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USERS' MANUAL FOR KALKER'S SIMPLIFIED NONLINEAR CREEP THEORY

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16. Abstract

The conversion of the computer program, "Simplified Theory of Rolling Contact," (used for calculation of a nonlinear creep force-creepage relationship) from the original Algol language to Fortran is considered. The Algol program was written by Professor J. J. Kalker and was derived from the paper, "Simplified Theory of Rolling Contact," Delft Progr. Rep., Series C: Mechanical and Aeronautical Engineering and Shipbuilding, 1 (1973), pp. 1-10. A significant number of changes was made in the program for more convenient use; however, the fundamental equations remain unchanged. The results were checked in detail to insure agreement with the original solution.

The program gives an appropriate solution for the resultant tangential creep forces and spin moment acting between two bodies of equal linearly elastic material properties. The creep forces and spin moment are due to lateral, longitudinal, and spin creepages. Assumptions corresponding to the Hertz contact theory are implied and two additional simplifying assumptions are made, resulting in a significant reduction in computation time as contrasted with previous solutions.

Two separate computer codes were developed, the first being the general solution with extended input and output, and the second a shortened version primarily intended for use as a subroutine.

Surprisingly good agreement is found to exist between the "Simplified Theory" and published experimental results for a wide range of contact ellipse eccentricity.

17. Key Words - nonlinear creep, creepage, creep forces, spin, spin moment, steady-state rolling, Hertz contact, railroads

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Last, but certainly not least, we would like to thank Professor J. J. Kalker of Delft who very graciously sent us copies of his papers and computer programs. We have tried, in this Users' Manual, to make the results of his work more widely available to the rail vehicle dynamics community. Although we have checked carefully the Fortran version of Professor Kalker's program, any errors in the conversion are ours and not Professor Kalker's.

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I. INTRODUCTION

Background

The forces and moments due to shear stresses in the contact area between wheel and rail play a major role in rail vehicle dynamics. These shear stresses arise, in part, due to relative linear and angular motions (lateral, longitudinal, and spin creepage) between the wheel and rail. Hobbs [1] presents a review of the analytical and experimental work concerned with the creep force/creepage phenomenon.

For many problems in rail vehicle dynamics a linear creep force/creepage relationship has been used. Typical of these are eigenvalue/eigenvector analyses of lateral stability, lateral forced response studies, and estimation of slip and flange contact boundaries for steady state curving. It is widely recognized that the best available linear creep law is that due to Kalker [2] and called the "linearized theory" (see equations (12) and (13) of [2]). Recently, however, more and more questions are being asked of rail vehicle dynamicists that require more sophisticated models of the wheel/rail interaction process. Factors that should be considered in these models are: (1) the nonlinear wheel/rail geometric constraint functions arising from curved or worn wheel and rail profiles; and, (2) the effects of adhesion limits on the creep force/creepage relationship (i.e. a nonlinear creep law).

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[1] A.E.W. Hobbs, "A Survey of Creep", DYN/52, April 1967, British Railways Research Dept., Derby, England.

[2] J.J. Kalker, "Simplified Theory of Rolling Contact," Delft. Progr. Rep., Series C: Mechanical and Aeronautical Engineering and Shipbuilding, 1 (1973), pp. 1-10.

Attempts have been made to formulate a nonlinear creep law. Johnson's theory [3,4] has been confirmed by laboratory experiments but does not account for spin creepage*. Unfortunately, the effects of spin creepage are expected to predominate for contact areas in the wheel flange region - precisely the situation where a nonlinear creep law is needed. The Levi-Chartet creep law [5,6] used by some researchers is empirically based and does not account for spin creepage.

Professor Kalker of Delft University has formulated two nonlinear creep laws that incorporate the effects of spin creepage and that have been found to compare well with results of laboratory experiments. These two creep laws are generally referred to as the "Simplified theory of rolling contact" [2] and the "exact solution for rolling contact" [7]. The differences in the solutions presented in [2] and [7] lie in two simplifying assumptions made in [2] concerning the tangential displacement-stress relations and the normal stress distribution on the contact surface. These assumptions shorten the computation time required by a factor of approximately 100.

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* Spin creepage is the nondimensional relative angular velocity between wheel and rail in the contact zone.

[3] K.L. Johnson, "Adhesion", <u>Proc. Inst. Mech. Engrs.</u>, Vol. 178, part 3E (1964), pp. 208, 209.

[4] P.J. Vermeulen and K.L. Johnson, "Contact of Nonspherical Elastic Bodies Transmitting Tangential Forces," <u>J. Appl. Mechanics</u>, Vol. 31 (1964), pp. 338-340.

[5] R. Levi, "Le roulement avec glissement", Compt. rend. Acad. Science 199, 1934, pp. 119-120.

[6] A. Chartet, "Proprietes generales des contacts de roulement. Theorie des similitudes." Compt. rend. Acad. Science 225, 1947, pp. 986-988.

[7] J.J. Kalker, "On the Rolling Contact Between Two Elastic Bodies in the Presence of Dry Friction," Ph.D. Thesis, Delft University of Technology (1967).

A portion of the work being conducted under contract DOT-OS-40018, Freight Car Dynamics, deals with developing models for the lateral dynamic response of North American freight cars during curve entry and negotiation. These models will be used to predict vehicle response and wheel/rail forces during hard curving where severe flange contact is anticipated. Consequently, it is expected that creep forces may approach the limits of adhesion and a nonlinear creep law will be required for accurate modeling.

The object of the work reported in this Users' Manual was to convert the Algol program developed by Professor Kalker for the "Simplified theory of rolling contact" to Fortran and to check the resulting program by direct comparison with the results calculated by the original Algol program and with available experimental results. It is anticipated that a Fortran version of this computer program will prove quite valuable to rail vehicle dynamics researchers in the United States where most scientific programs are written in Fortran.

To further aid in the use of Kalker's theory by rail vehicle dynamics researchers, a subroutine called FØRCES was developed based on the "Simplified theory of rolling contact". This subroutine can be included within Fortran programs that are used to obtain the lateral dynamic response of rail vehicles. This subroutine together with the program for the complete solution of the "Simplified theory of rolling contact" are discussed in this manual.

Summary of Users' Manual

It is intended that Kalker's original paper [2] be read concurrently with this manual. References to equations in [2] are made directly by equation number both in the present text and in the computer code.

The problem analysed in [2] and considered in the computer code is for steady rolling contact of two elastic bodies of equal linearly elastic material properties and having both longitudinal and lateral creepage and spin about an axis normal to the contact surface. The appropriate geometry is given in Figure 1 of [2].

Approximate solutions to three special problems of steady state rolling contact are presented in [2]. The first case is that of infinitesimal slip in which the area of slip is vanishingly small and the resultant tangential creep forces and torsional moment are linearly related to the creepage and spin parameters. This is Kalker's "Linearized theory" widely used by rail vehicle dynamics researchers. It is presented on page 4, equations (11) to (13) of [2]. The second solution, "Steady-state rolling with pure creepage", is presented on page 5, equations (15) to (21) and considers finite slip with a resulting nonlinear relationship between the resultant creep force and the creepage. The present computer code is for the third solution, "Combined creepage and spin: a numerical method", in which finite slip is assumed and the resultant creep forces and moment are nonlinearly related to the creepage and spin parameters.

The problem may be stated as follows. Given two bodies of equal elastic properties and known dimensions, normal force, rolling velocity, creepage and spin, determine the resultant creep forces tangent to the contact surface and the resultant moment about a normal to the contact surface. The region of slip within the contact surface is also determined. In the actual solution, the static Hertzian contact problem is first solved (see [7] page 55, or [8] page (414) to determine the dimensions of the contact ellipse, a and b. The resultant creep forces and moment, F_x, F_y, and M_z are then determined knowing the parameters a, b, N, G, ν, μ, ν_x, ν_y, and ϕ where:

F_x = longitudinal creep force (in the direction of rolling)
F_y = lateral creep force
M_z = spin creep moment about normal to contact surface
a = semi-axis of contact ellipse in longitudinal direction
b = semi-axis of contact ellipse in lateral direction
N = resultant normal load on the contact region
G = shear modulus
ν = Poisson's ratio
ν_x, ν_y = longitudinal and lateral creepage
ϕ = spin creepage

This is the same problem considered in [7] and referred to as the "exact" solution. The only differences in the solutions presented in [2] and [7] lie in two simplifying assumptions concerning the tangential displacement - stress relations and the normal stress distribution on the contact surface. These two assumptions considerably reduce the complexities in obtaining a numerical solution and shorten the computation time by approximately a factor of 100.

The first assumption regarding the tangential displacement-stress relation is common to all three solutions developed in [2]. This is:

$$u(x,y) = S_x X = - S_x \tau_{xz} \qquad \text{equation (9), [2]}$$
$$v(x,y) = S_y Y = -S_y \tau_{yz}$$

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[8] S.P. Timoshenko and J.N. Goodier, Theory of Elasticity, 3rd Ed., McGraw-Hill Book Company (1970).

where u(x,y) and v(x,y) are the tangential displacements in the longitudinal and lateral directions and τ_{xz} and τ_{yz} are the shear stresses. The "exact" relationships for the tangential displacements as given in [7] are

$$u(x,y) = \sum_{m=o}^{M} \sum_{n=o}^{M-m} a_{mn} x^m y^n$$

equation (2.2), [7]

$$v(x,y) = \sum_{m=o}^{M} \sum_{n=o}^{M-m} b_{mn} x^m y^n$$

The two elastic constants S_x and S_y of [2] are determined explicitly in terms of the elastic properties G and ν, the contact ellipse dimensions a and b and the creepage and spin coefficients C_{ij} (see equations (13) and (41) - (47) of [2]. It is important to note that S_x and S_y have different values if forces are to be computed than they have when the moment is to be determined.

The method of determination of the constants a_{mn} and b_{mn} in [7] is much more complicated than that used to determine S_x and S_y in [2] and is the significant difference in the solutions.

The simplified theory also may be used to investigate the effects of a very thin elastic layer covering the bodies and having a tangential displacement-stress relation as given by equation (45) of [2].

$$u_\ell = L_x X = - L_x \tau_{xz}, \text{and}$$

$$v_\ell = L_y Y = - L_y \tau_{yz},$$

where L_x and L_y are the inverse stiffnesses of the layer. The combined effective stiffnesses of the wheel-rail with an elastic layer are given by equations (46) and (47) of [2]. These are, for moments

$$S_x = 8b/(15C_{33}G) + L_x \qquad \text{and,}$$

$$S_y = \pi a^{3/2}/(4b^{1/2}C_{23}G) + L_y$$

and in the calculations of forces

$$S_x = 8a/(3C_{11}G) + L_x \qquad \text{and}$$

$$S_y = 8a/(3C_{22}G) + L_y.$$

If no layer is present one then takes $L_x = L_y = 0$.

The effect of changes in L_x and L_y on the resulting solution has not been investigated; however, some observations should be noted. First, the layer is assumed to be so thin that its presence does not influence the determination of the contact ellipse dimensions or the pressure distribution. That is, a and b are still computed from the static Hertz solution in terms of G, ν and N. The effect of a finite thickness work-hardened layer could not then be accounted for by including L_x and L_y. Further, it seems to the writers that if the effect of a contaminated rail is desired, it is more directly accounted for by an appropriate change in the coefficient of friction than in a layer as defined by equation (45). The utility of modifying the elastic properties by adding L_x and L_y is not clear to the writers at this time.

The additional simplification made in the combined creepage and spin solution of [2] is that the normal stress distribution over the contact region is assumed to be of the form given by (14.III) rather than the Hertz stress distribution of (14.I). It should be noted that the

Hertzian distribution is used to determine the contact region dimensions a and b. Equation (14.III) is chosen so as to have bounded derivatives at the edge of the contact region and to still be similar to the Hertzian distribution over most of the contact area. The functions A(y) and B(y) in (14.III) are

$$A(y) = 0.5\ (1-(y/b)^2)^{-\frac{1}{2}}(1-(0.9)^2)^{-\frac{1}{2}} \text{ and}$$

$$B(y) = -0.5\ (1-(y/b)^2)^{\frac{1}{2}}(1-(0.9)^2)^{\frac{1}{2}}.$$

Numerous changes were made in the computer code in order to make the program more convenient to use. The Algol version was, however, fundamentally correct and numerous checks were made to insure that the Fortran and Algol codes gave the same results. The use of the Fortran code is considered in the next sections. The complete solution is discussed first, followed by the subroutine, "SUBRØUTINE FØRCES".

II. DESCRIPTION OF COMPUTER CODE FOR COMPLETE SOLUTION

A. PURPOSE

This program and associated subroutines compute the lateral and longitudinal creep forces and the spin creep moment acting between two elastic bodies in steady state rolling contact. The bodies are of equal linearly elastic material properties and have longitudinal and lateral creepage and spin creepage about an axis normal to the contact region. Kalker's theory of simplified rolling contact [2] is the basis of the program.

B. PROGRAM DESCRIPTION

1) Usage: The program consists of a main program and three subroutines.

The main program, MAIN, coordinates the input and outputs the results. Subroutine MAAKZ computes the normal stress as given by equation (14.III). Subroutine R0L is the solution portion of the program and determines the region of slip or adhesion within the contact zone. Subroutine C0NST determines the linear creepage and spin coefficients, C_{ij}, and the normalized modulus GS by linear and quadratic interpolation from Kalker's table [7].

2) Subroutines Required:

SUBR0UTINE MAAKZ (P, Q, WZ, DZ, D2Z, A, B, MUZ) determines the assumed normal stress as given by equation (14.III).

SUBR0UTINE R0L (CS, GEL, MUZ, NX, NY, X, Y, VX, VY, G, FX, FY, MZ) determines the region of slip or adhesion and computes the tangential stresses and relative velocity at points within the contact zone. The resultant creep forces and moment are also computed. Uses subroutine MAAKZ.

SUBR0UTINE C0NST (A, B, NU, C11, C22, C23, C33, GS) determines the linear creepage and spin coefficients, C11, C22, C23, and C33 and the normalized modulus, GS, by linear and quadratic interpotation from Kalker's table, [7]. These values are used in MAIN to determine the normalized stiffness SXN and SYN and the spin constant HC.

