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USER'S MANUAL FOR PROGRAM CONWHEEL
(CONFORMAL WHEEL-RAIL CONTACT STRESS PRESSURES)

TECHNICAL REPORT NO. 10

BY

B. PAUL AND S. SINGH

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16. Abstract

CONWHEEL (CONformal WHEEL-rail contact stress problems) is an all FORTRAN computer program for the solution of normal contact stresses between two closely conforming (possibly nonHertzian) smooth elastic bodies. It can be used to determine: the boundary of the interface contact region; pressure distribution; stress within the critical subsurface region.

CONWHEEL is a much enhanced version of an earlier program CONFORM. The important changes include:

1. Calculation of subsurface stress and location of critically stressed points.
2. The rail and wheel profiles may now be specified either from engineering drawings (as before); or from tabulated offsets for the wheel and rail profiles (new).
3. The need to run two preliminary programs has been eliminated, resulting in a significantly reduced amount of user effort.
4. No FORTRAN coding or subroutine preparation is required of the user.
5. CONWHEEL is approximately 2.5 times the size of CONFORM, but is much more convenient to work with.

This manual includes: a brief description of the method of analysis, program structure, instructions for problem modelling, input preparation, and solution of sample problems.

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USER'S MANUAL FOR PROGRAM CONWHEEL[1]

(CONformal WHEEL-rail contact stress problems)

by

B. Paul[2] and S. Singh[3]

Department of Mechanical Engineering and Applied Mechanics
University of Pennsylvania
Philadelphia, PA 19104

[1]Supported by Federal Rail Administration under Contract DTFR53-81-C-00227

[2]Professor of Mechanical Engineering

[3]Research Fellow

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(CONFORMal WHEEL-rail contact stress problems)

Abstract

CONWHEEL (CONFORMal WHEEL-rail contact stress problems) is an all FORTRAN computer program for the solution of normal contact stresses between two closely conforming (possibly nonHertzian) smooth elastic bodies. It can be used to determine the following:

i) The boundary of the interface contact region

ii) The interface pressure distribution within the contact region.

iii) The state of stress within the critical subsurface region in the body

CONWHEEL is a much enhanced version of an earlier program CONFORM. The important changes are:

1. It is now possible to calculate subsurface stress states, and to determine those points which are most critical from the point of view of plastic flow or fatigue.

2. The rail and wheel profiles may now be specified either from engineering drawings (as before); or from a set of tabulated values of offsets for the wheel and rail profiles (new).

3. The need to run two preliminary programs, MIDSEP and INTERPEN, prior to running CONFORM has been eliminated, together with the need to do a preliminary graphical analysis based on their results. These preliminary analyses have been incorporated into CONWHEEL in such a way as to significantly reduce the amount of data that the user must prepare.

4. The program now specifically includes the IBM equation solver subroutine DGELG, instead of the subroutine LEQT1F which was not available to all potential users. This makes the program self-standing, and independent of any proprietary subroutine packages.

5. No FORTRAN coding or subroutine preparation is required of the user.

6. CONWHEEL is approximately 2.5 times the size of CONFORM, but is much more convenient to work with.

This manual includes: a brief description of the method of analysis, program structure, instructions for problem modelling, input preparation, and solution of sample problems.

1. PROGRAM SPECIFICATIONS

PROGRAM: CONWHEEL (CONformal WHEEL-rail contact stress problems)

AUTHORS: B. Paul and S. Singh

LANGUAGE: FORTRAN IV

MACHINES: Tested on Univac 90/70 (using BG-4 compiler)

PRECISION: Double precision

STORAGE REQUIREMENTS: 240 K Bytes

CAPABILITIES: CONWHEEL calculates the following, for a given rigid-body approach:

1. The shape of the contact region
2. The distribution of contact pressure
3. The resultant interfacial force and moments
4. The subsurface stresses.

NUMBER OF CARDS: Approximately 2,500.

2. PURPOSE

The program CONWHEEL is intended to solve the interface and subsurface contact stress problem for two elastic bodies of closely conforming surface geometry, brought into elastic contact under a given rigid-body approach. This program is a major enhancement of, and supersedes, program CONFORM*.

3. METHOD OF ANALYSIS

The program first solves for the normal contact pressure, and the contact region, and then it finds the subsurface stresses. The detailed theory of the analysis is given in Paul and Hashemi [1981], Paul and Hashemi [1980], Hashemi and Paul [1979], and Paul and Singh [1982].

3.1 Initial separation:

Figure 1 shows two elastic bodies (1 and 2) with arbitrary and closely conforming surface geometries. Let x_1, y_1, z_1 and x_2, y_2, z_2 be two coordinate systems with origins 0_1 and 0_2 respectively located on body 1 and 2. The bodies are brought in contact such that 0_1 and 0_2 touch at 0. A new coordinate system (x,y,z) is drawn at 0 such that z is the direction of the common normal to the surface, pointing into body 2. Then the <u>initial separation</u> of the two bodies along the z axis is defined by

$$f(x,y) = z_2(x,y) - z_1(x,y) \tag{3.1}$$

*See Paul and Hashemi [1978-a].

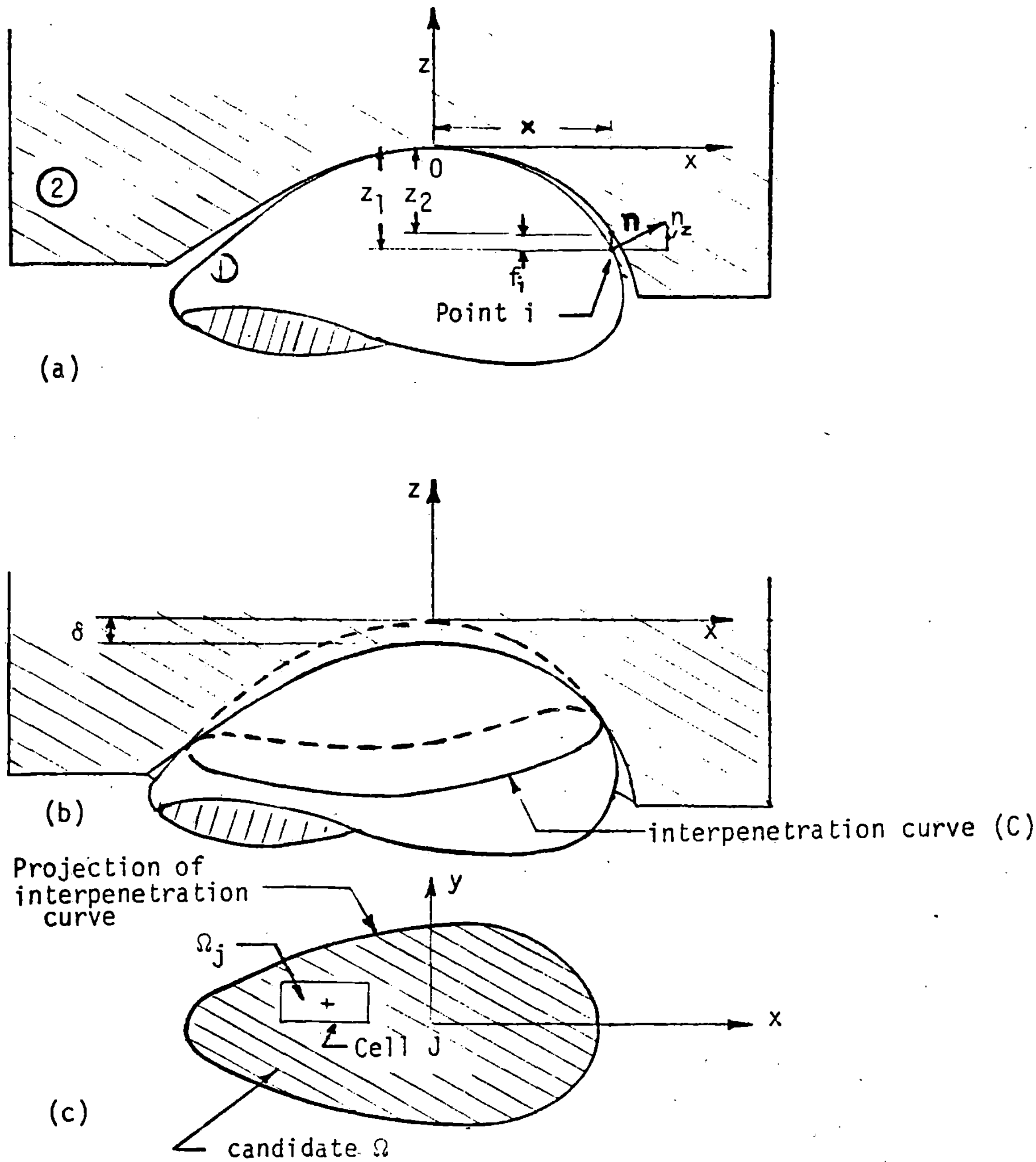

Fig. 1. Two bodies in conformal contact
(a) prior to deformation
(b) virtual penetration
(c) initial candidate contact patch

If the separation given by (3.1) is zero at 0 and positive everywhere else, the bodies contact at a single point, otherwise a possibility of multiple contact exists. This program only considers the case of singly-connected contact regions. As a first estimate of the contact region boundary, the program automatically finds the interpenetration curve -- defined below.

3.2 Interpenetration Curve:

If the bodies 1 and 2 undergo a rigid body relative displacement δ, the point of contact spreads over a region called the contact patch. An excellent first estimate of the contact patch boundary is the fictitious space curve of intersection which would arise if the bodies were to penetrate each other by the displacement δ as shown in Figures 1(b) and 1(c). The projection of this space curve on the x-y plane is given by setting the initial separation equal to δ in Eq. (3.1); i.e.

$$f(x,y) = \delta \tag{3.2}$$

The curve defined by eq. (3.2) will be referred to as the interpenetration curve, and it is determined by subroutine INTPEN.

