2-axle truck (TRKHNT) or a complete vehicle consisting of car body and two, 2-axle trucks (CARHNT). Each program computes the eigenvalues and normalized eigenvectors as a function of truck or vehicle speed.

The vehicle model shown in Figure 8-1 is modeled in the car hunting (CARHNT) program. This is a 17 degree-of-freedom model of one complete car with two, 2-axle trucks. The car body is rigid and has lateral, yaw, and roll degrees of freedom. Each truck includes lateral and yaw motions for each axle (wheelset) and lateral, yaw, and roll motions for the truck frame. The primary suspension (connecting the axles to the truck frame) is represented by linear springs in the vertical, lateral, and longitudinal directions. Each truck frame is attached to the car body through the secondary suspension which has lateral, yaw, and roll stiffness components. The secondary suspension includes both linear springs and parallel viscous dampers.

The vehicle model used for the truck hunting (TRKHNT) program is a 7 degree-of-freedom model of a single truck. The truck is connected to a car body through a secondary suspension system, and the car body is constrained to move at constant speed along the track centerline (no car body dynamic motions).

### 8.4 TRKV

The TRKV response programs use decoupled seven degree of freedom vertical and lateral models where the front trucks are modeled in detail while the rear trucks are represented only by a complex impedance. The vertical model includes one uniform free-free bending mode to indicate the effect of car body flexibility. The lateral model does not include the wheelset yaw degree of freedom and therefore does not properly model creep response. Track surface, alignment, and cross level irregularities drive the model.

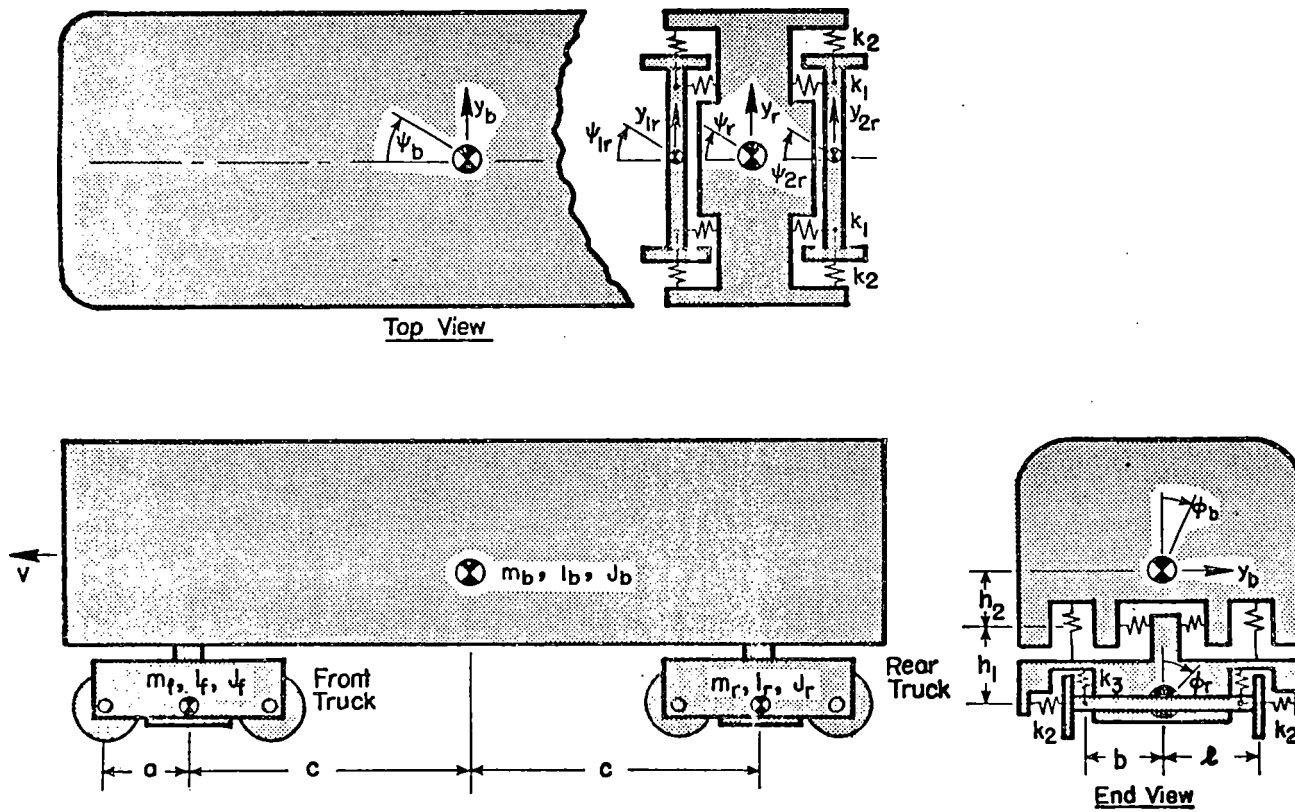


Figure 8-1. CARHNT Vehicle Model



In TRKVPSD, the track geometry irregularities are represented by power spectra of the form  $C\lambda^n$  in terms of wavelength. TRKVEH is a variation of the program with a rectified sine track input to simulate the effect of rail joints. Track structure is modeled as spring damper impedances with an effective mass lumped with the truck unsprung mass.

These programs have been acquired from BCL and are operational on TSC equipment.

## REFERENCES

1. J.E. Nickles and H. Weinstock, "Preliminary Vibration Measurements on Mark I Vehicle," Report No. DOT-TSC-UMTA-72-2, Oct. 1971.
2. H. Weinstock, "Analyses of Rail Vehicle Dynamics in Support of Development of the Wheel Rail Dynamics Research Facility," Report No. MA-06-0025-73, June 1973.
3. T.D. Hasselman, H.D. Reed, and A. Kaplan, "Dynamic Analysis of Multiple Car Vehicles Using Component Modes: Volume I - Analysis," FRA-RT-71-43, PB 195345, July 1970.
4. T.D. Hasselman and A. Kaplan, "Dynamic Analysis of Multiple Car Vehicles Using Component Modes: Volume II - Parametric Study," PB 194375, July 1970.
5. R.H. Prause, "User's Manual, Programs CARNHT and TRKNHT for Rail Vehicle Hunting Stability Analysis," Battelle Columbus Laboratories, May 2, 1974.

Frequency Domain Computer Programs for  
Prediction and Analysis of Rail Vehicle  
Dynamics, Volume I: Technical Report, 1975  
US DOT, FRA, AB Perlman, FP DiMasi

SMBAD.00VP333A

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