3) Description of Input Parameters:

NV1	NV1 is an integer denoting the number of complete problems to be solved. Input.
A,B	A = a/c, B = b/c, where a and b are the actual contact dimensions determined from the static Hertz solution and c = $\sqrt{ab}$ is the normalized unit of length. a is the longitudinal and b is the lateral semi-axis of the contact ellipse. Input.
NU	NU = ν = Poisson's ratio. Input.
LXN, LYN	LXN = $L_x \rho N/c^4$, LYN = $L_y \rho N/c^4$. Inverse stiffnesses of an elastic layer covering the bodies. N = resultant normal force and $1/\rho = 1/4\ (1/R_1^+ + 1/R_1^- + 1/R_2^+ + 1/R_2^-)$ with R_1^+, R_1^-, R_2^+, R_2^- being the principal radii of curvature of the two elastic bodies. See equation (45). For no layer, take LXN = LYN = 0. Input.
NX, NY	Lattice points in the normalized contact region with X = (I)(A)/NX, Y = (J)(B)/NY and $-NX \leq I \leq NX$, $-NY \leq J \leq NY$. Accuracy increases with increasing values of NX, NY. Maximum values NX, NY = 40. Typical values:

	A/B = 10.0, NX = 30, NY = 10, A/B = 0.1, NX = 10, NY = 30, A/B = 1.0, NX = NY = 20. Input.
DM	DM is an incremental step in the computation. Accuracy increases with decreasing values of DM. Typical value, DM = 0.02A. Input.
NF	If the resultant forces are desired, take NF = 1. For the resultant moment take NF = 2. The appropriate values of SXN, SYN and HC will then be computed. Input.
NS	To print all output including stresses and displacements on the contact region take NS = 1. To suppress all output except the resultant forces or moment take NS = 2. Input.
NV2	NV2 is the integral number of sets of UXN, UYN, PHN to be considered. Input.
UXN, UYN	UXN = $\nu_x\rho/\mu c$, UYN = $\nu_y\rho/\mu c$ where ν_x, ν_y are the longitudinal and lateral creepages, μ = coefficient of friction. See equation (6). Input.
PHN	PHN = $\phi\rho/\mu$ where ϕ is the spin creepage. See equation (6). Input.

4) Input Format:

A sample deck set up is listed in Appendix A of this manual. The program requires contact region dimensions, elastic properties, wheel/rail creepages and program control information. The following format is for NV1 = 1. If NV1 > 1, there would be NV1 sets of the group of cards after the first card.

Card Number	Input Data
1	NV1 = Integer. Program solves NV1 complete problems, Typical card: 1
2	A, B, NU, LXN, LYN Typical card: 2.5980 0.3849 0.28 0.250 0.125
3	NX, NY, DM, NF, NS Typical card: 30 10 0.04 1 2
4	NV2 = Integer. Program solves NV2 problems for different values of creepage and spin given on NV2 cards starting with 5. Typical card: 1
5 to NV2	UXN, UYN, PHN Typical card: 0.0 2.0 0.4

Note: The input is free format with a space needed between each input parameter.

5) Description of Other Parameters in Program:

C11, C22, C23, C33	Longitudinal, lateral, lateral/spin, and spin creepage coefficients, respectively; tabulated in [7].
GS	GS = $Gc^3/\rho N$ where G = shear modulus. GS may also be computed from GS = $3(1-\nu)\,\tilde{E}/(4\pi\sqrt{g})$ where $\tilde{E}$ = complete elliptic integral of the second kind, see [7] page 58, and g = axial ratio of the contact ellipse = min (a/b, b/a). GS is determined within the computer program in terms of A, B and NU.

MU MU = μ = coefficient of friction. All variables are normalized so that μ does not explicitly appear.

SXN, SYN SXN = $S_x \rho N/c^4$, SYN = $S_y \rho N/c^4$. Inverse stiffnesses of the elastic bodies. See equations (9), (43), (44), and (47) for the form of S_x, S_y to be used to determine the resultant forces and equations (42) and (46) for the appropriate form to determine the resultant moment.

For forces let

$$\mathrm{SXN1} = 8A/(3C_{11}GS),\ \mathrm{SYN1} = 8A/(3C_{22}GS)$$

and

$$\mathrm{HC1} = 32\sqrt{B/A}\ C_{23}/(3\pi C_{22}),\ \text{then}$$

$$\mathrm{SXN} = \mathrm{SXN1} + \mathrm{LXN}$$

$$\mathrm{SYN} = \mathrm{SYN1} + \mathrm{LYN}$$

$$\mathrm{HC} = (\mathrm{SYN1} + \mathrm{LYN})/(\mathrm{SYN1}/\mathrm{HC1} + \mathrm{LYN}).$$

For moments,

$$\mathrm{SXN} = 8B/(15C_{33}GS) + \mathrm{LYN},$$

$$\mathrm{SYN} = \pi A/(4\sqrt{B/A}\ C_{23}GS) + \mathrm{LYN},\ \text{and}$$

$$\mathrm{HC} = 1.0$$

The C_{ij} are the linear creepage and spin coefficients, [7]. SXN, SYN, HC and the C_{ij} are determined within the program in terms of A, B, NU, LXN and LYN.

6) Output: NV2 sub-cases of NV1 cases are calculated. For each of the NV1 cases, the input parameters A, B, NU, LXN, LYN, NX, NY, DM, NF, and NS are printed. The linear creepage

coefficients, calculated within the program, are also printed out as are the normalized shear modulus, GS, and normalized inverse stiffness, SXN and SYN. For each of the NV1 cases, there will be NV2 sets of output corresponding to the NV2 sets of normalized creepages and spin, UXN, UYN, and PHN. For each of the NV2 cases, the inputs UXN, UYN, and PHN are printed out together with the computed values of the normalized longitudinal and lateral creep forces, FXN and FYN, or the computed value of the spin creep moment, MZN (depending on whether NF = 1 or NF = 2). If NS = 2, the output is as described above. If NS = 1, the normalized coordinate points X, Y over the contact region and the values of the stresses (TX, TY, TZH, TZK) and slip components (VX, VY) are given at each point.

The Fortran names used in the program output are the following:

UXN, UYN, PHN	Repeated program input variables.
FXN, FYN	FXN = $F_x/\mu N$, FYN = $F_y/\mu N$. Normalized resultant longitudinal and lateral forces. Computed.
MZN	MZN = $M_z c/\mu N$. Normalized resultant moment. Computed.
X, Y	X = x/c, Y = y/c. $-A \leq X \leq A$, $-B \leq Y \leq B$. Normalized coordinates where x and y are longitudinal and lateral distances from the center of the contact ellipse.

TX, TY	Normalized shear stresses. $TX, TY = -\tau_{xz}c^3/\rho N, -\tau_{yz}c^3/\rho N,$ $\sqrt{TX^2 + TY^2} < TZK$ for no slip, $\sqrt{TX^2 + TY^2} = TZK$ for slip.
TZH	$TZH = 3/(2\pi)\sqrt{1-(X/A)^2-(Y/B)^2}$ = Normalized Hertzian stress on the contact region. Given for reference only. See equation (14.I).
TZK	TZK is the assumed normal stress distribution over the contact region, see equation (14.III). $TZK = (F)(A)\,A1(Y)(1-(X/A)^2-(Y/B)^2), X \geq 0.9L(Y)$ $TZK = (F)(A)(\sqrt{1-(X/A)^2-(Y/B)^2} + B1(Y)), X \leq 0.9L(Y)$ where $L(Y) = A\sqrt{1-(Y/B)^2}$ $A1(Y) = 0.5\,(1-(Y/B)^2)^{-1/2}(1-(0.9)^2)^{-1/2}$ $B1(Y) = -\,0.5(1-(Y/B)^2)^{1/2}(1-(0.9)^2)^{1/2}$ and (F)(A) = 0.656773, such that the resultant normal force = 1.0.
VX, VY	Normalized relative slip components. VX, VY = $v_x\rho/(V\mu c)$, $v_y\rho/(V\mu c)$ where V is the rolling velocity and v_x and v_y are the longitudinal and lateral components of the relative slip velocity.

7) Summary of User Requirements and Recommendations

All input data is on cards in free format as shown. As A and B are normalized, the product of A and B must be unity. LXN and LYN are taken as zero if no elastic layer is to be considered. Maximum values for NX and NY are 40. Accuracy increases with increasing values of NX and NY. Typical values are:

A/B = 10.0 NX = 30, NY = 10

A/B = 1.0 NX = NY = 20

A/B = 0.1 NX = 10, NY = 30

DM is an incremental step size in the computation. Accuracy is improved with smaller sizes of DM. A typical value is DM = 0.2*A.

C. TEST PROBLEM

The following test problem is given to demonstrate the program. The calculation were performed on an IBM-370/3165-II computer.

A = 2.598, B = 0.3849, NU = 0.28, LXN = 0, LYN = 0

NX = 10, NY = 10, DM = 0.04, NF = 1, NS = 1

UXN = 0, UYN = -1.4, PHN = 0.8

D. PROGRAM LISTINGS WITH EXAMPLE INPUT AND OUTPUT

A listing of the program for the sample problem with input and output is given in Appendix B.

III. DESCRIPTION OF COMPUTER CODE FOR SUBROUTINE FORCES

The subroutine FØRCES is a version of the complete code discussed in Chapter II that has been converted to subroutine form. All the WRITE statements have been deleted as has the calculation of the pure spin creep moment. In almost all cases of interest to rail vehicle dynamicists, the pure spin creep moment contributions from the two wheels comprising the wheelset are much smaller than the yaw moment about the wheelset center of gravity due to the longitudinal creep forces. Thus, this calculation was deleted in the interests of computational time savings.

The subroutine and its argument list are:

```
SUBRØUTINE FØRCES (A, B, NU, UXN, UYN, PHN, NX, NY, DM, FXN, FYN)
```

The input parameters are: A, B, NU, UXN, UYN, PHN, NX, NY and DM and are as defined in Chapter II. The outputs are FXN and FYN and are as defined in Chapter II. Stresses and slip values over the contact region are not returned. All discussion of users' requirements and other program descriptive material is as outlined for the complete code in Chapter II.

The purpose of FØRCES is to compute lateral and longitudinal creep forces acting between two elastic bodies in steady rolling contact. The bodies have relative longitudinal and lateral creepage as well as spin creepage about an axis normal to the contact region.

FØRCES may be used as a subroutine within other Fortran programs developed for calculating the lateral dynamic response of rail vehicles. It addresses only one wheel and must be called for each wheel separately.

The input parameters must be evaluated for each wheel/rail contact condition considered and the outputs FXN and FYN are appropriate obviously for only those input parameters.