3.3 Initial Meshwork:

The main section of the program viz. subroutine CONFOM accepts the interpenetration curve, as a first estimate of the contact region boundary and refines it by iteration to find the true contact patch. The estimated contact region is divided into bands, strips and cells as illustrated in Fig. 2 and described further in Sec. 4.

3.4 Contact Pressure and Boundary Iterations:

For the currently defined meshwork of n cells, the pressure in cell i is treated as a constant p_i. The program automatically generates and solves a set of n equations for the n values of p_i. Then it checks to see whether the pressures are all positive within the contact patch. If not, the contact patch boundary is adjusted by a scheme described in Paul and Hashemi [1981], and Paul and Hashemi [1979]. The procedure is repeated until the governing equations are satisfied, and the variation of the contact patch boundary is within a specified tolerance limit for two consecutive iterations. At this point, the boundary of the contact patch, and the interface pressure distribution over it is determined.

3.5 Subsurface Stresses:

Stresses beneath, but close to the surface, are determined by subroutine SUBSIG, which is based on the analysis given in Paul and Singh [1982]. The user may specify a surface point whose neighborhood is to be probed, he may ask the program to probe beneath the surface point where the contact pressure is maximum, or he may specify arbitrary subsurface locations to be examined. For all subsurface points specified, the program will calculate all six stress components ($\sigma_{\bar{x}}, \sigma_{\bar{y}}, \sigma_{\bar{z}}, \sigma_{\bar{x}\bar{y}}, \sigma_{\bar{y}\bar{z}}, \sigma_{\bar{z}\bar{x}}$) referred to the global axes ($\bar{x},\bar{y},\bar{z}$) defined in Sec. 5.4. In addition the program will calculate at such points the equivalent stress σ_{eq}. The significance of this term is that the body undergoes permanent (plastic) deformation when σ_{eq} reached the yield stress of the material.

The choice of subsurface points to be evaluated for stresses depends upon the value of a user supplied constant IOPT. If:

IOPT = 0, the program probes beneath the surface point where the contact pressure is maximum.

IOPT = 1, the program probes beneath a surface point ($\bar{x},\bar{y}$) specified by the user.

IOPT = 2, the program probes at a particular set of points ($\bar{x},\bar{y},\bar{z}$) specified by the user

IOPT = 3, no subsurface stresses are evaluated.

4. DISCRETIZATION OF THE CONTACT PATCH

When dealing with the initially estimated contact region, or any subsequent iteration thereof, the current region is subdivided into a meshwork of rectangular cells as follows:

1. The $\bar{x}$-diameter is divided into any number (NSEG) of segments called bands (see Fig. 2). The ratio of the width of a typical band I, to that of the $\bar{x}$-diameter is designated by RAT(I).
2. Each band I is further divided in NX(I) strips.
3. Each strip is divided into cells of width HY(J) in the $\bar{y}$-direction.

The user has the option of choosing the number of bands NSEG, the ratios RAT(I), and the number of strips NX(I) in each band. However, he may also elect to accept the default values of these mesh parameters, which are:

NSEG = 3

RAT(1) = 0.2, RAT(2) = 0.6, RAT(3) = 0.2

NX(1) = 4 , NX(2) = 5 , NX(3) = 4

These values were used in drawing Fig. 2.

The vertical widths HY(J) of the cells are automatically chosen by the program to make the cells nearly square in shape and to always have at least three cells in each strip with one cell always centered on the x axis. It is suggested that the default values of the mesh parameters be used for all problems. If the mesh so generated is unsatisfactory (e.g. not enough cells in certain regions) the user can rerun the problem with different mesh parameters.

The interpenetration curve is taken as the first approximation of the contact patch boundary as shown in Fig. 2. The program performs iterations which continually change the contact boundary.

The program will consider that convergence has occurred when the relative change in y_{max} is less than a certain fraction EPS*. The user may chose his own value for EPS or he may accept the default value of EPS = 0.01.

5. WHEEL AND RAIL GEOMETRY

A "wheel profile" is the curve traced out by the intersection of the wheel surface and any plane through the axis of the wheel. Any point on the wheel profile can be specified by coordinates x_w (axial direction) and z_w (radial direction) relative to axes fixed in the wheel as shown in Fig. 3. Similarly, the profile of the railhead is the curve of intersection of the rail surface with a plane transverse to the rail axis. The coordinates of the "rail profile" are x_r and z_r relative to axes fixed in the rail as shown in Fig. 4.

5.1 Data From Engineering Drawings:

The standard new rail or wheel profile is a collection of contiguous straight lines and circular arcs (see Figs. 3 and 4). The following Notation is used for the parameters which define the profile of segment number I:

---

* y_{max} is the largest strip length in the contact patch. For example, y_{max} is the value of $\bar{y}$ at the top of strip number 10 in Fig. 2.

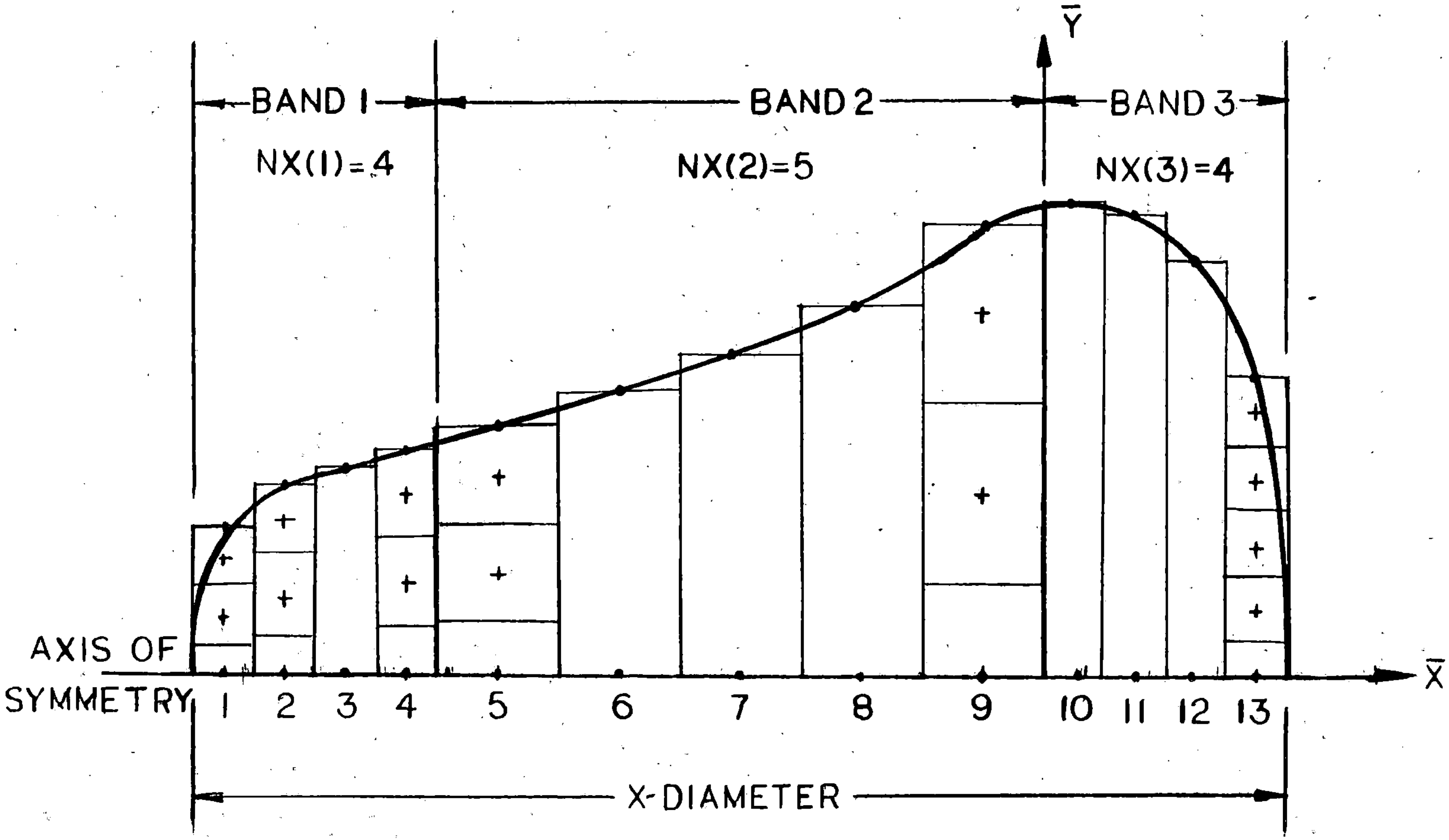

Figure 2. Mesh arrangment for sample interpenetration curve. (Using the defaulted values.) User may alter the mesh if he so desires.

FORTRAN name for parameters of Rail	Wheel	Meaning for Circular Arc	Straight Line
AR(I)	AW(I)	x-coord. of center	slope $\frac{dz}{dx}$
BR(I)	BW(I)	y-coord. of center	z intercept
CR(I)	CW(I)	radius	no meaning (leave blank)
XCR(I)	XCW(I)	x-cordinate of right-hand end of segment (whether circular or straight)	

When the above information is to be read in from engineering drawings (such as Fig. 4) the FORTRAN constant NOFF should be set to zero (or blank).