A listing and a sample test problem using SUBROUTINE FORCES is given in Appendix B.

```
C                     THEN SXN1=8*B/(15*C33*GS), SYN1=PI*A/(4*SQRT(B/A)
C                     *C23*GS), AND SXN=SXN1+LXN, SYN=SYN1+LYN, HC=1.0
C                     SEE EQUATIONS (42),(43),(46), AND (47).
C                     NS (TO PRINT OUTPUT ON THE CONTACT REGION,NS=1,
C                     TO SUPPRESS ALL OUTPUT EXCEPT THE RESULTANT
C                     FORCES OR MOMENT, TAKE NS=2), INTEGER
C                     NOTE: FXN=FX/(MU*N), FYN=FY/(MU*N),
C                     MZN=MZ*C/(MU*N)
C
C     DATA CARD #4    NV2
C                     TYPICAL CARD: 1
C
C                     SOLVES NV2 PROBLEMS FOR DISTINCT VALUES OF
C                     CREEPAGE AND SPIN GIVEN ON NV2 CARDS 5), INTEGER
C
C     DATA CARD #5    UXN,UYN,PHN
C                     TYPICAL CARD: 0.0 2.0 0.4
C
C                     UXN AND UYN ARE NORMALIZED CREEPAGES, PHN
C                     IS THE NORMALIZED SPIN), REAL
C                     UXN=UX*RHO/(MU*C), UYN=UY*RHO/(MU*C),
C                     PHN=PH*RHO/MU
C
C
C               ***** NOTE: ALL VARIABLES HAVE BEEN NORMALIZED SUCH
C               ***** THAT THE COEFFICIENT OF FRICTION, MU, DOES NOT
C               ***** EXPLICITLY APPEAR.
C
C
      COMMON A,B
      DATA X,Y,Z,ZH,VX,VY,G/6561*0.0,6561*0.0,6561*0.0,6561*0.0,6561*0.0
     $,6561*0.0,810*0.0/
      PI=3.14159
      GR=PI/180.0
      READ(1,*)NV1
      DO 999 III=1,NV1
      WRITE(3,968)
 1040  READ(1,*,END=9999)A,B,NU,LXN,LYN
      IF(A/B.LT.0.1) GO TO 998
C
C
C                     SUBROUTINE CONST COMPUTES THE LINEAR CREEPAGE AND
C                     SPIN COEFFICIENTS, AND THE NORMALIZED MODULUS
C                     FROM KALKER'S TABLES AND ASYMPTOTIC EXPANSIONS.
C                     VALID FOR A/B EQUAL TO OR GREATER THAN 0.1 .
C                     THESE VALUES ARE USED BELOW TO COMPUTE THE
C                     INVERSE STIFFNESSES SXN AND SYN.
C
C
      CALL CONST(A,B,NU,C11,C22,C23,C33,GS)
       READ(1,*,END=9999)   NX,NY,DM,NF,NS
      IF(NS.EQ.1) GO TO 1021
      MX=3*NX
      MY=3*NY
      LY=3*NY
      GO TO 1022
 1021 MX=1
      MY=1
```

```
      LY=1
 1022 MUZ=3.0/(2.0*PI)
      IF(NF.EQ.2) GO TO 1025
      SXN=8.0*A/(3.0*C11*GS)+LXN
      SYN1=8.0*A/(3.0*C22*GS)
      SYN=SYN1+LYN
      HC1=32.0*SQRT(B/A)*C23/(3.0*PI*C22)
      HC=(SYN1+LYN)/(SYN1/HC1+LYN)
      GO TO 1027
 1025 SXN=8.0*B/(15.0*C33*GS)+LXN
      SYN=PI*A/(4.0*SQRT(B/A)*C23*GS)+LYN
      HC=1.0
 1027 GEL(5)=DM
      WRITE(3,969)
      WRITE(3,970)A,B,NU,LXN,LYN
      WRITE(3,972)NX,NY,DM,NF,NS
      WRITE(3,973)C11,C22,C23,C33,GS,SXN,SYN
      GEL(1)=SXN
      GEL(2)=SYN
      GEL(3)=A
      GEL(4)=B
      I=-NX-1
1074  I=I+1
      IF(I.GT.NX)GO TO 1078
      J=-NY-1
1075  J=J+1
      IF(J.GT.NY)GO TO 1074
      P=1.-FLOAT(I*I)/FLOAT(NX)/FLOAT(NX)-FLOAT(J*J)/FLOAT(NY)/FLOAT(NY)
      IF(P.GT.0.0) GO TO 1076
      GO TO 1075
1076  ZH(I+NX+1,J+NY+1)=MUZ*SQRT(P)
      P=FLOAT(I)*A/FLOAT(NX)
      Q=FLOAT(J)*B/FLOAT(NY)
      CALL MAAKZ(P,Q,WZ,DZ,D2Z,A,B,MUZ)
      Z(I+NX+1,J+NY+1)=WZ
      GO TO 1075
1078  CONTINUE
      READ(1,*)NV2
      WRITE(3,974) NV2
      DO 997 II2=1,NV2
1090  READ(1,*,END=9999)UXN,UYN,PHN
      WRITE(3,975)UXN,UYN,PHN
      PHNN=HC*PHN
      CS(1)=UXN
      CS(2)=UYN
      CS(3)=PHNN
      CALL ROL(CS,GEL,MUZ,NX,NY,X,Y,VX,VY,G,FX,FY,MZ)
      IF(NF.EQ.2) GO TO 1091
      RES=SQRT(FX*FX+FY*FY)
      WRITE(3,977)FX,FY,RES
      GO TO 1092
 1091 WRITE(3,978)MZ
 1092 IF(NS.EQ.2) GO TO 1530
      WRITE(3,9004)
      J=NY
1260  J=J-LY
      IF(J.LT.-NY+1)GO TO 1345
      Q=FLOAT(J)*B/FLOAT(NY)
```

```
      IF(G(J+NY+1,1).LT.0.0) WRITE(3,9007)C
      IF(G(J+NY+1,1).GE.0.0) WRITE(3,9008)C
C DOOR:
1340  GO TO 1260
 1345 CONTINUE
      IF(MY.GT.2*NY)GO TO 1530
      J=NY
1350  J=J-MY
      IF(J.LT.1-NY)GO TO 1520
      C=FLOAT(J)*B/FLOAT(NY)
      WRITE(3,9005)
      WRITE(3,9006)C
      WRITE(3,9009)
      I=-NX-MX
1400  I=I+MX
      IF(I.GT.NX)GO TO 1515
      IF(Z(I+NX+1,J+NY+1).LT.1.E-8*MUZ)GO TO 1510
      P=FLOAT(I)*A/FLOAT(NX)
      FIX1=Z(I+NX+1,J+NY+1)
      FIX6=ZH(I+NX+1,J+NY+1)
      TX=X(I+NX+1,J+NY+1)
      TY=Y(I+NX+1,J+NY+1)
      UX=VX(I+NX+1,J+NY+1)
      UY=VY(I+NX+1,J+NY+1)
      FIX2=SQRT(TX*TX+TY*TY)
      ARG=1.0
      IF(TX.LT.0.0)ARG=-1.0
      ARG=ARG*ATAN(TY/(ABS(TX)+1.E-8))/GR+90.0*(1.0-ARG)
      FIX3=ARG
      ARG=1.0
      IF(UX.LT.0.0) ARG=-1.0
      ARG=ARG*ATAN(UY/(ABS(UX)+1.E-8))/GR+90.0*(1.0-ARG)
      FIX4=SQRT(UX*UX+UY*UY)
      FIX5=ARG
      WRITE(3,9010)P,FIX6,FIX1,FIX2,FIX3,FIX4,FIX5
C VERDER:
1510  GO TO 1400
1515  GO TO 1350
1520  WRITE(3,9001)
C NEXT:
1530  CONTINUE
 997  CONTINUE
      GO TO 999
 998  WRITE(3,979)
 999   CONTINUE
9999  WRITE(3,9998)
      STOP
 968  FORMAT('1',///,T63,'PROGRAM WITA-SRT',/,T53,'SIMPLIFIED THEORY OF
     $ROLLING CONTACT',/,T64,'BY J.J. KALKER',/,T56,'MODIFIED AT CLEMSON
     $ UNIVERSITY',/,T61,'DEPT. OF MECH. ENGR.',/,T66,'CLEMSON, SC',//)
 969  FORMAT(////,58X,'***** INPUT PARAMETERS *****',//)
 970  FORMAT(16X,'NORMALIZED CONTACT DIMENSIONS          A=',1PE11.4,10X,'(
     $ A=A1/C1, B=B1/C1, WHERE C1=SQRT(A1*B1),',/,32X,'(CARD #2)'
     $,11X,'B=',1PE11.4,10X,'( A1,B1 ARE ACTUAL CONTACT DIMENSIONS',//,
     $                30X,'POISSON S RATIO        NU=',1PE11.4,/,33X,'(CARD #2)
     $',//,28X,'LAYER STIFFNESSES      LXN=',1PE11.4,/,33X,'(CARD #2)',
     $ 8X,'LYN=',1PE11.4,/)
 972  FORMAT( 26X,'NUMERICAL CONSTANTS       NX=',I3,/,31X,'(CARD #3)',
```

```
     $11X,'NY=',I3,/,51X,'DM=',1PE11.4,/,51X,                           00002330
     $'NF=',I3,10X,'(NF=1,FORCES COMPUTED; NF=2,MOMENTS COMPUTED)',     00002340
     $/,51X,                                                            00002350
     $'NS=',I3,10X,'(NS=1, FULL OUTPUT; NS=2 ONLY FORCES OR MOMENTS)',/ 00002360
     $///)                                                              00002370
 973 FORMAT(47X,'***** PARAMETERS COMPUTED AND USED IN PROGRAM *****'   00002380
     $,//,     15X,'CREEPAGE AND SPIN COEFFICIENTS      C11=',1PE11.4,/, 00002390
     $50X,'C22=',1PE11.4,/,50X,'C23=',1PE11.4,/,50X,'C33=',1PE11.4,//,  00002400
     $21X,'NORMALIZED SHEAR MODULUS         GS=',1PE11.4,//,15X,'       00002410
     $NORMALIZED INVERSE STIFFNESSES      SXN=',1PE11.4,/,50X,'SYN=',   00002420
     $1PE11.4,///////)                                                  00002430
 974 FORMAT(42X,'***** NV2=',I2,' DISTINCT PROBLEMS FOLLOW FOR DIFFEREN00002440
     $T *****',/,45X,'***** VALUES OF NORMALIZED CREEPAGE AND SPIN *****00002450
     $',//)                                                             00002460
 975 FORMAT(//,17X,'NORMALIZED CREEPAGE AND SPIN      UXN=',1PE11.4,/,  00002470
     $23X,'(INPUT ON CARD #5)',                                         00002480
     $ 9X,'UYN=',1PE11.4,/,50X,'PHN=',1PE11.4,//)                       00002490
 977 FORMAT( 24X,'NORMALIZED FORCES ARE      FXN=',1PE11.4,/,29X,       00002500
     $'(COMPUTED)',11X,'FYN=',1PE11.4,//,24X,'RESULTANT FORCE           00002510
     $RES=',1PE11.4,/,24X,'(RES=SQRT(FXN**2+FYN**2))',//)               00002520
 978 FORMAT( 25X,'NORMALIZED MOMENT IS      MZN=',1PE11.4,/,            00002530
     $30X,'(COMPUTED)',//)                                              00002540
 979  FORMAT(//,58X,'***** A/B LESS THAN 0.1 *****',/,                  00002550
     $58X,'***** WORK NEXT PROBLEM *****',//)                           00002560
9001 FORMAT(1H1)                                                        00002570
9004 FORMAT(//,53X,'***** CONTACT REGION FOLLOWS *****',/,              00002580
     $10X,'X AND Y ARE NORMALIXED COORDINATES, X IN THE ROLLING',/,     00002590
     $10X,'DIRECTION, X,Y=X1/C1,Y1/C1 WHERE X1,Y1 ARE DIM. COORD.',/,   00002600
     $10X,'TZN=HERTZ STRESS =3/(2*PI)*SQRT(P), FOR REFERENCE ONLY',/,   00002610
     $10X,'TZK=KALKER NORMAL STRESS AS ASSUMED IN THE PROGRAM',/,       00002620
     $10X,'TZK=F*A*A1(Y)*P, FOR X.GE.0.9*L(Y)',/,                       00002630
     $10X,'TZK=F*A*(SQRT(P)+B1(Y)), FOR X.LE.0.9*L(Y)',/,               00002640
     $10X,'WHERE P=1.0-X*X/(A*A)-Y*Y/(B*B), L(Y)=A*SQRT(1.0-Y*Y/(B*B))',00002650
     $/,10X,'A1(Y)=0.5/SQRT((1.0-Y*Y/(B*B))*(1.0-(0.9)**2))',/,         00002660
     $10X,'B1(Y)=-0.25/A1(Y), F*A=0.656773,SUCH THAT RES NORMAL FORCE=1'00002670
     $,/,10X,'TX AND TY ARE NORMALIZED SHEAR STRESSES',/,10X,'TX=-TAUXZ*00002680
     $C**3/(RHO*N), TY=-TAUYZ*C**3/(RHO*N)',/,                          00002690
     $10X,'ABS(TX,TY) LESS THAN TZK FOR NO SLIP, EQUAL TO TZK FOR SLIP',00002700
     $/,10X,'VX,VY ARE NORMALIZED SLIP COMPONENTS, VX=VX1/V*RHO/(MU*C)',00002710
     $/,10X,'VY=VY1/V*RHO/(MU*C), WHERE VX1,VX2=REL. VEL. BETWEEN',/,10X00002720
     $,'ADJACENT POINTS AND V=ROLLING VEL.',/////)                      00002730
9005 FORMAT(1H )                                                        00002740
9006 FORMAT( 1X,'***** Y=',1F11.4)                                      00002750
9007 FORMAT(10X,'AT Y=',1F11.4,5X,'THE LEADING EDGE SLIPS')             00002760
9008 FORMAT(10X,'AT Y=',1F11.4,5X,'THE LEADING EDGE STICKS')            00002770
9009 FORMAT( 7X,'X',9X,'TZH',9X,'TZK',5X,'ABS(TX,TY)',1X,'ARG(TX,TY)', 00002780
     $      1X,'ABS(VX,VY)',1X,'ARG(VX,VY)')                            00002790
9010 FORMAT(10F11.4)                                                    00002800
9998 FORMAT(////16H INPUT EXHAUSTED///)                                 00002810
     END                                                                00002820
```

MAAKZ

```
      SUBROUTINE MAAKZ(P,C,WZ,DZ,D2Z,A,B,MUZ)
C     MAAKZ COMPUTES THE NORMAL STRESS AS GIVEN BY EQUATION 14.III
C     AW=L(Y) OF EQ. 14.III
C     WZ=TZK=ASSUMED NORMAL STRESS OF EQ. 14.III
C     DZ= FIRST DER. OF TZK WITH RESPECT TO X
      REAL MUZ
      AL=0.9
      S=SQRT(1.0-AL*AL)
      AW=A*SQRT(1.0-C*C/B/B)
      PI=3.1415926536
      F=MUZ*PI/2.0/A/(ATAN(AL/S)+(2.0/3.0-AL+AL*AL*AL/3.0)/S)
      IF(P.LE.AL*AW) GO TO 10
      WZ=F*(AW*AW-P*P)/2.0/AW/S
      DZ=F*P/AW/S
      D2Z=F/AW/S
      GO TO 11
10    WZ=F*SQRT(AW*AW-P*P)-F/2.0*AW*S
      IF(ABS(AW).EQ.ABS(P)) GO TO 12
      DZ=F*P/SQRT(AW*AW-P*P)
      GO TO 11
   12 DZ=1.0
   11 RETURN
      END
```