5.2 Data from Table of Offsets:

When the rail and wheel profiles are to be supplied as pairs of (x,z) coordinates (e.g. when readings are made with a profilometer) the input constant NOFF must be set equal to 1, and the number of points to be described on the wheel (NW) and on the rail (NR) are also to be entered (see Card B in Sec. 7). The FORTRAN names corresponding to point I are:

	x-coordinate	z-coordinate
For Wheel:	XOFFW(I)	ZOFFW(I)
For Rail:	XOFFR(I)	ZOFFR(I)

These values are entered on Cards F-2 for the rail and on cards G-2 for the wheel (see Sec. 7). When offsets are specified the program automatically converts the offset data into circular arcs or straight lines between offset points.

5.3 Location of Initial Point of Contact

Figure 5 shows a wheel and rail in contact (under zero load) at a point C. The location of point C on the wheel is defined by the x_w-coordinate x_w^c [FORTRAN name XWC] as shown in Figs. 5 and 3(a). Similarly, the x_r coordinate of point C on the railhead is denoted by x_r^c [XRC] as shown in Figs. 5 and 4(a). The corresponding Z values are denoted by ZWC and ZRC. The values of XWC and XRC are set by the user at any values that are geometrically meaningful; the values of ZWC and ZRC are then calculated internally by the program, but are not needed directly by the user.

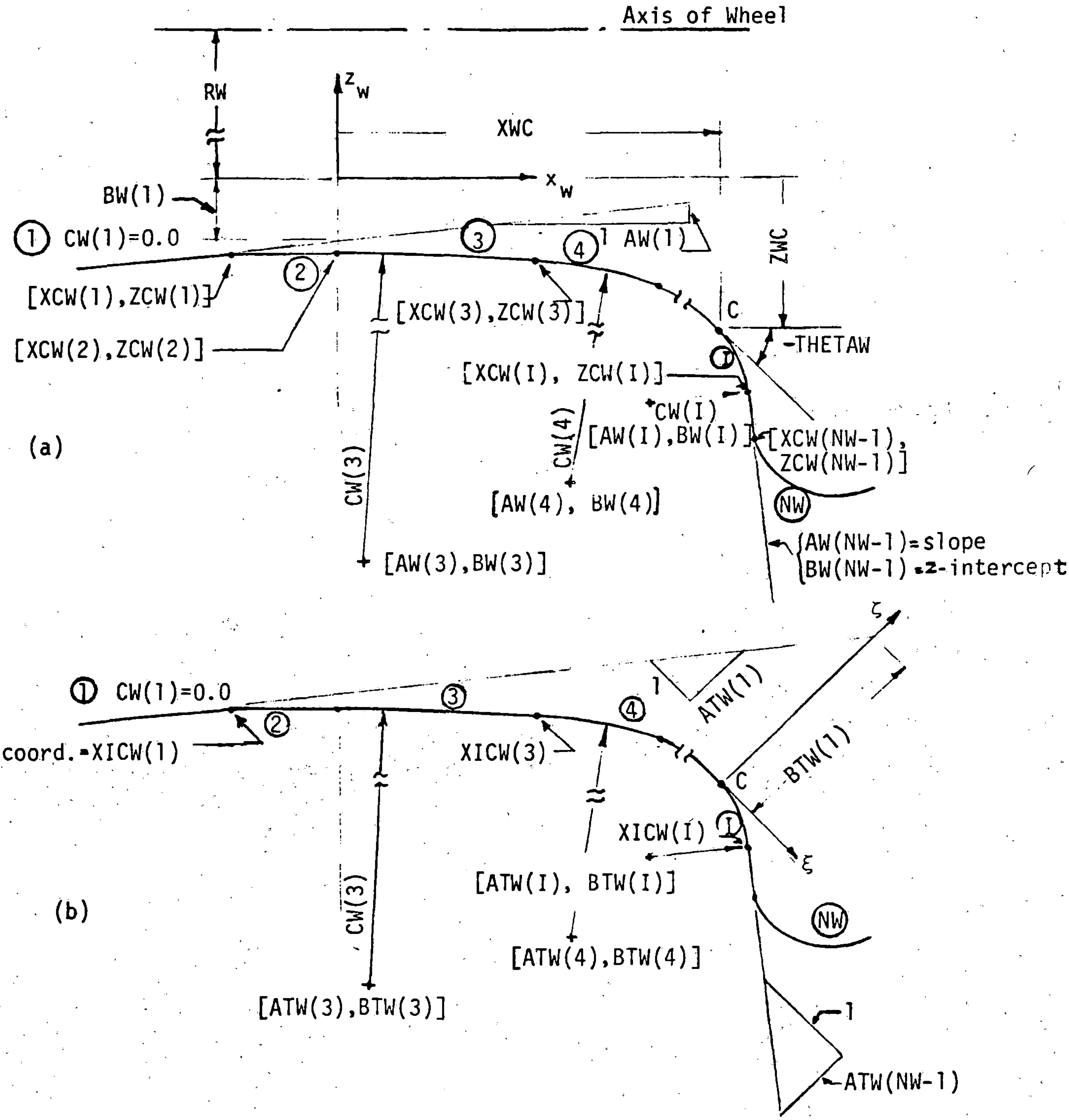

Fig. 3 Standard wheel parameters. Note: wheel axes are x_w, y_w, z_w, where x_w is parallel to wheel axis, and the global axes are ξ, ζ, η, where ζ is normal to the wheel at C and η is parallel to y.

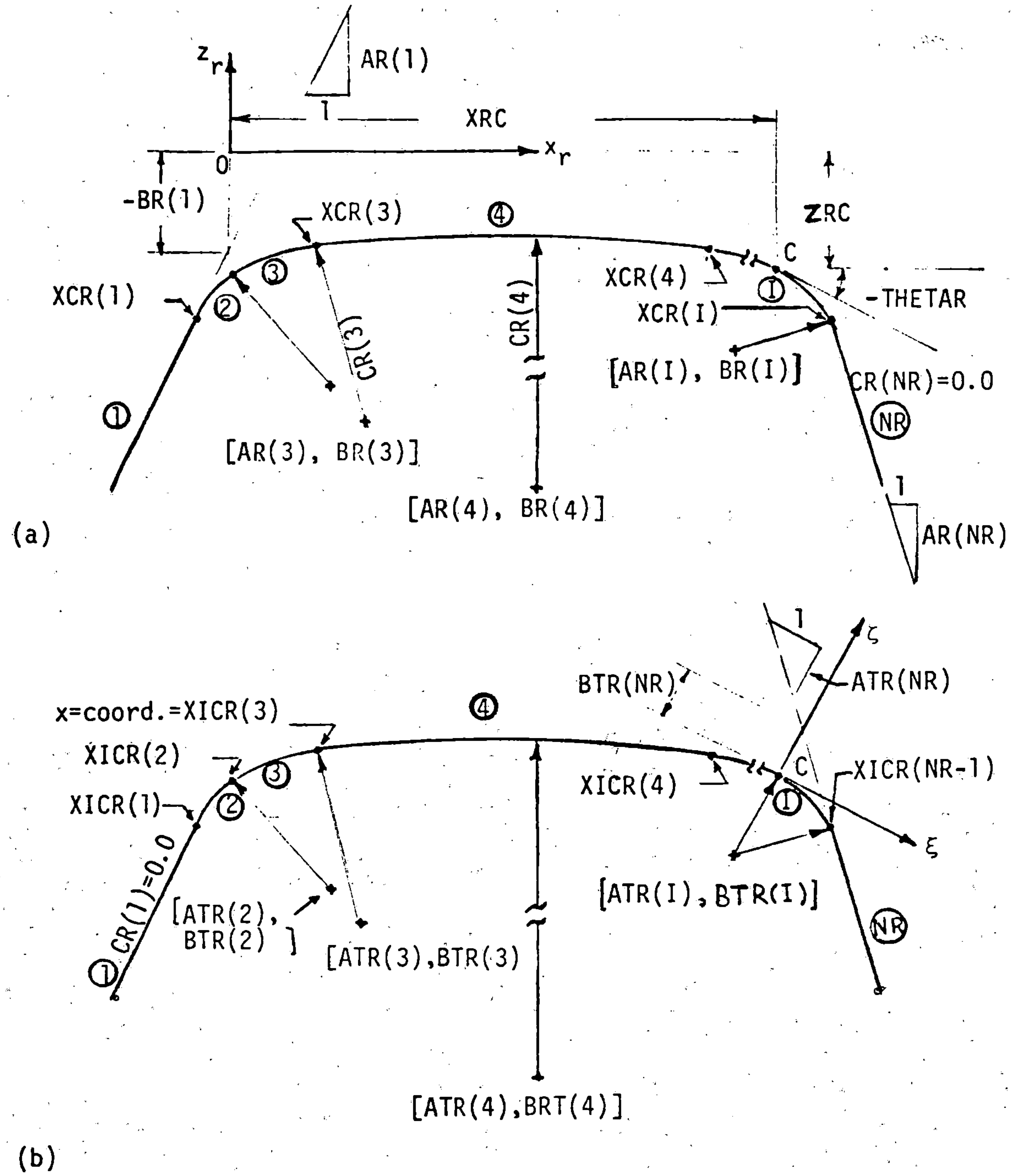

Fig. 4 Standard rail parameters. Note: Rail reference axes are x_R, y_R, z_R, where y_R is parallel to the track direction. The global axes are ξ, ζ, η, where ζ is normal to the rail at C and η is parallel to y_R.

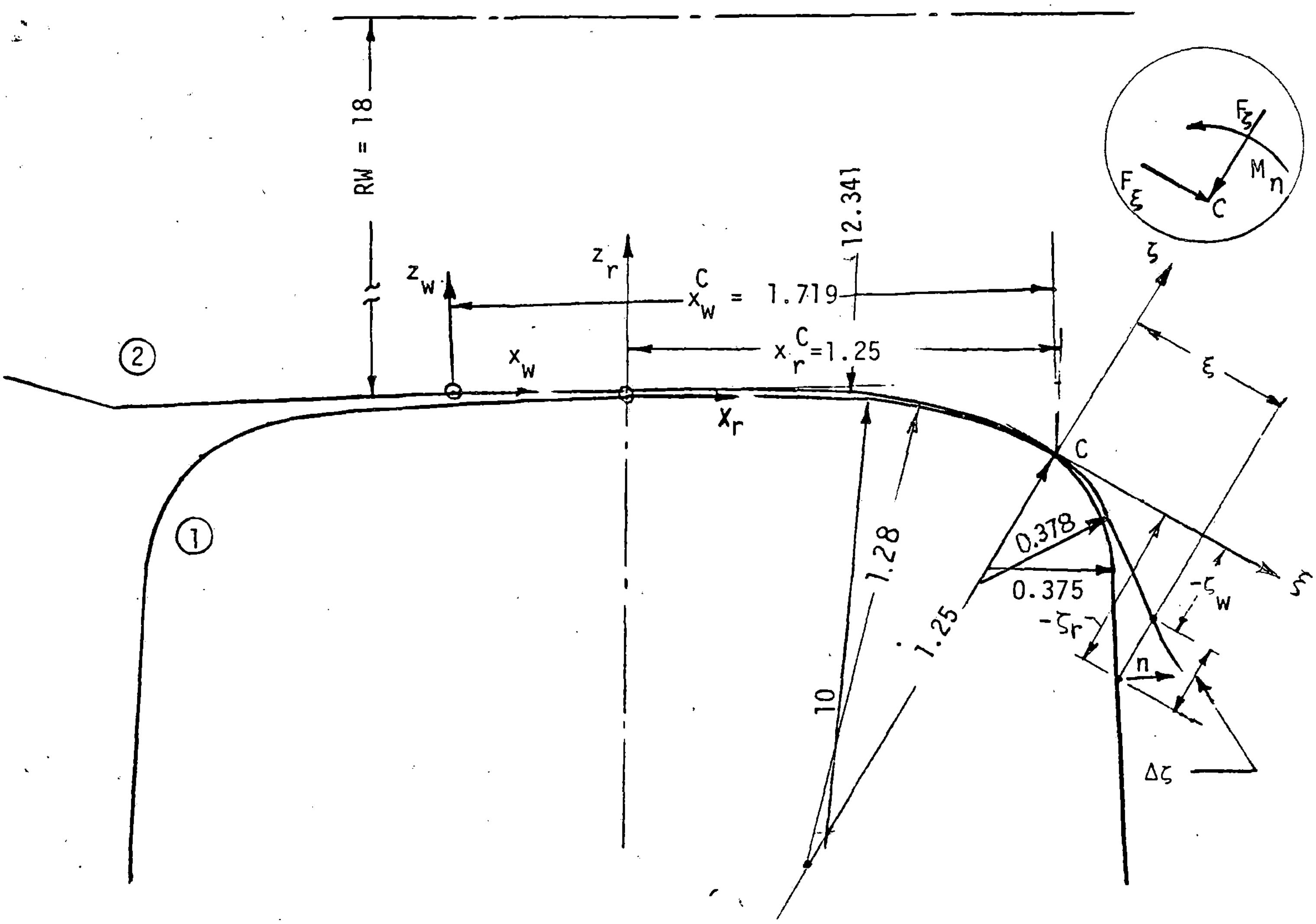

Fig. 5. Rail and wheel position, before deformation under applied load
(x_w, z_w) wheel reference coordinate system
(x_R, z_R) rail reference coordinate system
(ξ, ζ) global coordinate system

5.4 Global Coordinates $(\xi,\eta,\zeta) \doteq (\bar{x},\bar{y},\bar{z})$

Once the initial contact point C has been defined by the user's choice of x_w^c and x_r^c, it is possible to establish a global (fixed set of orthogonal coordinate axes (ξ,η,ζ) with origin at point C. As shown in Fig. 5, ζ points normally outward from the railhead, ξ is tangent to the rail (and wheel) profile, and η is parallel to the rail axis, such that (ξ,η,ζ) form a right-handed system. Occasionally, the coordinates (ξ,η,ζ) will be designated as $(\bar{x},\bar{y},\bar{z})$. At numerous points in the analysis, the program will have to transform quantities from global to local coordinates and vice-versa. The user need not make any such transformations (they are done automatically where needed). For general information it is shown in Fig. 3(b) how the ("transformed") wheel parameters ATW(4) and BTW(4), referred to as global coordinates are related to the "untransformed" parameters AW(4), BW(4) shown in Fig. 4(a), for profile segment number 4. The transformed parameter CTW(4) is the radius of arc segment number 4, which is the same as the untransformed radius CW(4). The program is set up such that the user need only supply the untransformed profile parameters (Figs. 3(a) and 4(a) for both wheels and rails.

6. STRUCTURE OF THE PROGRAM CONWHEEL

The main program CONWHEEL reads in the required input data and controls the flow of information between all the subprograms. In MIDSEP, the input parameters for wheel and rail coordinates are transformed to the global coordinates for the problem, and the initial separation function is determined in the"mid-plane", i.e. with $y = 0$ in Eq. (3.1). Subroutine INTPEN determines the interpenetration curve defined in SEc. 3.2. Then subroutine CONFOM is the heart of the program, as it governs the iteration process on the true contact boundary and the contact pressure distribution. Subroutine SUBSIG finds the subsurface stresses. A brief description of all subprograms follows:

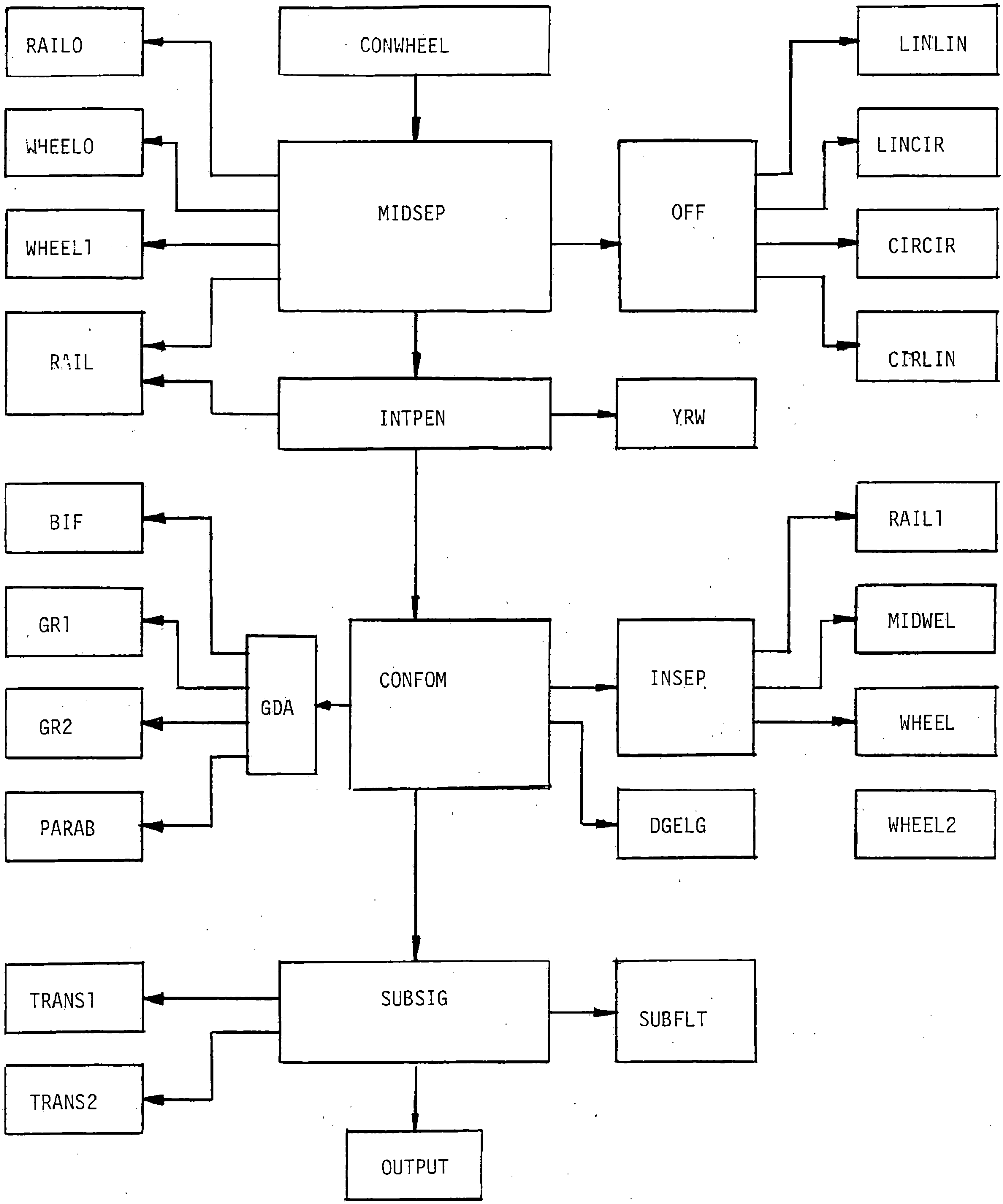

Figure 6. Struture of CONWHEEL
Arrows run from calling subprogram to called subprogram