ROL

```
      SUBROUTINE ROL(CS,GEL,MUZ,NX,NY,X,Y,VX,VY,G,FX,FY,MZ)
      REAL MUZ,MZ,K,NU
      INTEGER PIJL
      REAL LE(81)
      INTEGER E(81)
      COMMON A,B
      DIMENSION X(81,81),Y(81,81),VX(81,81),VY(81,81),
     $ G(81,10),CS(3),GEL(5)
      UX=CS(1)
      UY=CS(2)
      PH=CS(3)
      DM=GEL(5)
      SX=GEL(1)
      SY=GEL(2)
      A=GEL(3)
      B=GEL(4)
      H=A/FLOAT(NX)
      K=B/FLOAT(NY)
      PI=3.1415926536
      CZP=MUZ*2.0/A/A
      J=-NY
380   J=J+1
      IF(J.GT.NY-1)GO TO 100
      LE(J+NY+1)=A*SQRT(1.0-FLOAT(J*J)/FLOAT(NY)/FLOAT(NY))
      P=LE(J+NY+1)
      E(J+NY+1)=0.99*P/H
      I=-NX-1
401   I=I+1
      IF(I.GT.NX)GO TO 200
      IF(I.LT.-E(J+NY+1).OR.I.GT.E(J+NY+1))GO TO 15
      GO TO 10
15    VY(I+NX+1,J+NY+1)=0.0
      VX(I+NX+1,J+NY+1)=0.0
      Y(I+NX+1,J+NY+1)=0.0
      X(I+NX+1,J+NY+1)=0.0
10    GO TO 401
200   CONTINUE
      DO 20 I=1,10
 20   G(J+NY+1,I)=-3.0*A
      GO TO 380
100   CONTINUE
      THV=-1.0
      IF(UX-PH*B.GE.0.0) THV=1.0
      THV=THV*ATAN(UY/(ABS(UX-PH*B)+1.E-08))+PI/2.0*(1.0-THV)
C
      J=-NY
470   J=J+1
      IF(J.GT.NY-1)GO TO 300
      Q=J*K
      P=LE(J+NY+1)
      PG=P
      CUX=UX-PH*Q
      PIJL=2
      CUY=UY+PH*P
      CALL MAAKZ(P,Q,ZN,CZ,CZP,A,B,MUZ)
      XG=0.0
      YG=0.0
      XV=0.0
```

```
      YV=0.0
      VV=0.0
      I=E(J+NY+1)
      G(J+NY+1,1)=1.0
      IF(CZ*CZ-CUX*CUX/SX/SX-CUY*CUY/SY/SY.GT.0.0) GO TO 1001
C     SLIP AT THE LEADING EDGE. DETERMINE THETA.
      G(J+NY+1,1)=-1.0
      T=-1.0
      IF(CUX.GE.0.0) T=1.0
      T=T*ATAN(CUY/(ABS(CUX)+1.E-08))+PI/2.0*(1.0-T)
C BEPT
1002  S=SIN(T)
      C=COS(T)
      NU=(CUX*S-CUY*C-CZ*(SX-SY)*C*S)/(CUX*C+CUY*S-CZ*(SX-SY)*(C*C-S*S))
      T=T-NU
      IF(ABS(NU).GT.1.E-03) GO TO 1002
      THV=T
      C=COS(T)
      S=SIN(T)
C     THE STARTING VALUE OF T HAS BEEN FOUND. THE
C     DERIVATIVE IS DETERMINED IN A SPECIAL WAY.
      TP=(PH*C+CZP*(SX-SY)*C*S)/(CUX*C+CUY*S-CZ*((3.0*C*C-1.0)*(SX-SY)
     $ -SX))
C NEXTGL
1003  D=-DM
      IF(P-DM.LE.I*H+1.E-06) D=I*H-P
      PN=P+D
      CALL MAAKZ(PN,G,ZN,CZ,CZP,A,B,MUZ)
      TN=T+D*TP
      S=SIN(TN)
      C=COS(TN)
      CUY=UY+PH*PN
      TPN=(CUX*S-CUY*C-CZ*C*S*(SX-SY))/ZN/(SY*C*C+SX*S*S)
      T=T+0.5*D*(TP+TPN)
      S=SIN(T)
      C=COS(T)
      XN=ZN*C
      YN=ZN*S
      V=CUX*SY*C+CUY*SX*S-CZ*SX*SY
      IF(V.GE.-4.E-05) GO TO 1004
      AN=VV/(VV-V)
      IF(ABS(VV).LT.1.E-10) AN=0.9
      VV=0.0
      AV=1.0-AN
      G(J+NY+1,PIJL)=AV*P+AN*PN
      PG=G(J+NY+1,PIJL)
      PIJL=PIJL+1
      P=PG
      IF(PIJL.GT.10)PIJL=10
      XV=AV*XV+AN*XN
      XG=XV
      YV=AV*YV+AN*YN
      YG=YV
      GO TO 1006
1004  CONTINUE
C     SLIP IN THE NEW POINT
      XV=XN
      YV=YN
```

```
      P=PN
C SLIP
1005  TP=(CUX*S-CUY*C-CZ*C*S*(SX-SY))/ZN/(SY*C*C+SX*S*S)
      IF(P.GT.I*H+1.E-06) GO TO 1003
      X(I+NX+1,J+NY+1)=XN
      Y(I+NX+1,J+NY+1)=YN
      XP=-ZN*S*TP-CZ*C
      YP=ZN*C*TP-CZ*S
      VX(I+NX+1,J+NY+1)=CUX+SX*XP
      VY(I+NX+1,J+NY+1)=CUY+SY*YP
      VV=V
      I=I-1
      IF(I.LT.-E(J+NY+1)) GO TO 1007
      GO TO 1003
C ADH
1001  T=-1.0
      IF(CUX.GT.0.0) T=1.0
      THV=T*ATAN((UY+PH*P)/(ABS(CUX)+1.E-08))+PI*(1.0-T)/2.0
C ADHERE
1006  PN=I*H
      XN=CUX*(PG-PN)/SX+XG
      YN=(UY+0.5*PH*(PG+PN))*(PG-PN)/SY+YG
      CALL MAAKZ(PN,G,ZN,CZ,CZP,A,E,MUZ)
      V=ZN-SQRT(XN*XN+YN*YN)
      IF(V.GE.(-4.E-05)*MUZ)GO TO 1008
      AN=VV/(VV-V)
      IF(ABS(VV).LT.1.E-10) AN=0.9
      AV=1.0-AN
      P=AV*P+AN*PN
      G(J+NY+1,PIJL)=P
      VV=0.0
      PIJL=PIJL+1
      XV=AV*XV+AN*XN
      YV=AV*YV+AN*YN
      IF(PIJL.GT.10) PIJL=10
      T=-1.0
      IF(XV.GE.0.0) T=1.0
      CUY=UY+PH*P
      T=T*ATAN(YV/(ABS(XV)+1.E-08))+PI/2.0*(1.0-T)
      C=COS(T)
      S=SIN(T)
      CALL MAAKZ(P,G,ZN,CZ,CZP,A,E,MUZ)
      GO TO 1005
1008  VV=V
      X(I+NX+1,J+NY+1)=XN
      XV=XN
      Y(I+NX+1,J+NY+1)=YN
      YV=YN
      VY(I+NX+1,J+NY+1)=0.0
      VX(I+NX+1,J+NY+1)=0.0
      I=I-1
      P=PN
      IF(I.GE.-E(J+NY+1)) GO TO 1006
1007  GO TO 470
300   CONTINUE
C     THE ARRAYS ARE FILLED. THE INTEGRALS ARE
C     DETERMINED
      T=4.0*K/3.0
```

```
      TN=2.0*K
      FX=0.0
      FY=0.0
      MZ=0.0
      YN=2.0*H
      J=-NY
940   J=J+1
      IF(J.GT.NY-1)GO TO 400
      I=E(J+NY+1)
      PIJL=I
      P=(LE(J+NY+1)-I*H)/2.0-H/3.0
      YP=J*K
      C=P*(X(I+NX+1,J+NY+1)+X(NX+1-I,J+NY+1))
      S=P*(Y(I+NX+1,J+NY+1)+Y(NX+1-I,J+NY+1))
      D=P*I*H*(Y(I+NX+1,J+NY+1)-Y(NX+1-I,J+NY+1))
      P=2.0*H/3.0
      I=-PIJL-1
500   I=I+1
      IF (I.GT.PIJL)GO TO 550
      C=C+P*X(I+NX+1,J+NY+1)
      D=D+P*I*H*Y(I+NX+1,J+NY+1)
      S=S+P*Y(I+NX+1,J+NY+1)
      P=YN-P
      GO TO 500
550   FX=FX+T*C
      FY=FY+T*S
      MZ=MZ+T*(D-YP*C)
      T=TN-T
      GO TO 940
400   CONTINUE
      RETURN
      END
```

```
      SUBROUTINE CONST(A,B,NU,C11,C22,C23,C33,GS)                       00005120
      DIMENSION AL(5),BE(5),D(15),E(15,20),AR(20),CNT(5),D1(15,9),      00005130
     $D2(15,9),D3(15),D4(15)                                            00005140
C       ***** DATA E(I,J) GIVES LINEAR CREEPAGE AND SPIN COEFFICIENTS****00005150
C       *****AND GS FROM KALKER REPORT TABLE 1 ***********************  00005160
C       ***** VALID FOR A/B GREATER THAN OR EQUAL TO 0.1                00005170
      REAL NU                                                           00005180
      DATA D1/                                                          00005190
     $  2.51, 3.31, 4.85, 2.51, 2.52, 2.53, 0.334, 0.473, 0.731, 6.42,  00005200
     $ 8.28, 11.7, 0.7670, 0.5752, 0.3835,                              00005210
     $ 2.59, 3.37, 4.81, 2.59, 2.63, 2.66, 0.483, 0.603, 0.809, 3.46,   00005220
     $ 4.27, 5.66, 0.5608, 0.4206, 0.2804,                              00005230
     $ 2.68, 3.44, 4.80, 2.68, 2.75, 2.81, 0.607, 0.715, 0.889, 2.49,   00005240
     $ 2.96, 3.72, 0.4779, 0.3584, 0.2390,                              00005250
     $ 2.78, 3.53, 4.82, 2.78, 2.88, 2.98, 0.720, 0.823, 0.977, 2.02,   00005260
     $ 2.32, 2.77, 0.4343, 0.3257, 0.2172,                              00005270
     $ 2.88, 3.62, 4.83, 2.88, 3.01, 3.14, 0.827, 0.929, 1.07, 1.74,    00005280
     $ 1.93, 2.22, 0.4089, 0.3066, 0.2044,                              00005290
     $ 2.98, 3.72, 4.91, 2.98, 3.14, 3.31, 0.930, 1.03, 1.18, 1.56,     00005300
     $ 1.68, 1.86, 0.3934, 0.2950, 0.1967,                              00005310
     $ 3.09, 3.81, 4.97, 3.09, 3.28, 3.48, 1.03, 1.14, 1.29, 1.43,      00005320
     $ 1.50, 1.60, 0.3840, 0.2880, 0.1920,                              00005330
     $ 3.19, 3.91, 5.05, 3.19, 3.41, 3.65, 1.13, 1.25, 1.40, 1.34,      00005340
     $ 1.37, 1.42, 0.3785, 0.2839, 0.1892,                              00005350
     $ 3.29, 4.01, 5.12, 3.29, 3.54, 3.82, 1.23, 1.36, 1.51, 1.27,      00005360
     $ 1.27, 1.27, 0.3758, 0.2818, 0.1879/                              00005370
      DATA D2/                                                          00005380
     $ 3.40, 4.12, 5.20, 3.40, 3.67, 3.98, 1.33, 1.47, 1.63, 1.21,      00005390
     $ 1.19, 1.16, 0.3750, 0.2812, 0.1875,                              00005400
     $ 3.51, 4.22, 5.30, 3.51, 3.81, 4.16, 1.44, 1.59, 1.77, 1.16,      00005410
     $ 1.11, 1.06, 0.3758, 0.2818, 0.1879,                              00005420
     $ 3.65, 4.36, 5.42, 3.65, 3.99, 4.39, 1.58, 1.75, 1.94, 1.10,      00005430
     $ 1.04, 0.954, 0.3785, 0.2839, 0.1892,                             00005440
     $ 3.82, 4.54, 5.58, 3.82, 4.21, 4.67, 1.76, 1.95, 2.18, 1.05,      00005450
     $ 0.965, 0.852, 0.3840, 0.2880, 0.1920,                            00005460
     $ 4.06, 4.78, 5.80, 4.06, 4.50, 5.04, 2.01, 2.23, 2.50, 1.01,      00005470
     $ 0.892, 0.751, 0.3934, 0.2950, 0.1967,                            00005480
     $ 4.37, 5.10, 6.11, 4.37, 4.90, 5.56, 2.35, 2.62, 2.96, 0.958,     00005490
     $ 0.819, 0.650, 0.4089, 0.3066, 0.2044,                            00005500
     $ 4.84, 5.57, 6.57, 4.84, 5.48, 6.31, 2.88, 3.24, 3.70, 0.912,     00005510
     $ 0.747, 0.549, 0.4343, 0.3257, 0.2172,                            00005520
     $ 5.57, 6.34, 7.34, 5.57, 6.40, 7.51, 3.79, 4.32, 5.01, 0.868,     00005530
     $ 0.674, 0.446, 0.4779, 0.3584, 0.2390,                            00005540
     $ 6.96, 7.78, 8.82, 6.96, 8.14, 9.79, 5.72, 6.63, 7.89, 0.828,     00005550
     $ 0.601, 0.341, 0.5608, 0.4206, 0.2804/                            00005560
      DATA D3/                                                          00005570
     $ 10.7, 11.7, 12.9, 10.7, 12.8, 16.0, 12.2, 14.6, 18.0, 0.795,     00005580
     $ 0.562, 0.228, 0.7670, 0.5752, 0.3835 /                           00005590
      DATA D4/                                                          00005600
     $ 11.08, 12.01, 13.10, 11.08, 13.38, 16.90, 13.72, 16.34, 20.20,   00005610
     $ 0.785, 0.552, 0.208, 0.7918, 0.5938, 0.3959/                     00005620
      DATA AR / 0.1,0.2,0.3,0.4,0.5,0.6,0.7,0.8,0.9,1.0,1.111111,       00005630
     $1.25,1.428571,1.666667,2.0,2.5,3.333333,5.0,10.0,11.0/            00005640
      DO 6 I=1,15                                                       00005650
      DO 5 J=1,9                                                        00005660
      E(I,J)=D1(I,J)                                                    00005670
    5 E(I,J+9)=D2(I,J)                                                  00005680
      E(I,19)=D3(I)                                                     00005690
```

```
6     E(I,20)=D4(I)
      PI=3.14159
      RG=A/B
      IF(RG.GT.AR(20)) GO TO 14
      GO TO 15
14    SG=B/A
      GAM=ALOG(16.0/(SG*SG))
      C11=2.0*PI/(SG*(GAM-2.0*NU))*(1.0+( 1.613706    )/(GAM-2.0*NU))
      C22=(( 1.613706 )*(1.0-NU))/(2.0*NU+GAM*(1.0-NU))
      C22=2.0*PI*(1.0+C22)/(SG*(2.0*NU+GAM*(1.0-NU)))
      C23=2.0*PI/((SQRT(SG)*SG*3.0)*((1.0-NU)*GAM-2.0+4.0*NU))
      C33=PI/4.0*(GAM*(1.0-2.0*NU)-2.0+6.0*NU)/(GAM*(1.0-NU)-2.0+4.0*NU)
      GS=3.0*(1.0-NU)/(4.0*PI*SQRT(SG))
      GO TO 80
15    DO 20 I=2,20
      IF(RG.LE.AR(I)) GO TO 25
20    CONTINUE
25    J=I
      DO 30 I=1,15
30    D(I)=E(I,J-1)+(E(I,J)-E(I,J-1))*(RG-AR(J-1))/(AR(J)-AR(J-1))
      DO 40 I=1,5
      AL(I)=8.0*(D(3*I)-2.0*D(3*I-1)+D(3*I-2))
      BE(I)=2.0*(-D(3*I)+4.0*D(3*I-1)-3.0*D(3*I-2))
40    CNT(I)=AL(I)*NU**2+BE(I)*NU+D(3*I-2)
      C11=CNT(1)
      C22=CNT(2)
      C23=CNT(3)
       C33=CNT(4)
      GS=CNT(5)
80    CONTINUE
      RETURN
      END
```

```
                                        PROGRAM WITA-SRT
                                 SIMPLIFIED THEORY OF ROLLING CONTACT
                                          BY J.J. KALKER
                                   MODIFIED AT CLEMSON UNIVERSITY
                                        DEPT. OF MECH. ENGR.
                                            CLEMSON, SC




                                   ***** INPUT PARAMETERS *****


 NORMALIZED CONTACT DIMENSIONS      A= 2.5980E+00          ( A=A1/C1, B=B1/C1, WHERE C1=SQRT(A1*B1),
            (CARD #2)               B= 3.8490E-01          ( A1,B1 ARE ACTUAL CONTACT DIMENSIONS

             POISSON S RATIO       NU= 2.8000E-01
                 (CARD #2)

           LAYER STIFFNESSES      LXN= 0.0
                 (CARD #2)        LYN= 0.0

         NUMERICAL CONSTANTS       NX= 10
              (CARD #3)            NY= 10
                                   DM= 4.0000E-02
                                   NF=  1         (NF=1,FORCES COMPUTED; NF=2,MOMENTS COMPUTED)
                                   NS=  1         (NS=1, FULL OUTPUT; NS=2 ONLY FORCES OR MOMENTS)



                           ***** PARAMETERS COMPUTED AND USED IN PROGRAM *****

 CREEPAGE AND SPIN COEFFICIENTS   C11= 9.2721E+00
                                  C22= 9.9975E+00
                                  C23= 9.6298E+00
                                  C33= 5.5604E-01

     NORMALIZED SHEAR MODULUS      GS= 4.5572E-01

 NORMALIZED INVERSE STIFFNESSES   SXN= 1.6396E+00
                                  SYN= 1.5206E+00
```

```
                ***** NV2= 1 DISTINCT PROBLEMS FOLLOW FOR DIFFERENT *****
                   ***** VALUES OF NORMALIZED CREEPAGE AND SPIN *****



       NORMALIZED CREEPAGE AND SPIN      UXN= 0.0
            (INPUT ON CARD #5)           UYN=-1.4000E+00
                                         PHN= 8.0000E-01


             NORMALIZED FORCES ARE       FXN= 8.9034E-07
                  (COMPUTED)             FYN=-3.4703E-01

             RESULTANT FORCE             RES= 3.4703E-01
             (RES=SQRT(FXN**2+FYN**2))



                                          ***** CONTACT REGION FOLLOWS *****
X AND Y ARE NORMALIXED COORDINATES, X IN THE ROLLING
DIRECTION, X,Y=X1/C1,Y1/C1 WHERE X1,Y1 ARE DIM. COORD.
TZN=HERTZ STRESS =3/(2*PI)*SQRT(P), FOR REFERENCE ONLY
TZK=KALKER NORMAL STRESS AS ASSUMED IN THE PROGRAM
TZK=F*A*A1(Y)*P, FOR X.GE.0.9*L(Y)
TZK=F*A*(SQRT(P)+B1(Y)), FOR X.LE.0.9*L(Y)
WHERE P=1.0-X*X/(A*A)-Y*Y/(B*B), L(Y)=A*SQRT(1.0-Y*Y/(B*B))
A1(Y)=0.5/SQRT((1.0-Y*Y/(B*B))*(1.0-(0.9)**2))
B1(Y)=-0.25/A1(Y), F*A=0.656773,SUCH THAT RES NORMAL FORCE=1
TX AND TY ARE NORMALIZED SHEAR STRESSES
TX=-TAUXZ*C**3/(RHO*N), TY=-TAUYZ*C**3/(RHO*N)
ABS(TX,TY) LESS THAN TZK FOR NO SLIP, EQUAL TO TZK FOR SLIP
VX,VY ARE NORMALIZED SLIP COMPONENTS, VX=VX1/V*RHO/(MU*C)
VY=VY1/V*RHO/(MU*C), WHERE VX1,VX2=REL. VEL. BETWEEN
ADJACENT POINTS AND V=ROLLING VEL.



AT Y=      0.3464     THE LEADING EDGE STICKS
AT Y=      0.3079     THE LEADING EDGE STICKS
AT Y=      0.2694     THE LEADING EDGE STICKS
AT Y=      0.2309     THE LEADING EDGE STICKS
AT Y=      0.1924     THE LEADING EDGE SLIPS
AT Y=      0.1540     THE LEADING EDGE SLIPS
AT Y=      0.1155     THE LEADING EDGE SLIPS
AT Y=      0.0770     THE LEADING EDGE SLIPS
AT Y=      0.0385     THE LEADING EDGE SLIPS
AT Y=      0.0        THE LEADING EDGE SLIPS
AT Y=     -0.0385     THE LEADING EDGE SLIPS
AT Y=     -0.0770     THE LEADING EDGE SLIPS
AT Y=     -0.1155     THE LEADING EDGE SLIPS
AT Y=     -0.1540     THE LEADING EDGE SLIPS
AT Y=     -0.1924     THE LEADING EDGE SLIPS
AT Y=     -0.2309     THE LEADING EDGE STICKS
AT Y=     -0.2694     THE LEADING EDGE STICKS
AT Y=     -0.3079     THE LEADING EDGE STICKS
AT Y=     -0.3464     THE LEADING EDGE STICKS
```

```
***** Y=      0.3464
     X          TZH          TZK        ABS(TX,TY) ARG(TX,TY)  ABS(VX,VY) ARG(VX,VY)
  -1.0392     0.0827       0.0514       0.0514     261.5010     3.3606    261.5010
  -0.7794     0.1510       0.1453       0.1453     260.1367     2.5782    260.1367
  -0.5196     0.1849       0.1920       0.1920     258.2253     2.1538    258.2253
  -0.2598     0.2026       0.2163       0.2163     255.5179     1.7885    255.5178
   0.0        0.2081       0.2239       0.2239     251.7460     1.4411    251.7460
   0.2598     0.2026       0.2163       0.2163     246.7664     1.0954    246.7663
   0.5196     0.1849       0.1920       0.1920     240.7860     0.7374    240.7860
   0.7794     0.1510       0.1453       0.1263     233.5065     0.0         0.0
   1.0392     0.0827       0.0519       0.0273     223.4534     0.0         0.0

***** Y=      0.3079
     X          TZH          TZK        ABS(TX,TY) ARG(TX,TY)  ABS(VX,VY) ARG(VX,VY)
  -1.2990     0.1584       0.1319       0.1319     263.0781     3.3061    263.0781
  -1.0392     0.2135       0.2078       0.2078     262.0066     2.8105    262.0066
  -0.7794     0.2481       0.2554       0.2554     260.4182     2.4291    260.4182
  -0.5196     0.2701       0.2856       0.2856     257.9551     2.0834    257.9551
  -0.2598     0.2825       0.3027       0.3027     254.1517     1.7515    254.1517
   0.0        0.2865       0.3082       0.3082     248.5337     1.4219    248.5337
   0.2598     0.2825       0.3027       0.3027     240.8285     1.0879    240.8285
   0.5196     0.2701       0.2856       0.2856     231.2778     0.7493    231.2778
   0.7794     0.2481       0.2554       0.1864     217.7482     0.0         0.0
   1.0392     0.2135       0.2078       0.1032     197.7118     0.0         0.0
   1.2990     0.1584       0.1319       0.0496     172.2837     0.0         0.0
   1.5588     0.0002       0.0000       0.0          0.0        0.0         0.0

***** Y=      0.2694
     X          TZH          TZK        ABS(TX,TY) ARG(TX,TY)  ABS(VX,VY) ARG(VX,VY)
  -1.5588     0.1849       0.1521       0.1521     264.4822     3.5782    264.4822
  -1.2990     0.2435       0.2327       0.2327     263.6963     3.0991    263.6963
  -1.0392     0.2825       0.2863       0.2863     262.4902     2.7217    262.4902
  -0.7794     0.3094       0.3234       0.3234     260.5039     2.3791    260.5039
  -0.5196     0.3273       0.3480       0.3480     257.2070     2.0507    257.2070
  -0.2598     0.3376       0.3622       0.3622     251.9587     1.7242    251.9587
   0.0        0.3410       0.3668       0.3668     244.1799     1.3889    244.1798
   0.2598     0.3376       0.3622       0.3622     233.6683     1.0382    233.6683
   0.5196     0.3273       0.3480       0.2846     219.0531     0.0         0.0
   0.7794     0.3094       0.3234       0.1855     196.2500     0.0         0.0
   1.0392     0.2825       0.2863       0.1385     167.1360     0.0         0.0
   1.2990     0.2435       0.2327       0.1150     143.1956     0.0         0.0
   1.5588     0.1849       0.1521       0.0792     128.2595     0.0         0.0
   1.8186     0.0675       0.0211       0.0125     119.2189     0.0         0.0

***** Y=      0.2309
     X          TZH          TZK        ABS(TX,TY) ARG(TX,TY)  ABS(VX,VY) ARG(VX,VY)
  -1.8186     0.1849       0.1399       0.1399     265.6860     3.9349    265.6860
  -1.5588     0.2527       0.2330       0.2330     265.1428     3.4150    265.1428
  -1.2990     0.2982       0.2956       0.2956     264.3091     3.0261    264.3091
  -1.0392     0.3308       0.3405       0.3405     262.8872     2.6793    262.8872
  -0.7794     0.3541       0.3726       0.3726     260.3867     2.3505    260.3867
  -0.5196     0.3698       0.3942       0.3942     256.1316     2.0266    256.1316
  -0.2598     0.3790       0.4068       0.4068     249.3674     1.6948    249.3674
   0.0        0.3820       0.4109       0.4109     239.5138     1.3409    239.5138
   0.2598     0.3790       0.4068       0.3707     225.9109     0.0         0.0
   0.5196     0.3698       0.3942       0.2403     203.0654     0.0         0.0
   0.7794     0.3541       0.3726       0.1872     169.7594     0.0         0.0
   1.0392     0.3308       0.3405       0.1876     141.7920     0.0         0.0
   1.2990     0.2982       0.2956       0.1896     125.6612     0.0         0.0
   1.5588     0.2527       0.2330       0.1648     116.5637     0.0         0.0
   1.8186     0.1849       0.1399       0.1029     110.9889     0.0         0.0
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```
***** Y=      0.1924
     X          TZH          TZK        ABS(TX,TY) ARG(TX,TY) ABS(VX,VY) ARG(VX,VY)
  -2.0784     0.1584       0.0939       0.0939    266.7043    4.4259   266.7043
  -1.8186     0.2435       0.2109       0.2109    266.3472    3.7651   266.3472
  -1.5588     0.2982       0.2862       0.2862    265.8198    3.3456   265.8198
  -1.2990     0.3376       0.3404       0.3404    264.9233    2.9870   264.9233
  -1.0392     0.3667       0.3805       0.3805    263.2856    2.6536   263.2856
  -0.7794     0.3879       0.4096       0.4096    260.3276    2.3308   260.3276
  -0.5196     0.4023       0.4294       0.4294    255.2953    2.0065   255.2953
  -0.2598     0.4107       0.4410       0.4410    247.4095    1.6657   247.4095
   0.0        0.4135       0.4448       0.4448    236.1813    1.2922   236.1813
   0.2598     0.4107       0.4410       0.2952    217.1395    0.0        0.0
   0.5196     0.4023       0.4294       0.2047    181.6965    0.0        0.0
   0.7794     0.3879       0.4096       0.2120    145.0798    0.0        0.0
   1.0392     0.3667       0.3805       0.2493    125.0430    0.0        0.0
   1.2990     0.3376       0.3404       0.2670    114.9075    0.0        0.0
   1.5588     0.2982       0.2862       0.2493    109.1405    0.0        0.0
   1.8186     0.2435       0.2109       0.1911    105.4864    0.0        0.0
   2.0784     0.1584       0.0957       0.0904    102.9887    0.0        0.0

***** Y=      0.1540
     X          TZH          TZK        ABS(TX,TY) ARG(TX,TY) ABS(VX,VY) ARG(VX,VY)
  -2.0784     0.2135       0.1625       0.1625    267.3408    4.1843   267.3408
  -1.8186     0.2825       0.2574       0.2574    267.0288    3.6901   267.0288
  -1.5588     0.3308       0.3238       0.3238    266.5320    3.3068   266.5320
  -1.2990     0.3667       0.3733       0.3733    265.6292    2.9627   265.6292
  -1.0392     0.3937       0.4104       0.4104    263.9319    2.6366   263.9319
  -0.7794     0.4135       0.4376       0.4376    260.8638    2.3174   260.8638
  -0.5196     0.4271       0.4562       0.4562    255.7088    1.9940   255.7088
  -0.2598     0.4350       0.4672       0.4672    247.7752    1.6519   247.7752
   0.0        0.4376       0.4708       0.3962    235.6525    0.0        0.0
   0.2598     0.4350       0.4672       0.2275    208.9920    0.0        0.0
   0.5196     0.4271       0.4562       0.1850    160.4578    0.0        0.0
   0.7794     0.4135       0.4376       0.2414    128.3532    0.0        0.0
   1.0392     0.3937       0.4104       0.2995    114.7184    0.0        0.0
   1.2990     0.3667       0.3733       0.3261    107.9860    0.0        0.0
   1.5588     0.3308       0.3238       0.3129    104.0802    0.0        0.0
   1.8186     0.2825       0.2574       0.2573    101.5555    0.0        0.0
   2.0784     0.2135       0.1625       0.1623     99.7412    0.0        0.0
   2.3382     0.0827       0.0247       0.0247     98.4597    0.0996    98.4598

***** Y=      0.1155
     X          TZH          TZK        ABS(TX,TY) ARG(TX,TY) ABS(VX,VY) ARG(VX,VY)
  -2.3382     0.1510       0.0711       0.0711    268.1663    4.8506   268.1663
  -2.0784     0.2481       0.2047       0.2047    267.9902    4.0869   267.9902
  -1.8186     0.3094       0.2891       0.2891    267.7378    3.6488   267.7378
  -1.5588     0.3541       0.3505       0.3505    267.3147    3.2831   267.3147
  -1.2990     0.3879       0.3970       0.3970    266.5259    2.9471   266.5259
  -1.0392     0.4135       0.4322       0.4322    265.0417    2.6257   265.0417
  -0.7794     0.4324       0.4582       0.4582    262.3899    2.3102   262.3899
  -0.5196     0.4453       0.4761       0.4761    258.0063    1.9924   258.0063
  -0.2598     0.4530       0.4865       0.4865    251.3702    1.6618   251.3702
   0.0        0.4555       0.4900       0.3397    239.8070    0.0        0.0
   0.2598     0.4530       0.4865       0.1707    206.7441    0.0        0.0
   0.5196     0.4453       0.4761       0.1645    144.5747    0.0        0.0
   0.7794     0.4324       0.4582       0.2510    117.4232    0.0        0.0
   1.0392     0.4135       0.4322       0.3206    107.6421    0.0        0.0
   1.2990     0.3879       0.3970       0.3525    102.9081    0.0        0.0
   1.5588     0.3541       0.3505       0.3422    100.1493    0.0        0.0
   1.8186     0.3094       0.2891       0.2891     98.3403    0.0273    98.3404
   2.0784     0.2481       0.2047       0.2047     97.0495    0.1093    97.0495
   2.3382     0.1510       0.0790       0.0790     96.1159    0.1288    96.1159
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***** Y=      0.0770
      X           TZH            TZK        ABS(TX,TY) ARG(TX,TY)  ABS(VX,VY) ARG(VX,VY)
   -2.3382      0.1849         0.1141        0.1141     268.7739     4.6488     268.7739
   -2.0784      0.2701         0.2313        0.2313     268.6521     4.0376     268.6521
   -1.8186      0.3273         0.3100        0.3100     268.4727     3.6249     268.4727
   -1.5588      0.3698         0.3685        0.3685     268.1626     3.2686     268.1626
   -1.2990      0.4023         0.4132        0.4132     267.5769     2.9373     267.5769
   -1.0392      0.4271         0.4472        0.4472     266.4773     2.6189     266.4773
   -0.7794      0.4453         0.4723        0.4723     264.5308     2.3070     264.5308
   -0.5196      0.4580         0.4897        0.4897     261.3491     1.9956     261.3491
   -0.2598      0.4654         0.4999        0.4999     256.5828     1.6790     256.5828
    0.0         0.4678         0.5033        0.2951     247.0065     0.0          0.0
    0.2598      0.4654         0.4999        0.1167     208.0296     0.0          0.0
    0.5196      0.4580         0.4897        0.1483     127.7151     0.0          0.0
    0.7794      0.4453         0.4723        0.2570     107.7684     0.0          0.0
    1.0392      0.4271         0.4472        0.3341     101.4192     0.0          0.0
    1.2990      0.4023         0.4132        0.3695      98.3826     0.0          0.0
    1.5588      0.3698         0.3685        0.3613      96.6089     0.0          0.0
    1.8186      0.3273         0.3100        0.3100      95.4402     0.0438      95.4402
    2.0784      0.2701         0.2313        0.2313      94.5983     0.1530      94.5983
    2.3382      0.1849         0.1153        0.1153      93.9837     0.1473      93.9837

***** Y=      0.0385
      X           TZH            TZK        ABS(TX,TY) ARG(TX,TY)  ABS(VX,VY) ARG(VX,VY)
   -2.3382      0.2026         0.1362        0.1362     269.3855     4.5703     269.3857
   -2.0784      0.2825         0.2461        0.2461     269.3232     4.0131     269.3232
   -1.8186      0.3376         0.3220        0.3220     269.2300     3.6122     269.2300
   -1.5588      0.3790         0.3789        0.3789     269.0657     3.2606     269.0657
   -1.2990      0.4107         0.4226        0.4226     268.7532     2.9318     268.7532
   -1.0392      0.4350         0.4559        0.4559     268.1680     2.6154     268.1680
   -0.7794      0.4530         0.4806        0.4806     267.1382     2.3059     267.1382
   -0.5196      0.4654         0.4977        0.4977     265.4653     1.9999     265.4653
   -0.2598      0.4727         0.5077        0.5077     262.9739     1.6942     262.9739
    0.0         0.4751         0.5111        0.2654     257.3679     0.0          0.0
    0.2598      0.4727         0.5077        0.0668     219.0571     0.0          0.0
    0.5196      0.4654         0.4977        0.1378     109.3834     0.0          0.0
    0.7794      0.4530         0.4806        0.2605      98.7453     0.0          0.0
    1.0392      0.4350         0.4559        0.3419      95.6179     0.0          0.0
    1.2990      0.4107         0.4226        0.3792      94.1317     0.0          0.0
    1.5588      0.3790         0.3789        0.3722      93.2624     0.0          0.0
    1.8186      0.3376         0.3220        0.3220      92.6873     0.0521      92.6873
    2.0784      0.2825         0.2461        0.2461      92.2718     0.1741      92.2718
    2.3382      0.2026         0.1363        0.1363      91.9662     0.1576      91.9662

***** Y=      0.0
      X           TZH            TZK        ABS(TX,TY) ARG(TX,TY)  ABS(VX,VY) ARG(VX,VY)
   -2.3382      0.2081         0.1431        0.1431     -90.0000     4.5484     -90.0000
   -2.0784      0.2865         0.2509        0.2509     -90.0000     4.0056     -90.0000
   -1.8186      0.3410         0.3259        0.3259     -90.0000     3.6082     -90.0000
   -1.5588      0.3820         0.3823        0.3823     -90.0000     3.2581     -90.0000
   -1.2990      0.4135         0.4256        0.4256     -90.0000     2.9301     -90.0000
   -1.0392      0.4376         0.4588        0.4588     -90.0000     2.6143     -90.0000
   -0.7794      0.4555         0.4834        0.4834     -90.0000     2.3058     -90.0000
   -0.5196      0.4678         0.5004        0.5004     -90.0000     2.0017     -90.0000
   -0.2598      0.4751         0.5103        0.5103     -90.0000     1.7003     -90.0000
    0.0         0.4775         0.5136        0.2548     -90.0000     0.0          0.0
    0.2598      0.4751         0.5103        0.0380     -89.9998     0.0          0.0
    0.5196      0.4678         0.5004        0.1342      90.0000     0.0          0.0
    0.7794      0.4555         0.4834        0.2616      90.0000     0.0          0.0
    1.0392      0.4376         0.4588        0.3443      90.0000     0.0          0.0
    1.2990      0.4135         0.4256        0.3824      90.0000     0.0          0.0
    1.5588      0.3820         0.3823        0.3757      90.0000     0.0          0.0
    1.8186      0.3410         0.3259        0.3259      90.0000     0.0546      90.0000
    2.0784      0.2865         0.2509        0.2509      90.0000     0.1805      90.0000
    2.3382      0.2081         0.1431        0.1431      90.0000     0.1610      90.0000
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***** Y=    -0.0385
     X          TZH          TZK       ABS(TX,TY)  ARG(TX,TY)  ABS(VX,VY)  ARG(VX,VY)
  -2.3382     0.2026       0.1362       0.1362    -89.3857     4.5703     -89.3857
  -2.0784     0.2825       0.2461       0.2461    -89.3234     4.0131     -89.3234
  -1.8186     0.3376       0.3220       0.3220    -89.2301     3.6122     -89.2301
  -1.5588     0.3790       0.3789       0.3789    -89.0656     3.2606     -89.0656
  -1.2990     0.4107       0.4226       0.4226    -88.7532     2.9318     -88.7532
  -1.0392     0.4350       0.4559       0.4559    -88.1679     2.6154     -88.1679
  -0.7794     0.4530       0.4806       0.4806    -87.1380     2.3059     -87.1380
  -0.5196     0.4654       0.4977       0.4977    -85.4652     1.9999     -85.4652
  -0.2598     0.4727       0.5077       0.5077    -82.9735     1.6942     -82.9735
   0.0        0.4751       0.5111       0.2654    -77.3669     0.0          0.0
   0.2598     0.4727       0.5077       0.0668    -39.0543     0.0          0.0
   0.5196     0.4654       0.4977       0.1378     70.6146     0.0          0.0
   0.7794     0.4530       0.4806       0.2605     81.2535     0.0          0.0
   1.0392     0.4350       0.4559       0.3419     84.3813     0.0          0.0
   1.2990     0.4107       0.4226       0.3792     85.8676     0.0          0.0
   1.5588     0.3790       0.3789       0.3722     86.7368     0.0          0.0
   1.8186     0.3376       0.3220       0.3220     87.3120     0.0521      87.3120
   2.0784     0.2825       0.2461       0.2461     87.7278     0.1741      87.7278
   2.3382     0.2026       0.1363       0.1363     88.0336     0.1576      88.0336

***** Y=    -0.0770
     X          TZH          TZK       ABS(TX,TY)  ARG(TX,TY)  ABS(VX,VY)  ARG(VX,VY)
  -2.3382     0.1849       0.1141       0.1141    -88.7741     4.6488     -88.7741
  -2.0784     0.2701       0.2313       0.2313    -88.6521     4.0376     -88.6521
  -1.8186     0.3273       0.3100       0.3100    -88.4728     3.6249     -88.4728
  -1.5588     0.3698       0.3685       0.3685    -88.1628     3.2686     -88.1628
  -1.2990     0.4023       0.4132       0.4132    -87.5770     2.9373     -87.5770
  -1.0392     0.4271       0.4472       0.4472    -86.4774     2.6189     -86.4774
  -0.7794     0.4453       0.4723       0.4723    -84.5309     2.3070     -84.5309
  -0.5196     0.4580       0.4897       0.4897    -81.3491     1.9956     -81.3491
  -0.2598     0.4654       0.4999       0.4999    -76.5824     1.6790     -76.5824
   0.0        0.4678       0.5033       0.2951    -67.0059     0.0          0.0
   0.2598     0.4654       0.4999       0.1167    -28.0290     0.0          0.0
   0.5196     0.4580       0.4897       0.1483     52.2839     0.0          0.0
   0.7794     0.4453       0.4723       0.2570     72.2309     0.0          0.0
   1.0392     0.4271       0.4472       0.3341     78.5804     0.0          0.0
   1.2990     0.4023       0.4132       0.3695     81.6169     0.0          0.0
   1.5588     0.3698       0.3685       0.3613     83.3906     0.0          0.0
   1.8186     0.3273       0.3100       0.3100     84.5593     0.0438      84.5592
   2.0784     0.2701       0.2313       0.2313     85.4013     0.1530      85.4013
   2.3382     0.1849       0.1153       0.1153     86.0161     0.1473      86.0161

***** Y=    -0.1155
     X          TZH          TZK       ABS(TX,TY)  ARG(TX,TY)  ABS(VX,VY)  ARG(VX,VY)
  -2.3382     0.1510       0.0711       0.0711    -88.1665     4.8506     -88.1665
  -2.0784     0.2481       0.2047       0.2047    -87.9903     4.0869     -87.9903
  -1.8186     0.3094       0.2891       0.2891    -87.7378     3.6488     -87.7378
  -1.5588     0.3541       0.3505       0.3505    -87.3148     3.2831     -87.3148
  -1.2990     0.3879       0.3970       0.3970    -86.5260     2.9471     -86.5260
  -1.0392     0.4135       0.4322       0.4322    -85.0418     2.6257     -85.0418
  -0.7794     0.4324       0.4582       0.4582    -82.3900     2.3102     -82.3900
  -0.5196     0.4453       0.4761       0.4761    -78.0063     1.9924     -78.0063
  -0.2598     0.4530       0.4865       0.4865    -71.3700     1.6618     -71.3700
   0.0        0.4555       0.4900       0.3397    -59.8068     0.0          0.0
   0.2598     0.4530       0.4865       0.1707    -26.7440     0.0          0.0
   0.5196     0.4453       0.4761       0.1645     35.4246     0.0          0.0
   0.7794     0.4324       0.4582       0.2510     62.5763     0.0          0.0
   1.0392     0.4135       0.4322       0.3206     72.3575     0.0          0.0
   1.2990     0.3879       0.3970       0.3525     77.0915     0.0          0.0
   1.5588     0.3541       0.3505       0.3422     79.8503     0.0          0.0
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      1.8186      0.3094      0.2891      0.2891     81.6592     0.0273     81.6590
      2.0784      0.2481      0.2047      0.2047     82.9503     0.1093     82.9503
      2.3382      0.1510      0.0790      0.0790     83.8840     0.1288     83.8840

***** Y=     -0.1540
        X          TZH          TZK     ABS(TX,TY) ARG(TX,TY) ABS(VX,VY) ARG(VX,VY)
     -2.0784      0.2135      0.1625      0.1625    -87.3408     4.1843    -87.3408
     -1.8186      0.2825      0.2574      0.2574    -87.0290     3.6901    -87.0290
     -1.5588      0.3308      0.3238      0.3238    -86.5320     3.3068    -86.5320
     -1.2990      0.3667      0.3733      0.3733    -85.6292     2.9627    -85.6292
     -1.0392      0.3937      0.4104      0.4104    -83.9320     2.6366    -83.9320
     -0.7794      0.4135      0.4376      0.4376    -80.8637     2.3174    -80.8637
     -0.5196      0.4271      0.4562      0.4562    -75.7088     1.9940    -75.7088
     -0.2598      0.4350      0.4672      0.4672    -67.7751     1.6519    -67.7751
      0.0         0.4376      0.4708      0.3962    -55.6524     0.0         0.0
      0.2598      0.4350      0.4672      0.2275    -28.9918     0.0         0.0
      0.5196      0.4271      0.4562      0.1850     19.5421     0.0         0.0
      0.7794      0.4135      0.4376      0.2414     51.6465     0.0         0.0
      1.0392      0.3937      0.4104      0.2995     65.2814     0.0         0.0
      1.2990      0.3667      0.3733      0.3261     72.0138     0.0         0.0
      1.5588      0.3308      0.3238      0.3129     75.9196     0.0         0.0
      1.8186      0.2825      0.2574      0.2573     78.4443     0.0         0.0
      2.0784      0.2135      0.1625      0.1623     80.2587     0.0         0.0
      2.3382      0.0827      0.0247      0.0247     81.5402     0.0996     81.5401

***** Y=     -0.1924
        X          TZH          TZK     ABS(TX,TY) ARG(TX,TY) ABS(VX,VY) ARG(VX,VY)
     -2.0784      0.1584      0.0939      0.0939    -86.7044     4.4259    -86.7044
     -1.8186      0.2435      0.2109      0.2109    -86.3472     3.7651    -86.3472
     -1.5588      0.2982      0.2862      0.2862    -85.8198     3.3456    -85.8198
     -1.2990      0.3376      0.3404      0.3404    -84.9235     2.9870    -84.9235
     -1.0392      0.3667      0.3805      0.3805    -83.2858     2.6536    -83.2858
     -0.7794      0.3879      0.4096      0.4096    -80.3276     2.3308    -80.3276
     -0.5196      0.4023      0.4294      0.4294    -75.2952     2.0065    -75.2952
     -0.2598      0.4107      0.4410      0.4410    -67.4096     1.6657    -67.4096
      0.0         0.4135      0.4448      0.4448    -56.1814     1.2922    -56.1814
      0.2598      0.4107      0.4410      0.2952    -37.1395     0.0         0.0
      0.5196      0.4023      0.4294      0.2047     -1.6965     0.0         0.0
      0.7794      0.3879      0.4096      0.2120     34.9202     0.0         0.0
      1.0392      0.3667      0.3805      0.2493     54.9570     0.0         0.0
      1.2990      0.3376      0.3404      0.2670     65.0925     0.0         0.0
      1.5588      0.2982      0.2862      0.2493     70.8595     0.0         0.0
      1.8186      0.2435      0.2109      0.1911     74.5116     0.0         0.0
      2.0784      0.1584      0.0957      0.0904     77.0113     0.0         0.0

***** Y=     -0.2309
        X          TZH          TZK     ABS(TX,TY) ARG(TX,TY) ABS(VX,VY) ARG(VX,VY)
     -1.8186      0.1849      0.1399      0.1399    -85.6862     3.9349    -85.6862
     -1.5588      0.2527      0.2330      0.2330    -85.1429     3.4150    -85.1429
     -1.2990      0.2982      0.2956      0.2956    -84.3092     3.0261    -84.3092
     -1.0392      0.3308      0.3405      0.3405    -82.8873     2.6793    -82.8873
     -0.7794      0.3541      0.3726      0.3726    -80.3867     2.3505    -80.3867
     -0.5196      0.3698      0.3942      0.3942    -76.1317     2.0266    -76.1317
     -0.2598      0.3790      0.4068      0.4068    -69.3675     1.6948    -69.3675
      0.0         0.3820      0.4109      0.4109    -59.5139     1.3409    -59.5139
      0.2598      0.3790      0.4068      0.3707    -45.9109     0.0         0.0
      0.5196      0.3698      0.3942      0.2403    -23.0654     0.0         0.0
      0.7794      0.3541      0.3726      0.1872     10.2406     0.0         0.0
      1.0392      0.3308      0.3405      0.1876     38.2080     0.0         0.0
      1.2990      0.2982      0.2956      0.1896     54.3388     0.0         0.0
      1.5588      0.2527      0.2330      0.1648     63.4363     0.0         0.0
      1.8186      0.1849      0.1399      0.1029     69.0111     0.0         0.0
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***** Y=     -0.2694
      X          TZH           TZK       ABS(TX,TY) ARG(TX,TY) ABS(VX,VY) ARG(VX,VY)
   -1.5588     0.1849        0.1521       0.1521    -84.4823    3.5782    -84.4823
   -1.2990     0.2435        0.2327       0.2327    -83.6963    3.0991    -83.6963
   -1.0392     0.2825        0.2863       0.2863    -82.4903    2.7217    -82.4903
   -0.7794     0.3094        0.3234       0.3234    -80.5040    2.3791    -80.5040
   -0.5196     0.3273        0.3480       0.3480    -77.2072    2.0507    -77.2072
   -0.2598     0.3376        0.3622       0.3622    -71.9587    1.7242    -71.9587
    0.0        0.3410        0.3668       0.3668    -64.1800    1.3890    -64.1800
    0.2598     0.3376        0.3622       0.3622    -53.6684    1.0382    -53.6684
    0.5196     0.3273        0.3480       0.2846    -39.0531    0.0         0.0
    0.7794     0.3094        0.3234       0.1855    -16.2500    0.0         0.0
    1.0392     0.2825        0.2863       0.1385     12.8640    0.0         0.0
    1.2990     0.2435        0.2327       0.1150     36.8044    0.0         0.0
    1.5588     0.1849        0.1521       0.0792     51.7405    0.0         0.0
    1.8186     0.0675        0.0211       0.0125     60.7811    0.0         0.0

***** Y=     -0.3079
      X          TZH           TZK       ABS(TX,TY) ARG(TX,TY) ABS(VX,VY) ARG(VX,VY)
   -1.2990     0.1584        0.1319       0.1319    -83.0782    3.3061    -83.0782
   -1.0392     0.2135        0.2078       0.2078    -82.0067    2.8105    -82.0067
   -0.7794     0.2481        0.2554       0.2554    -80.4182    2.4291    -80.4182
   -0.5196     0.2701        0.2856       0.2856    -77.9553    2.0834    -77.9553
   -0.2598     0.2825        0.3027       0.3027    -74.1517    1.7515    -74.1517
    0.0        0.2865        0.3082       0.3082    -68.5338    1.4219    -68.5338
    0.2598     0.2825        0.3027       0.3027    -60.8286    1.0879    -60.8286
    0.5196     0.2701        0.2856       0.2856    -51.2779    0.7493    -51.2779
    0.7794     0.2481        0.2554       0.1864    -37.7482    0.0         0.0
    1.0392     0.2135        0.2078       0.1032    -17.7118    0.0         0.0
    1.2990     0.1584        0.1319       0.0496      7.7163    0.0         0.0
    1.5588     0.0002        0.0000       0.0          0.0      0.0         0.0

***** Y=     -0.3464
      X          TZH           TZK       ABS(TX,TY) ARG(TX,TY) ABS(VX,VY) ARG(VX,VY)
   -1.0392     0.0827        0.0514       0.0514    -81.5012    3.3606    -81.5012
   -0.7794     0.1510        0.1453       0.1453    -80.1367    2.5782    -80.1367
   -0.5196     0.1849        0.1920       0.1920    -78.2255    2.1538    -78.2255
   -0.2598     0.2026        0.2163       0.2163    -75.5179    1.7885    -75.5179
    0.0        0.2081        0.2239       0.2239    -71.7461    1.4411    -71.7460
    0.2598     0.2026        0.2163       0.2163    -66.7664    1.0954    -66.7664
    0.5196     0.1849        0.1920       0.1920    -60.7860    0.7374    -60.7860
    0.7794     0.1510        0.1453       0.1263    -53.5065    0.0         0.0
    1.0392     0.0827        0.0519       0.0273    -43.4534    0.0         0.0
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APPENDIX B

LISTING AND TEST PROBLEM FOR SUBRØUTINE FØRCES

(FØRTRAN IV G1 RELEASE 2.0)