CONWHEEL	The main program controls the flow of information to and from subprograms.
MIDSEP	Subroutine to determine the transformed rail-wheel profile parameters, and the midplane separation.
RAILO	Subroutine to determine z and $\frac{dz}{dx}$ of the rail profile with reference to the rail reference coordinates (x_r, z_r)
WHEELO	Subroutine to determine the z and $\frac{dz}{dx}$ of the wheel profile (at any point in the x-z plane with reference to the wheel reference coordinates (x_w, z_w)
RAIL	Subroutine to find the ζ component of the rail profile for given ξ
WHEEL1	Subroutine to find the ζ component of the wheel-profile for given ξ
INTPEN	Subroutine to determine the interpenetration curve for a given rigid-body approach
YRW	Subfunction to determine the η coordinate of the wheel surface for given ξ and ζ
CONFOM	Subroutine to determine the contact patch boundary, the interface pressure, and the corresponding force and moment components
BIF*	Subfunction to find $\int G dA$ when the integrand is singular
GDA*	Subfunction to find $\int G(\underset{\sim}{x};\underset{\sim}{x}')dA'$
GR1*	Subfunction to find $G(\underset{\sim}{x};\underset{\sim}{x}')$ for body 1 (Rail)
GR2*	Subfunction to find $G(\underset{\sim}{x};\underset{\sim}{x}')$ for body 2 (Wheel)
INSEP	Subroutine to find the separation between the two points $\underset{\sim}{x}_1$ and $\underset{\sim}{x}_2$ along with the unit normal components at $\underset{\sim}{x}_1$
MIDWEL	Subfunction to find ζ-component of the wheel for any ξ in midplane

*The symbol G (which appears in the explanation of subroutines BIF, GDA, GR1 and GR2) is a Green's function that is described in Paul and Hashemi [1981].

PARAB	Subfunction to interpolate or extrapolate between two points
RAIL1	Subroutine to find ζ-component of a point on surface 1 (Rail) along with components of normal to the surface 1 (Rail) along with the components of the normal to the surface at that point
WHEEL	Subfunction to find ζ component of any point on wheel surface
WHEEL2	Subroutine to find z and $\frac{dz}{dx}$ of the wheel profile at any point with coordinate x in mid-plane with respect to wheel reference coordinates (x_w, z_w)
DGELG*	Subroutine to solve the linear algebraic equations
SUBSIG	Subroutine to determine the state of stress at a subsurface point within the body
SUBFLT	Subroutine to determine the state of stress at a point in a semi-infinite body
TRANS1	Subroutine to determine the elements of the transformation matrix from global coordinate system to local coordinate system
TRANS2	Subroutine to transform the stress tensor from local coordinate system to global coordinate system
OFF	Subroutine to reduce data of wheel and rail from values of offsets, to parameters of straight line or circular arc segments
LINLIN	Subroutine to determine the segment parameters when the profile transition occurs from a straight line to another straight line
LINCIR	Subroutine to determine the segment parameters when the profile transition occurs from a straight line to a circular arc
CIRCIR	Subroutine to determine the segment parameters when the profile transition occurs from a circular arc to another circular arc
CIRLIN	Subroutine to determine the segment parameters when the profile transition occurs from a circular arc to a straight line

*IBM

7. INPUT PREPARATION FOR CONWHEEL

Input consists of the following punched cards with the field format indicated in parentheses*. Although not needed by the user the FORTRAN names of the input variables are given in brackets.

(A) Title Card (20A4)

(B) Program control card (8I5,F10.0)

Col. 1-5 [NW] No. of segments in wheel profile (see Fig. 3) if NOFF=0; or no. of offset values specified for wheel profile if NOFF=1

Col. 6-10 [NR] No. of segments in rail profile (see Fig. 4) if NOFF=0; or no. of offset values specified for rail profile if NOFF=1

Col. 11-15 [NOFF] NOFF=0 (or blank) when wheel and rail profile parameters are specified as in Figs. 3 and 4. NOFF=1 when offsets are given

Col. 16-20 [NSEG] No. of bands along the x axis (see Fig. 2). Use 0 or blank for default. See Sec. 4 for discussion of default parameters.

Col. 21-25 [ITM] Maximum number of iterations [=10 by default when left blank or set = 0]

Col. 26-30 [IOPT] Variable controlling the location at which the sub-surface stresses are to be calculated

If IOPT=0 (or blank), the program determines the location of maximum surface pressure, and calculates the stresses beneath this point, up to a depth equal to the $\bar{x}$-diameter of the contact zone, at as many points as the number of cells along the $\bar{x}$ axis. When this option is exercised, no additional cards are required.

If IOPT=1, the program calculates the stresses at $(\bar{x},\bar{y})$ locations specified by the user, at as many points below the surface and up to a depth specified by the user in the card group [H1]

---

*Integer names begin with letters I,J,K,L,M,N. On input integers they are to be right justified in field of 5 characters. Real constants may be placed anywhere in a field of 10. The user's decimal point will override the specified "F10.0" used in the program.

If IOPT=2, the program calculates the stresses at($\bar{x}$,$\bar{y}$,$\bar{z}$) location specified by the user in card group H2.

If IOPT=3, no subsurface stresses are calculated.

Col. 31-35 [IBUG] Variable controlling the output. When left blank or made = 0, only the final results of interface pressures and subsurface stresses are printed, otherwise the full output of transformed rail wheel coordinates, midplane separation, interpenetration curve, interface pressures, and subsurface stresses are printed out (see examples in sec. 9).

Col. 41-50 [EPS] Relative tolerance for convergence (0.01 by default if left blank) (See sec. 4)

(C) Approach, Nominal Wheel Radius, elasticity parameters, convergence tolerance (8F10.0)

Col. 1-10 [DELTA] Displacement (or rigid body approach δ)

Col. 11-20 [XWC] x coordinate of initial point of contact on wheel, in wheel reference coordinate system (see Figs. 3 and 5)

Col. 21-30 [XRC] x coordinate of initial point of contact on rail, in rail reference coordinate system (see Figs. 4 and 5)

Col. 31-40 [RW] Reference radius of the wheel (see Fig. 3)

Col. 41-50 [E1] Young's modulus of elasticity for wheel

Col. 51-60 [E2] Young's modulus of elasticity for rail

Col. 61-70 [ANU1] Poisson's ratio for wheel

Col. 71-80 [ANU2] Poisson's ratio for rail

(D) Mesh Generating Card (16I5)

This card to be omitted for default case (NSEG=0)

Col. 1-5 [NX(1)] No. of strips in band no. 1

Col. 6-10 [NX(2)] No. of strips in band no. 2

....

(E) Mesh Generating Card (8F10.0) See Sec. 4

Omit this group of cards when default is desired (NSEG=0)

Col. 1-10 [RAT(1)] Ratio of band 1 width to total width

Col. 11-20 [RAT(2)] Ratio of band 2 width to total width

....

(Eight values are entered per card. Use as many cards as needed.)

(F.1) Rail Profile Parameter Group (when NOFF=0); See Sec. 5.1:

1st card (8F10.0)

Col. 1-10 [AR(1)]

Col. 11-20 [BR(1)]

Col. 21-30 [CR(1)]

Col. 31-40 [XCR(1)]

Col. 41-50 [AR(2)]

Col. 51-60 [BR(2)]

Col. 61-70 [CR(2)]

Col. 71-80 [XCR(2)]

2nd card (8F10.0)

Col. 1-10 [AR(3)]

....

....

Two sets of (AR,BR,CR,XCR) are entered per card. Use as many cards as needed.

(F.2) Rail Profile by Offsets (when NOFF=1); see Sec. 5.2

1st card (8F10.0)

Col. 1-10 [XOFFR(1)] X-offset (i.e. x-coordinate) For point 1 on rail

Col. 11-20 [ZOFFR(1)] Z-offset (i.e. z-coordinate) For point 1 on rail

Col. 21-30 [XOFFR(2)]

Col. 31-40 [ZOFFR(2)]

....

Col. 71-80 [ZOFFR(4)]

2nd card (8F10.0)

Col. 1-10 [XOFFR(5)]

....

....

Col. 71-80 [ZOFFR(8)]

Use as many cards as needed, with eight values per card. Last card can have fewer than eight values.

(G.1) Wheel Profile Specification (when NOFF=0); see Sec. 5.1:

1st card (8F10.0) (See Fig. 3 for definitions)

Col. 1-10 [AW(1)]
Col. 11-20 [BW(1)]
Col. 21-30 [CW(1)]
Col. 31-40 [ZCW(1)]
Col. 41-50 [AW(2)]
Col. 51-60 [BW(2)]
Col. 61-70 [CW(2)]
Col. 71-80 [XCW(2)]

2nd card

Col. 1-10 [AW(3)]

....

Two sets of (AW,BW,CW, XCW) are entered per card. Use as many cards as needed.

(G.2) Wheel Profile by Offsets (when NOFF=1); see Sec. 5.2:

1st card (8F10.0)

Col. 1-10 [XOFFW(1)] X-offset (i.e. x-coordinates) for Point 1 on wheel
Col. 11-20 [ZOFFW(1)] Z-offset (i.e. z-coordinates) for Point 1 on wheel
Col. 21-30 [XOFFW(2)]
Col. 31-40 [ZOFFW(2)]

....

Col. 71-80 [ZOFFW(4)]

2nd card (8F10.0)

Col. 1-10 [XOFFW(5)]

....

Col. 71-80 ZOFFW(8)]

Use as many cards as needed, with eight values per card.

(H.1) Specification of $(\bar{x},\bar{y})$ Location of Depth Probe required with IOPT=1

Card 1 (I10)

Col. 1-10	[NP]	No. of points along the depth at which the stresses are to be calculated.

Card 2 (3F10.4)

Col. 1-10	[XFM]	The $\bar{x}$-coordinate of the location at which the depth must be probed.
Col. 11-20	[YFM]	The $\bar{y}$-coordinate of the location at which the depth must be probed.
Col. 21-30	[ZFM]	The maximum value of the depth to be probed.

(H.2) Specification of $\bar{x},\bar{y},\bar{z}$ Location Card Group [required when IOPT=2]

Card 1 (I10)

Col. 1-10	[NP]	No. of points at which the stresses are to be calculated

Card 2 (6F10.0)

Col. 1-10	[XFM]	x-coordinate of a point
Col. 11-20	[YFM]	y-coordinate of a point
Col. 21-30	[ZFM]	z-coordinate of a point
Col. 31-40	[XFM]	x-coordinate of the next point
Col. 41-50	[YFM]	y-coordinate of the next point
Col. 51-60	[ZFM]	z-coordinate of the next point

Two sets of XFM, YFM, ZFM values are specified per card. Use as many cards as needed.

8. COMPUTER PROGRAM OUTPUT

The program gives the results (depending on the print options used) in the following format:

1. The first section gives an echo of the primary input data.
2. The second section (optional if IBUG=1) gives the results of subroutine MIDSEP and contains:
   - Wheel parameters referred to global coordinates (see section 5.4)
   - Rail parameters
   - The midplane separation, i.e. table of ξ and $\Delta\zeta$ (see Fig. 5) denoted by XI and DELTA ZETA.
3. The third section (optional; if IBUG=1) gives the results of subroutine INTPEN and contains:
   - The value of rigid body approach (repeated for convenience)
   - The value of the leftmost (XBL) and rightmost (XBR) x-coordinates of the interpenetration curve.
   - The interpenetration curve, i.e. table of ξ-η (denoted by XI-ETA) coordinates.
4. The fourth section (standard) gives the results of normal interface pressures calculated by subroutine CONFOM and contains
   - The iteration number
   - Co-ordinates of the boundary of contact patch, i.e. table of ξ-η (XI,ETA) coordinates
   - The surface pressure distribution, i.e. table of ξ,η,ζ (XI,ETA, ZETA) coordinates and p (normal pressure)

- The resultant forces in normal (ζ) and tangential (ξ) directions (ZETA-FORCE, XI-FORCE) and moment about η direction (ETA-MOMENT)
- The left and right boundary of the contact patch (XBL,XBR)

5. The last section gives results of subsurface stresses calculated by subroutine SUBSIG and contains:
   - (optional; if IBUG=1) Coordinates of surface points and the components of applied tractions acting at these points.
   - Location of field points $[(\bar{x},\bar{y},\bar{z} = (\xi,\eta,\zeta)]$ probed
   - $\sigma_{\bar{x}\bar{x}},\sigma_{\bar{y}\bar{y}},\sigma_{\bar{z}\bar{z}},\sigma_{\bar{x}\bar{y}},\sigma_{\bar{y}\bar{z}},\sigma_{\bar{z}\bar{x}}$ stresses at the points probed
   - The equivalent stress σ_{eq} at the points probed (see Sec. 3.5).

Samples of computer output are shown in Sec. 9.

9. EXAMPLES

The use of the program is illustrated by two examples.

The profiles of the rail and wheel selected are given in Fig. 7. The wheel is positioned on the railhead so that the initial contact takes place at C, as shown in Fig. 5. Observe that there is a jump in curvature at the initial point of contact. The wheel and rail are made of steel with the following elastic properties

$E = 30 \times 10^6$ psi	Modulus of Elasticity
$\nu = 0.3$	Poisson's ratio

9.1 Example 1

In the first example the rigid body approach assumed is δ = 0.0005", and the full output is requested. The data is as follows:

Card (A) Rail and Wheel Contact Stress Analysis - Example 1 CONWHEEL

Card (B) NW=5 NR=7 NOFF=0 NSEG=0 ITM=0
IOPT=0 IBUG=1 EPS=0

Card (C) DELTA=0.0005 XWC=1.719 XRC=1.25 RW=180
E1=30000000.0 E2=30000000.0 ANU1=0.3 ANU2=0.3

Card (D) Not required as NSEG=0

Card (E) Not required as NSEG=0

Card (F1)

AR(1)=14.32809	BR(1)=20.03716	CR(1)= 0.0	XCR(1)=-1.43284
AR(2)=-1.05874	BR(2)=-0.518812	CR(2)= 0.375	XCR(2)=-1.25
AR(3)=-0.6125	BR(3)=-1.271464	CR(3)=10.0	XCR(3)=-0.7
AR(4)= 0.0	BR(4)=-10.0	CR(4)=10.0	XCR(4)= 0.7
AR(5)= 0.6125	BR(5)=-1.271464	CR(5)= 1.25	XCR(5)= 1.25
AR(6)= 1.05875	BR(6)=-0.518812	CR(6)= 0.325	XCR(6)= 1.43284
AR(7)=-14.32809	BR(7)=20.03716	CR(7)= 0.0	XCR(7)= 1.500

Card (G1)

AW(1)= 0.0	BW(1)= 0.0	CW(1)= 0.0	XCW(1)=0.0
AW(2)= 0.0	BW(2)=-12.341	CW(2)=12.341	XCW(2)=1.171
AW(3)=1.049545	BW(3)=-1.329907	CW(3)= 1.28	XCW(3)=1.719
AW(4)=1.521301	BW(4)=-0.5611092	CW(4)= 0.378	XCW(4)=1.876
AW(5)=-2.714595	BW(5)= 4.662134	CW(5)= 0.0	XCW(5)=2.000

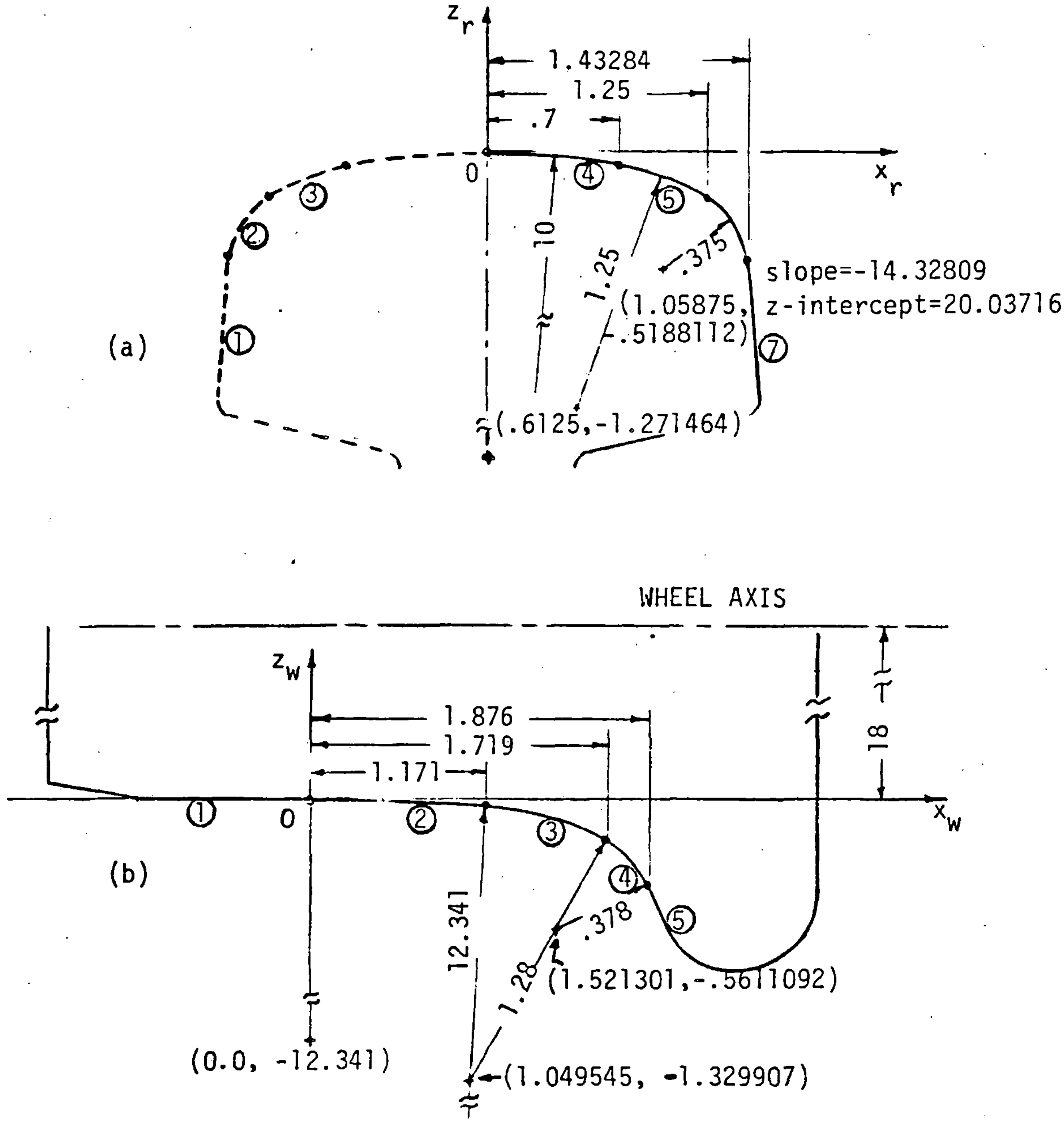

Fig. 7. Examples of standard wheel and rail

(a) Rail-140 RE

(b) Metroliner wheel (SIG Schweizerische Industrie-Gesellschaft)

The input data deck is illustrated in Fig.8.