```
      REAL NU
C
C
C     TO USE SUBROUTINE FORCES, THE PARAMETERS TRANSFERRED FROM THE MAIN
C     PROGRAM ARE (A,B,NU,UXN,UYN,PHN,NX,NY,DM) THE SUBROUTINE THEN
C     DETERMINES THE NORMALIZED RESULTANT FORCES FXN AND FYN.
C     THIS PROGRAM CONTAINS SUBROUTINES FORCES,MAAKZ,RCL,AND CONST
C
C
C                         ( A AND B ARE THE NORMALIZED CONTACT ELLIPSE
C                         DIMENSIONS, WHERE IF A1 AND B1 ARE THE ACTUAL
C                         DIMENSIONS THEN A=A1/SQRT(A1*B1) AND
C                         B=B1/SQRT(A1*B1). NOTE A/B >= 0.1
C                         NU IS POISSON'S RATIO. THIS IS THE ONLY
C                         INFORMATION NEEDED TO COMPUTE (INTERNALLY) FROM
C                         KALKER'S TABLES AND ASYMPTOTIC EXPANSIONS, ( SEE
C                         SUBROUTINE CONST ), THE LINEAR CREEPAGE AND SPIN
C                         COEFFICIENTS, CIJ, AND THE NORMALIZED MODULUS, GS.
C                         THE CONSTANT GS=G*(C***3)/(RHO*N),WHERE C=
C                         SQRT(A1*B1), 1/RHO=1/4*(1/R1+ + 1/R1- + 1/R2+ +
C                         1/R2-),AND N=RESULTANT NORMAL FORCE.
C                         THESE ARE USED IN THE PROGRAM TO COMPUTE THE INVERSE
C                         STIFFNESSES SX AND SY.
C                         THEN SX=8*A/(3*C11*GS), SY=8*A/(3*C22*GS),
C                         HC=32*SQRT(B/A)*C23/(3*PI*C22) AND PHNN=HC*PHN
C                         NOTE, THE OPERATION PHNN=HC*PHN IS DONE IN THE
C                         PROGRAM. THE NORMALIZED SPIN PHN=PH*RHO/MU IS
C                         THE VALUE TRANSFERED TO THE SUBROUTINE
C                         UXN AND UYN ARE NORMALIZED CREEPAGES, PHN
C                         IS THE NORMALIZED SPIN),
C                         UXN=UX*RHO/(MU*C), UYN=UY*RHO/(MU*C), PHN=PH*RHO/MU
C                         NX,NY (LATTICE POINTS IN CONTACT REGION,
C                         (I*A/NX,J*B/NY), -NX<I<NX, -NY<J<NY, ACCURACY
C                         INCREASES WITH INCREASING NX,NY TYPICAL VALUES NX
C                         =30,NY=10. MAXIMUM VALUES; NX,NY=40. FOR A/B=10
C                         NX=40,NY=10, FOR A/B=0.1 NX=10,NY=40, FOR A/B=1
C                         NX=NY=20, ARE TYPICAL VALUES.
C                         DM (AN INCREMENTAL STEP IN THE COMPUTATION,
C                         ACCURACY INCREASES WITH DECREASING DM, TYPICAL
C                         VALUE=0.02*A )
C                         THE NORMALIZED FORCES RETURNED ARE
C                         FXN=FX/(MU*N), FYN=FY/(MU*N)
C
C                   ***** NOTE: ALL VARIABLES HAVE BEEN NORMALIZED SUCH
C                   ***** THAT THE COEFFICIENT OF FRICTION, MU, DOES NOT
C                   ***** EXPLICITLY APPEAR.
C
C
      READ(1,*) NV1
      DO 999 I=1,NV1
      READ(1,*) A,B,NU,UXN,UYN,PHN,NX,NY,DM
       CALL FORCES(A,B,NU,UXN,UYN,PHN,NX,NY,DM,FXN,FYN)
      WRITE(3,850)A,B,NU,UXN,UYN,PHN,NX,NY,DM,FXN,FYN
  850 FORMAT(//,12X,'A=',1PE11.4,/,12X,'B=',1PE11.4,/,11X,'NU=',1PE11.4,
     $/,10X,'UXN=',1PE11.4,/,10X,'UYN=',1PE11.4,/,10X,'PHN=',1PE11.4,/,
     $11X,'NX=', I3    ,/,11X,'NY=', I3    ,/,11X,'DM=',1PE11.4,/,10X,'F
     $XN=',1PE11.4,/,10X,'FYN=',1PE11.4,//)
  999 CONTINUE
      STOP
      END
```

FORCES

```
C      SEE "SIMPLIFIED THEORY OF ROLLING CONTACT", BY J.J. KALKER
C      DELFT PROGR. REP., SERIES C: MECHANICAL AND AERONAUTICAL
C      ENGINEERING AND SHIPBUILDING, 1 (1973) PP.1-10
       SUBROUTINE FORCES(A,B,NU,UXN,UYN,PHN,NX,NY,DM,FXN,FYN)
C      ***** A AND B ARE THE NORMALIZED CONTACT ELLIPSE DIMENSIONS, NU
C      ***** IS POISSON'S RATIO. NOTE, A/B MUST BE .GE. 0.1
C      ***** UXN AND UYN ARE THE NORMALIZED CREEPAGES AND PHN IS THE
C      ***** NORMALIZED SPIN.  UXN=UX*RHO/(MU*C), UYN=UY*RHO/(MU*C),
C      ***** PHN=PH*RHO/MU
C      ***** NX AND NY ARE LATTICE POINTS IN THE CONTACT REGION,
C      ***** (I*A/NX,J*B/NY),-NX .GE. I .LE. NX, -NY .GE. J .LE. NY
C      ***** ACCURACY INCREASES WITH INCREASING NX,NY. TYPICAL VALUES
C      ***** NX=30,NY=10, MAXIMUM VALUES NX,NY=40.  DM IS A STEP
C      ***** ACCURACY INCREASES WITH DECREASING DM, TYPICAL VALUE=0.02*A
C      ***** NORMALIZED FORCES ARE RETURNED WHERE FXN=FX/(MU*N)
C      ***** FYN=FY/(MU*N) WITH N=RESULTANT NORMAL FORCE
       REAL X(81,81),Y(81,81),VX(81,81),VY(81,81),G(81,10)
        REAL CS(3),GEL(5)
       REAL CNT(5)
       REAL UX,UY,PH,SX,SY,A,B,MUZ,FX,FY,MZ,U,UB,UE,UD,DM,AB,AD,AE,TX,TY,
      $P,Q,ARG,GR,AL,NU
       INTEGER NX,NY,I,J,MX,MY,LY,MF,NS
       DATA X,Y,VX,VY,G/6561*0.0,6561*0.0,6561*0.0,6561*0.0,
      $ 810*0.0/
       CALL CONST(A,B,NU,C11,C22,C23,C33,GS)
       PI=3.14159
       GR=PI/180.0
       MX=3*NX
       MY=3*NY
       LY=3*NY
  1022 MUZ=3.0/(2.0*PI)
       SX=8.0*A/(3.0*C11*GS)
       SY=8.0*A/(3.0*C22*GS)
       HC=32.0*SQRT(B/A)*C23/(3.0*PI*C22)
  1027 GEL(5)=DM
       GEL(1)=SX
       GEL(2)=SY
       GEL(3)=A
       GEL(4)=B
       PHNN=HC*PHN
       CS(1)=UXN
       CS(2)=UYN
       CS(3)=PHNN
       CALL ROL(CS,GEL,MUZ,NX,NY,X,Y,VX,VY,G,FX,FY,MZ)
       FXN=FX
       FYN=FY
       RETURN
       END
```

MAAKZ