The output of the program is illustrated in Fig. 9 (a through h).

Figure 9(a) and (b) illustrate the output of subroutine MIDSEP, giving the transformed wheel and rail parameters, and the table of ξ vs $\Delta\zeta$.

Figure 9(c) illustrates the output of the subroutine INTPEN giving the values of the rigid body approach, the extremities of the interpenetration curve, and the table of ξ-η the coordinates of the interpenetration curve.

Fig.9(d) (e) illustrate the output of subroutine CONFOM, giving the iteration number, the coordinates (ξ-η) of the contact region and the table of surface point coordinates (ξ,η,ζ) and the associated contact pressures, along with the values of net force and moments.

Figure 9(f) and (g) illustrate the output of subroutine SUBSIG. 9(f) gives the location and tractions of the source cells, while 9(g) gives the state of stress at a series of subsurface points along the depth ($-\zeta$) axis, at $\xi=0$, $\eta=0$.

Figure 9(h) gives a graphical interpretation of the results.

9.2 Example 2

In the second example, the rigid body approach is taken to be 0.003, and only results of final interface pressures and subsurface stresses is requested.

Figure 10 illustrates the input data deck, and Figures 11(a through d) illustrate the results as before.

```
ECON

EXAMPLE # 1. RAIL WHEEL CONTACT STRESS ANALYSIS (CONWHEEL)
    5      7      0      0      0      0      1               0.0
 0.0005    1.719      1.25        18.0       30000000. 30000000. 0.3        0.3
 14.328609  20.0376    0.0        -1.43284   -1.05874   -0.518812  0.375      -1.25
-0.6125    -1.271464   1.25       -0.7        0.0       -10.0      10.0        0.7
 0.6125    -1.271464   1.25        1.25       1.05875   -0.518812  0.375      1.433
-14.328609  20.03716   0.0         1.50
 0.0        0.0        0.0         0.0        0.0       -12.341    12.341     1.171
 1.049545  -1.329907   1.28        1.719      1.521301  -0.561109  0.378      1.876
-2.714595   4.662134   0.0         2.0
```

Fig. 8. Illustrating input data deck for example 1 with approach $\delta = 0.0005$

```
        EXAMPLE 1. RAIL WHEEL CONTACT ANALYSIS

WHEEL PARAMETERS

       XWC=  0.1719000D 01   ZWC= -0.2389304D 00   THETAW= -0.5503807D 00

       AW              BW              CW                     AW             BW             CW
 0.0000000D 00   0.0000000D 00   0.0000000D 00       0.0000000D 00 -0.1234100D 02  0.1234100D 02
 0.1049545D 01  -0.1329907D 01   0.1280000D 01       0.1521301D 01 -0.5611090D 00  0.3780000D 00
-0.2714595D 01   0.4662134D 01   0.0000000D 00

      ATW             BTW              ATW             BTW                ATW             BTW
 0.6136291D 00   0.2803276D 00     0.4864377D 01 -0.1121396D 02     0.0000000D 00 -0.1280000D 01
-0.6991739D-06  -0.3780000D 00    -0.7881318D 00  0.1032862D 00

      XCW             ZCW              XCW             ZCW                XCW             ZCW
 0.0000000D 00   0.0000000D 00     0.1171000D 01 -0.5568193D-01     0.1719000D 01 -0.2389304D 00
 0.1876000D 01  -0.4304466D 00

      XICW            XICW            XICW            XICW            XICW            XICW
-0.1590111D 01  -0.5629154D 00   0.0000000D 00   0.2339804D 00

RAIL PARAMETERS

      XICR            XICR            XICR            XICR            XICR            XICR
-0.2156518D 01  -0.2150432D 01  -0.1764914D 01  -0.5606702D 00   0.0000000D 00   0.3084683D 00

      ATR             BTR             CR                      ATR            BTR            CR
-0.1990748D 01  -0.5916336D 01   0.0000000D 00      -0.1821410D 01 -0.1454922D 01  0.3750000D 00
-0.1053713D 01  -0.1874751D 01   0.1250000D 01       0.3924697D 01 -0.9070440D 01  0.1000000D 02
 0.0000000D 00  -0.1250000D 01   0.1250000D 01       0.1122411D-05 -0.3750019D 00  0.3750000D 00
-0.1446545D 01   0.2844557D 00   0.0000000D 00

       XRC=  0.1250000D 01   ZRC= -0.1962444D 00   THETAR= -0.5351848D 00

       AR              BR               AR             BR                  AR             BR
 0.1432809D 02   0.2003716D 02    -0.1058740D 01 -0.5188120D 00    -0.6125000D 00 -0.1271464D 01
 0.0000000D 00  -0.1000000D 02     0.6125000D 00 -0.1271462D 01     0.1058750D 01 -0.5188120D 00
-0.1432809D 02   0.2003716D 02

      XCR             XCR             XCR             XCR             XCR             XCR
-0.1432840D 01  -0.1250000D 01  -0.7000000D 00   0.7000000D 00   0.1250000D 01   0.1432840D 01

   XIL= -0.9000000D 00    XIR=  0.4500000D 00    N= 0    NW= 5    NR= 7
```

Fig. 9(a). Part 1 of output giving the transformed wheel and rail parameters

XI	DELTA ZETA	XI	DELTA ZETA
-0.9000000D 00	0.9376436D-02	-0.8100000D 00	0.7472630D-021
-0.7200000D 00	0.5816925D-02	-0.6300000D 00	0.4401972D-021
-0.5400000D 00	0.3175119D-02	-0.4500000D 00	0.2100070D-021
-0.3600000D 00	0.1294192D-02	-0.2700000D 00	0.7077204D-031
-0.1800000D 00	0.3084434D-03	-0.9000000D-01	0.7622710D-041
-0.2775558D-16	-0.2220446D-15	0.4500000D-01	0.2338348D-041
0.9000000D-01	0.9106515D-04	0.1350000D 00	0.2149423D-031
0.1800000D 00	0.4166019D-03	0.2250000D 00	0.7422489D-031
0.2700000D 00	0.5251054D-02	0.3150000D 00	0.2623064D-011
0.3600000D 00	0.5585923D-01	0.4050000D 00	0.8548782D-011
0.4500000D 00	0.1151164D 00	0.4950000D 00	0.1447450D 001

Fig. 9(b). Part 2 of output giving the separation $\Delta\zeta$ vs ξ.

```
EXAMPLE 1. RAIL WHEEL CONTACT ANALYSIS (CONWHEEL)

DELTA= 0.50000D-03      XBL=-0.22810D 00      XBR= 0.19380D 00

WHEEL PARAMETERS

              XWC=  0.1719000D 01  ZWC= -0.2389304D 00  TETAW= -0.5503807D 00

        AW               BW               CW                           AW               BW               CW
 0.0000000D 00   0.0000000D 00   0.0000000D 00          0.0000000D 00 -0.1234100D 02   0.1234100D 02
 0.1049545D 01  -0.1329907D 01   0.1280000D 01          0.1521301D 01 -0.5611090D 00   0.3780000D 00
-0.2714595D 01   0.4662134D 01   0.0000000D 00

       XCW              XCW              XCW              XCW              XCW              XCW
 0.0000000D 00   0.1171000D 01   0.1719000D 01   0.1876000D 01

RAIL PARAMETERS

       XICR             XICR             XICR             XICR             XICR             XICR
-0.2156518D 01  -0.2150432D 01  -0.1764914D 01  -0.5606702D 00   0.0000000D 00   0.3084683D 00

       ATR              BTR              CR                           ATR              BTR              CR
-0.1990748D 01  -0.5916376D 01   0.0000000D 00        -0.1821410D 01  -0.1454922D 01   0.3750000D 00
-0.1053713D 01  -0.1874751D 01   0.1250000D 01         0.3924697D 01  -0.9070440D 01   0.1000000D 02
 0.0000000D 00  -0.1250000D 01   0.1250000D 01         0.1122411D-05  -0.3750019D 00   0.3750000D 00
-0.1446545D 01   0.2844557D 00   0.0000000D 00

  RW=  0.1800000D 02    NW= 5    NR= 7

        XI               ETA                   XI               ETA
  -0.2280991D 00   0.0000000D 00       -0.2052892D 00   0.6125932D-01
  -0.1824793D 00   0.8460282D-01       -0.1596694D 00   0.1010593D 00
  -0.1368595D 00   0.1136478D 00       -0.1140496D 00   0.1235394D 00
  -0.9123965D-01   0.1313240D 00       -0.6842974D-01   0.1373350D 00
  -0.4561983D-01   0.1417690D 00       -0.2280991D-01   0.1447370D 00
   0.6591949D-16   0.1460173D 00        0.1937954D-01   0.1478367D 00
   0.3875907D-01   0.1485406D 00        0.5813861D-01   0.1480014D 00
   0.7751814D-01   0.1459981D 00        0.9689768D-01   0.1421692D 00
   0.1162772D 00   0.1359153D 00        0.1356567D 00   0.1261851D 00
   0.1550363D 00   0.1109102D 00        0.1744158D 00   0.8490115D-01
```