```
      SUBROUTINE MAAKZ(P,Q,WZ,DZ,D2Z,A,B,MUZ)
      REAL MUZ
      AL=0.9
      S=SQRT(1.0-AL*AL)
      AW=A*SQRT(1.0-Q*Q/B/B)
      PI=3.1415926536
      F=MUZ*PI/2.0/A/(ATAN(AL/S)+(2.0/3.0-AL+AL*AL*AL/3.0)/S)
      IF(P.LE.AL*AW) GO TO 10
      WZ=F*(AW*AW-P*P)/2.0/AW/S
      DZ=F*P/AW/S
      D2Z=F/AW/S
      GO TO 11
 10   WZ=F*SQRT(AW*AW-P*P)-F/2.0*AW*S
      DZ=F*P/SQRT(AW*AW-P*P)
 11   RETURN
      END
```

```
      SUBROUTINE ROL(CS,GEL,MUZ,NX,NY,X,Y,VX,VY,G,FX,FY,MZ)
      REAL MUZ,MZ,K,MU
      INTEGER PIJL
      REAL LE(81)
      INTEGER E(81)
      COMMON A,B
      DIMENSION X(81,81),Y(81,81),VX(81,81),VY(81,81),
     $ G(81,10),CS(3),GEL(5)
      UX=CS(1)
      UY=CS(2)
      PH=CS(3)
      DM=GEL(5)
      SX=GEL(1)
      SY=GEL(2)
      A=GEL(3)
      B=GEL(4)
      H=A/FLOAT(NX)
      K=B/FLOAT(NY)
      PI=3.1415926536
      CZP=MUZ*2.0/A/A
      J=-NY
380   J=J+1
      IF(J.GT.NY-1)GO TO 100
      LE(J+NY+1)=A*SQRT(1.0-FLOAT(J*J)/FLOAT(NY)/FLOAT(NY))
      P=LE(J+NY+1)
      E(J+NY+1)=0.99*P/H
      I=-NX-1
401   I=I+1
      IF(I.GT.NX)GO TO 200
      IF(I.LT.-E(J+NY+1).OR.I.GT.E(J+NY+1))GO TO 15
      GO TO 10
15    VY(I+NX+1,J+NY+1)=0.0
      VX(I+NX+1,J+NY+1)=0.0
      Y(I+NX+1,J+NY+1)=0.0
      X(I+NX+1,J+NY+1)=0.0
10    GO TO 401
200   CONTINUE
      DO 20 I=1,10
20    G(J+NY+1,I)=-3.0*A
      GO TO 380
100   CONTINUE
      THV=-1.0
      IF(UX-PH*B.GE.0.0) THV=1.0
      THV=THV*ATAN(UY/(ABS(UX-PH*B)+1.E-08))+PI/2.0*(1.0-THV)
C
      J=-NY
470   J=J+1
      IF(J.GT.NY-1)GO TO 300
      C=J*K
      P=LE(J+NY+1)
      PG=P
      CUX=UX-PH*C
      PIJL=2
      CUY=UY+PH*P
      CALL MAAKZ(P,C,ZN,CZ,CZP,A,B,MUZ)
      XG=0.0
      YG=0.0
      XV=0.0
```

```
      YV=0.0
      VV=0.0
      I=E(J+NY+1)
      G(J+NY+1,1)=1.0
      IF(CZ*CZ-CUX*CUX/SX/SX-CUY*CUY/SY/SY.GT.0.0) GO TO 1001
C     SLIP AT THE LEADING EDGE. DETERMINE T(HETA).
      G(J+NY+1,1)=-1.0
      T=-1.0
      IF(CUX.GE.0.0) T=1.0
      T=T*ATAN(CUY/(ABS(CUX)+1.E-08))+PI/2.0*(1.0-T)
C BEPT
1002  S=SIN(T)
      C=COS(T)
      NU=(CUX*S-CUY*C-CZ*(SX-SY)*C*S)/(CUX*C+CUY*S-CZ*(SX-SY)*(C*C-S*S))
      T=T-NU
      IF(ABS(NU).GT.1.E-03) GO TO 1002
      THV=T
      C=COS(T)
      S=SIN(T)
C     THE STARTING VALUE OF T HAS BEEN FOUND. THE
C     DERIVATIVE IS DETERMINED IN A SPECIAL WAY.
      TP=(PH*C+CZP*(SX-SY)*C*S)/(CUX*C+CUY*S-CZ*((3.0*C*C-1.0)*(SX-SY)
     $ -SX))
C NEXTGL
1003  D=-DM
      IF(P-DM.LE.I*H+1.E-06) D=I*H-P
      PN=P+D
      CALL MAAKZ(PN,G,ZN,CZ,CZP,A,B,MUZ)
      TN=T+D*TP
      S=SIN(TN)
      C=COS(TN)
      CUY=UY+PH*PN
      TPN=(CUX*S-CUY*C-CZ*C*S*(SX-SY))/ZN/(SY*C*C+SX*S*S)
      T=T+0.5*D*(TP+TPN)
      S=SIN(T)
      C=COS(T)
      XN=ZN*C
      YN=ZN*S
      V=CUX*SY*C+CUY*SX*S-CZ*SX*SY
      IF(V.GE.-4.E-05) GO TO 1004
      AN=VV/(VV-V)
      IF(ABS(VV).LT.1.E-10) AN=0.5
      VV=0.0
      AV=1.0-AN
      G(J+NY+1,PIJL)=AV*P+AN*PN
      PG=G(J+NY+1,PIJL)
      PIJL=PIJL+1
      P=PG
      IF(PIJL.GT.10)PIJL=10
      XV=AV*XV+AN*XN
      XG=XV
      YV=AV*YV+AN*YN
      YG=YV
      GO TO 1006
1004  CONTINUE
C     SLIP IN THE NEW POINT
      XV=XN
      YV=YN
```

```
      P=PN
C SLIP
1005  TP=(CUX*S-CUY*C-CZ*C*S*(SX-SY))/ZN/(SY*C*C+SX*S*S)
      IF(P.GT.I*H+1.E-06) GO TO 1003
      X(I+NX+1,J+NY+1)=XN
      Y(I+NX+1,J+NY+1)=YN
      XP=-ZN*S*TP-CZ*C
      YP=ZN*C*TP-CZ*S
      VX(I+NX+1,J+NY+1)=CUX+SX*XP
      VY(I+NX+1,J+NY+1)=CUY+SY*YP
      VV=V
      I=I-1
      IF(I.LT.-E(J+NY+1)) GO TO 1007
      GO TO 1003
C ADH
1001  T=-1.0
      IF(CUX.GT.0.0) T=1.0
      THV=T*ATAN((UY+PH*P)/(ABS(CUX)+1.E-08))+PI*(1.0-T)/2.0
C ADHERE
1006  PN=I*H
      XN=CUX*(PG-PN)/SX+XG
      YN=(UY+0.5*PH*(PG+PN))*(PG-PN)/SY+YG
      CALL MAAKZ(PN,Q,ZN,CZ,CZP,A,B,MUZ)
      V=ZN-SQRT(XN*XN+YN*YN)
      IF(V.GE.(-4.E-05)*MUZ)GO TO 1008
      AN=VV/(VV-V)
      IF(ABS(VV).LT.1.E-10) AN=0.9
      AV=1.0-AN
      P=AV*P+AN*PN
      G(J+NY+1,PIJL)=P
      VV=0.0
      PIJL=PIJL+1
      XV=AV*XV+AN*XN
      YV=AV*YV+AN*YN
      IF(PIJL.GT.10) PIJL=10
      T=-1.0
      IF(XV.GE.0.0) T=1.0
      CUY=UY+PH*P
      T=T*ATAN(YV/(ABS(XV)+1.E-08))+PI/2.0*(1.0-T)
      C=COS(T)
      S=SIN(T)
      CALL MAAKZ(P,Q,ZN,CZ,CZP,A,B,MUZ)
      GO TO 1005
1008  VV=V
      X(I+NX+1,J+NY+1)=XN
      XV=XN
      Y(I+NX+1,J+NY+1)=YN
      YV=YN
      VY(I+NX+1,J+NY+1)=0.0
      VX(I+NX+1,J+NY+1)=0.0
      I=I-1
      P=PN
      IF(I.GE.-E(J+NY+1)) GO TO 1006
1007  GO TO 470
300   CONTINUE
C     THE ARRAYS ARE FILLED. THE INTEGRALS ARE
C     DETERMINED
      T=4.0*K/3.0
```

```
      TN=2.0*K
      FX=0.0
      FY=0.0
      MZ=0.0
      YN=2.0*H
      J=-NY
940   J=J+1
      IF(J.GT.NY-1)GO TO 400
      I=E(J+NY+1)
      PIJL=I
      P=(LE(J+NY+1)-I*H)/2.0-H/3.0
      YP=J*K
      C=P*(X(I+NX+1,J+NY+1)+X(NX+1-I,J+NY+1))
      S=P*(Y(I+NX+1,J+NY+1)+Y(NX+1-I,J+NY+1))
      D=P*I*H*(Y(I+NX+1,J+NY+1)-Y(NX+1-I,J+NY+1))
      P=2.0*H/3.0
      I=-PIJL-1
500   I=I+1
      IF (I.GT.PIJL)GO TO 550
      C=C+P*X(I+NX+1,J+NY+1)
      D=D+P*I*H*Y(I+NX+1,J+NY+1)
      S=S+P*Y(I+NX+1,J+NY+1)
      P=YN-P
      GO TO 500
550   FX=FX+T*C
      FY=FY+T*S
      MZ=MZ+T*(D-YP*C)
      T=TN-T
      GO TO 940
400   CONTINUE
      RETURN
      END
```

```
      SUBROUTINE CONST(A,B,NU,C11,C22,C23,C33,GS)                       00002710
      DIMENSION AL(5),BE(5),D(15),E(15,20),AR(20),CNT(5),D1(15,9),      00002720
     $D2(15,9),D3(15),D4(15)                                            00002730
C      ***** DATA E(I,J) GIVES LINEAR CREEPAGE AND SPIN COEFFICIENTS****00002740
C      *****AND GS FROM KALKER REPORT TABLE 1 **********************    00002750
C      ***** VALID FOR A/B GREATER THAN OR EQUAL TO 0.1                 00002760
      REAL NU                                                           00002770
      DATA D1/                                                          00002780
     $  2.51, 3.31, 4.85, 2.51, 2.52, 2.53, 0.334, 0.473, 0.731, 6.42,  00002790
     $ 8.28, 11.7, 0.7670, 0.5752, 0.3835,                              00002800
     $ 2.59, 3.37, 4.81, 2.59, 2.63, 2.66, 0.483, 0.603, 0.809, 3.46,   00002810
     $ 4.27, 5.66, 0.5608, 0.4206, 0.2804,                              00002820
     $ 2.68, 3.44, 4.80, 2.68, 2.75, 2.81, 0.607, 0.715, 0.889, 2.49,   00002830
     $ 2.96, 3.72, 0.4779, 0.3584, 0.2390,                              00002840
     $ 2.78, 3.53, 4.82, 2.78, 2.88, 2.98, 0.720, 0.823, 0.977, 2.02,   00002850
     $ 2.32, 2.77, 0.4343, 0.3257, 0.2172,                              00002860
     $ 2.88, 3.62, 4.83, 2.88, 3.01, 3.14, 0.827, 0.929, 1.07, 1.74,    00002870
     $ 1.93, 2.22, 0.4089, 0.3066, 0.2044,                              00002880
     $ 2.98, 3.72, 4.91, 2.98, 3.14, 3.31, 0.930, 1.03, 1.18, 1.56,     00002890
     $ 1.68, 1.86, 0.3934, 0.2950, 0.1967,                              00002900
     $ 3.09, 3.81, 4.97, 3.09, 3.28, 3.48, 1.03, 1.14, 1.29, 1.43,      00002910
     $ 1.50, 1.60, 0.3840, 0.2880, 0.1920,                              00002920
     $ 3.19, 3.91, 5.05, 3.19, 3.41, 3.65, 1.13, 1.25, 1.40, 1.34,      00002930
     $ 1.37, 1.42, 0.3785, 0.2839, 0.1892,                              00002940
     $ 3.29, 4.01, 5.12, 3.29, 3.54, 3.82, 1.23, 1.36, 1.51, 1.27,      00002950
     $ 1.27, 1.27, 0.3758, 0.2818, 0.1879/                              00002960
      DATA D2/                                                          00002970
     $ 3.40, 4.12, 5.20, 3.40, 3.67, 3.98, 1.33, 1.47, 1.63, 1.21,      00002980
     $ 1.19, 1.16, 0.3750, 0.2812, 0.1875,                              00002990
     $ 3.51, 4.22, 5.30, 3.51, 3.81, 4.16, 1.44, 1.59, 1.77, 1.16,      00003000
     $ 1.11, 1.06, 0.3758, 0.2818, 0.1879,                              00003010
     $ 3.65, 4.36, 5.42, 3.65, 3.99, 4.39, 1.58, 1.75, 1.94, 1.10,      00003020
     $ 1.04, 0.954, 0.3785, 0.2839, 0.1892,                             00003030
     $ 3.82, 4.54, 5.58, 3.82, 4.21, 4.67, 1.76, 1.95, 2.18, 1.05,      00003040
     $ 0.965, 0.852, 0.3840, 0.2880, 0.1920,                            00003050
     $ 4.06, 4.78, 5.80, 4.06, 4.50, 5.04, 2.01, 2.23, 2.50, 1.01,      00003060
     $ 0.892, 0.751, 0.3934, 0.2950, 0.1967,                            00003070
     $ 4.37, 5.10, 6.11, 4.37, 4.90, 5.56, 2.35, 2.62, 2.96, 0.958,     00003080
     $ 0.819, 0.650, 0.4089, 0.3066, 0.2044,                            00003090
     $ 4.84, 5.57, 6.57, 4.84, 5.48, 6.31, 2.88, 3.24, 3.70, 0.912,     00003100
     $ 0.747, 0.549, 0.4343, 0.3257, 0.2172,                            00003110
     $ 5.57, 6.34, 7.34, 5.57, 6.40, 7.51, 3.79, 4.32, 5.01, 0.868,     00003120
     $ 0.674, 0.446, 0.4779, 0.3584, 0.2390,                            00003130
     $ 6.96, 7.78, 8.82, 6.96, 8.14, 9.79, 5.72, 6.63, 7.89, 0.828,     00003140
     $ 0.601, 0.341, 0.5608, 0.4206, 0.2804/                            00003150
      DATA D3/                                                          00003160
     $ 10.7, 11.7, 12.9, 10.7, 12.8, 16.0, 12.2, 14.6, 18.0, 0.795,     00003170
     $ 0.562, 0.228, 0.7670, 0.5752, 0.3835 /                           00003180
      DATA D4/                                                          00003190
     $ 11.08, 12.01, 13.10, 11.08, 13.38, 16.90, 13.72, 16.34, 20.20,   00003200
     $ 0.785, 0.552, 0.208, 0.7918, 0.5938, 0.3959/                     00003210
      DATA AR / 0.1,0.2,0.3,0.4,0.5,0.6,0.7,0.8,0.9,1.0,1.111111,       00003220
     $1.25,1.428571,1.666667,2.0,2.5,3.333333,5.0,10.0,11.0/            00003230
      DO 6 I=1,15                                                       00003240
      DO 5 J=1,9                                                        00003250
      E(I,J)=D1(I,J)                                                    00003260
    5 E(I,J+9)=D2(I,J)                                                  00003270
      E(I,19)=D3(I)                                                     00003280
```

```
6    E(I,20)=D4(I)                                                     00003290
     PI=3.14159                                                        00003300
     RG=A/B                                                            00003310
     IF(RG.GT.AR(20)) GO TO 14                                         00003320
     GO TO 15                                                          00003330
14   SG=B/A                                                            00003340
     GAM=ALOG(16.0/(SG*SG))                                            00003350
     C11=2.0*PI/(SG*(GAM-2.0*NU))*(1.0+( 1.613706    )/(GAM-2.0*NU))   00003360
     C22=(( 1.613706 )*(1.0-NU))/(2.0*NU+GAM*(1.0-NU))                 00003370
     C22=2.0*PI*(1.0+C22)/(SG*(2.0*NU+GAM*(1.0-NU)))                   00003380
     C23=2.0*PI/((SQRT(SG)*SG*3.0)*((1.0-NU)*GAM-2.0+4.0*NU))          00003390
     C33=PI/4.0*(GAM*(1.0-2.0*NU)-2.0+6.0*NU)/(GAM*(1.0-NU)-2.0+4.0*NU)00003400
     GS=3.0*(1.0-NU)/(4.0*PI*SQRT(SG))                                 00003410
     GO TO 80                                                          00003420
15   DO 20 I=2,20                                                      00003430
     IF(RG.LE.AR(I)) GO TO 25                                          00003440
20   CONTINUE                                                          00003450
25   J=I                                                               00003460
     DO 30 I=1,15                                                      00003470
30   D(I)=E(I,J-1)+(E(I,J)-E(I,J-1))*(RG-AR(J-1))/(AR(J)-AR(J-1))      00003480
     DO 40 I=1,5                                                       00003490
     AL(I)=8.0*(D(3*I)-2.0*D(3*I-1)+D(3*I-2))                          00003500
     BE(I)=2.0*(-D(3*I)+4.0*D(3*I-1)-3.0*D(3*I-2))                     00003510
40   CNT(I)=AL(I)*NU**2+BE(I)*NU+D(3*I-2)                              00003520
     C11=CNT(1)                                                        00003530
     C22=CNT(2)                                                        00003540
     C23=CNT(3)                                                        00003550
      C33=CNT(4)                                                       00003560
     GS=CNT(5)                                                         00003570
80   CONTINUE                                                          00003580
     RETURN                                                            00003590
     END                                                               00003600
```

```
   A=  2.5980E+00
   B=  3.8490E-01
  NU=  2.8000E-01
 UXN=  0.0
 UYN=-1.4000E+00
 PHN=  8.0000E-01
  NX=  10
  NY=  10
  DM=  4.0000E-02
 FXN=  8.9034E-07
 FYN=-3.4703E-01
```

```
   A=  2.5980E+00
   B=  3.8490E-01
  NU=  2.8000E-01
 UXN=  0.0
 UYN=-1.4000E+00
 PHN=  8.0000E-01
  NX=  40
  NY=  20
  DM=  4.0000E-02
 FXN=  8.6427E-07
 FYN=-3.4689E-01
```

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