Part 9(c). Part 3 of output giving the extremities of the interpenetration curve and the (ξ,η) coordinate of the interpenetration curve

```
EXAMPLE # 1. RAIL WHEEL CONTACT STRESS ANALYSIS (CONWHEEL)

ITM=10  NC= 9  MYOPT=0 MYMI=3  MYMA= 5  IBUG= 1  IOPT= 0
E1= .3000000+008  ANU1= .300  E2= .3000000+008  ANU2= .300
NSEG= 3    XBL= -.2281      XBR=  .1939
NX(I) ARE:
    4     5     4
THE FOLLOWING IS RAT(I)
 .200       .600       .200
       DELTA=  .50000-003             EPS=  .10000-001

THE FOLLOWING IS WHEEL DATA
    RADIUS,RW=  .18000+002       NO. OF SEGMENTS, NW= 5
    XWC=   .1719000+001  ZWC=  -.2389304+000  THETAW=  -.5503807+000

      AW              BW             CW                AW              BW             CW
   .0000000        .0000000       .0000000          .0000000     -.1234100+002   .1234100+002
   .1047545+001   -.1329907+001   .1280000+001      .1521301+001  -.5611090+000   .3780000+000
  -.2714595+001    .4662134+001   .0000000

      XCW             ZCW              XCW             ZCW             XCW             ZCW
   .0000000        .0000000        .1171000+001   -.5568193-001    .1719000+001   -.2389304+000
   .1876000+001   -.4304468+000

THE FOLLWING IS RAIL DATA
    NO. OF SEGMENTS, NR= 7

      XCR             XCR             XCR             XCR             XCR             XCR
 -.1432840+001   -.1250000+001   -.7000000+000    .7000000+000    .1250000+001    .1433000+001

      AR              BR             CR                AR              BR             CR
   .1432809+002    .2003760+002   .0000000         -.1058740+001  -.5188120+000   .3750000+000
  -.6125000+000   -.1271464+001   .1250000+001      .0000000      -.1000000+002   .1000000+002
   .6125000+000   -.1271464+001   .1250000+001      .1058750+001  -.5188120+000   .3750000+000
  -.1432809+002    .2003716+002   .0000000
```

Fig. 9(d). Part 4 of the output giving the mesh layout parameters, and the rail wheel coordinates

ITERATION =10

BOUNDARY OF CONTACT REGION

XI	ETA	XI	ETA	XI	ETA
-0.1641442D 00	[illegible]	-0.1483953D 00	0.4708296D-01	-0.1326444D 00	0.5895281D-01
-0.1168935D 00	[illegible]	-0.9011696D-01	0.8073997D-01	-0.5231480D-01	0.9135627D-01
-0.1451263D-01	0.9651915D-01	0.2329954D-01	0.9901235D-01	0.6109171D-01	0.9760710D-01
0.8766824D-01	0.8653578D-01	0.1036191D 00	0.8025403D-01	0.1193700D 00	0.6773512D-01
0.1351210D 00	0.4480244D-01				

NODE	XI	ETA	ZETA	P
1	-0.1641D 00	0.0000D 00	-0.1082D-01	0.1303D 05
2	-0.1641D 00	0.1155D-01	-0.1082D-01	0.1192D 05
3	-0.1641D 00	0.2309D-01	-0.1082D-01	0.7244D 04
4	-0.1484D 00	0.0000D 00	-0.8840D-02	0.2261D 05
5	-0.1484D 00	0.1883D-01	-0.8840D-02	0.2088D 05
6	-0.1484D 00	0.3767D-01	-0.8840D-02	0.1284D 05
7	-0.1326D 00	0.0000D 00	-0.7058D-02	0.2711D 05
8	-0.1326D 00	0.1684D-01	-0.7058D-02	0.2624D 05
9	-0.1326D 00	0.3369D-01	-0.7058D-02	0.2244D 05
10	-0.1326D 00	0.5053D-01	-0.7058D-02	0.1288D 05
11	-0.1169D 00	0.0000D 00	-0.5478D-02	0.3113D 05
12	-0.1169D 00	0.1482D-01	-0.5478D-02	0.3031D 05
13	-0.1169D 00	0.2964D-01	-0.5478D-02	0.2302D 05
14	-0.1169D 00	0.4446D-01	-0.5478D-02	0.2346D 05
15	-0.1169D 00	0.5928D-01	-0.5478D-02	0.1354D 05
16	-0.9012D-01	0.0000D 00	-0.3253D-02	0.3618D 05
17	-0.9012D-01	0.3230D-01	-0.3253D-02	0.3319D 05
18	-0.9012D-01	0.6459D-01	-0.3253D-02	0.1909D 05
19	-0.5231D-01	0.0000D 00	-0.1095D-02	0.4041D 05
20	-0.5231D-01	0.3654D-01	-0.1095D-02	0.3715D 05
21	-0.5231D-01	0.7309D-01	-0.1095D-02	0.2128D 05
22	-0.1451D-01	0.0000D 00	-0.8425D-04	0.4265D 05
23	-0.1451D-01	0.3861D-01	-0.8425D-04	0.3913D 05
24	-0.1451D-01	0.7721D-01	-0.8425D-04	0.2243D 05
25	0.2330D-01	0.0000D 00	-0.7257D-03	0.4196D 05
26	0.2330D-01	0.3960D-01	-0.7257D-03	0.3852D 05
27	0.2330D-01	0.7921D-01	-0.7257D-03	0.2207D 05
28	0.6109D-01	0.0000D 00	-0.5011D-02	0.3830D 05
29	0.6109D-01	0.3904D-01	-0.5011D-02	0.3513D 05
30	0.6109D-01	0.7809D-01	-0.5011D-02	0.2019D 05
31	0.8767D-01	0.0000D 00	-0.1044D-01	0.3305D 05
32	0.8767D-01	0.1923D-01	-0.1044D-01	0.3280D 05
33	0.8767D-01	0.3846D-01	-0.1044D-01	0.2966D 05
34	0.8767D-01	0.5769D-01	-0.1044D-01	0.2586D 05
35	0.8767D-01	0.7692D-01	-0.1044D-01	0.1500D 05
36	0.1036D 00	0.0000D 00	-0.1460D-01	0.2570D 05
37	0.1036D 00	0.1783D-01	-0.1460D-01	0.2821D 05
38	0.1036D 00	0.3567D-01	-0.1460D-01	0.2603D 05
39	0.1036D 00	0.5350D-01	-0.1460D-01	0.2185D 05
40	0.1036D 00	0.7134D-01	-0.1460D-01	0.1270D 05
41	0.1194D 00	0.0000D 00	-0.1951D-01	0.2376D 05
42	0.1194D 00	0.1505D-01	-0.1951D-01	0.2318D 05
43	0.1194D 00	0.3010D-01	-0.1951D-01	0.2154D 05
44	0.1194D 00	0.4516D-01	-0.1951D-01	0.1801D 05
45	0.1194D 00	0.6021D-01	-0.1951D-01	0.1043D 05
46	0.1351D 00	0.0000D 00	-0.2519D-01	0.1368D 05
47	0.1351D 00	0.1792D-01	-0.2519D-01	0.1225D 05
48	0.1351D 00	0.3585D-01	-0.2519D-01	0.7396D 04

XI	ETA	XI	ETA	XI	ETA
-0.1641462D 00	0.2886665D-01	-0.1483953D 00	0.4711[illegible]	-0.1326444D 00	0.5881123D-01
-0.1168935D 00	0.6669449D-01	-0.9011696D-01	0.8055529D-01	-0.5231480D-01	0.9092406D-01
-0.1451263D-01	0.9609029D-01	0.2329954D-01	0.9857856D-01	0.6109171D-01	0.9734413D-01
0.8766824D-01	0.8666564D-01	0.1036191D 00	0.8026307D-01	0.1193700D 00	0.6775381D-01
0.1351210D 00	0.4497723D-01				

XI-FORCE= 67.1 ETA-FORCE= 1408.6 ETA-MOMENT= -8.0

Fig. 9(e). Part 5 of the output giving the iteration number, boundaries of the (ξ,η) coordinates of the final contact patch, the pressure distribution over the cells, the net ξ and ζ forces, and the η moment.

INPUT DATA

POINT INDICES		SOURCE POINT GLOBAL COORDINATES			SURFACE TRACTIONS LOCAL COORDINATES		
I	J	X S	Y S	Z S	TX	TY	TZ
1	1	-0.164	-0.023	-0.011	0.0000D 00	0.0000D 00	0.7244D 04
1	2	-0.164	-0.012	-0.011	0.0000D 00	0.0000D 00	0.1192D 05
1	3	-0.164	0.000	-0.011	0.0000D 00	0.0000D 00	0.1303D 05
1	4	-0.164	0.012	-0.011	0.0000D 00	0.0000D 00	0.1192D 05
1	5	-0.164	0.023	-0.011	0.0000D 00	0.0000D 00	0.7244D 04
2	1	-0.148	-0.038	-0.009	0.0000D 00	0.0000D 00	0.1284D 05
2	2	-0.148	-0.019	-0.009	0.0000D 00	0.0000D 00	0.2089D 05
2	3	-0.148	0.000	-0.009	0.0000D 00	0.0000D 00	0.2261D 05
2	4	-0.148	0.019	-0.009	0.0000D 00	0.0000D 00	0.2088D 05
2	5	-0.148	0.038	-0.009	0.0000D 00	0.0000D 00	0.1284D 05
3	1	-0.133	-0.051	-0.007	0.0000D 00	0.0000D 00	0.1288D 05
3	2	-0.133	-0.034	-0.007	0.0000D 00	0.0000D 00	0.2244D 05
3	3	-0.133	-0.017	-0.007	0.0000D 00	0.0000D 00	0.2624D 05
3	4	-0.133	0.000	-0.007	0.0000D 00	0.0000D 00	0.2711D 05
3	5	-0.133	0.017	-0.007	0.0000D 00	0.0000D 00	0.2624D 05
3	6	-0.133	0.034	-0.007	0.0000D 00	0.0000D 00	0.2244D 05
3	7	-0.133	0.051	-0.007	0.0000D 00	0.0000D 00	0.1288D 05
4	1	-0.117	-0.059	-0.005	0.0000D 00	0.0000D 00	0.1354D 05
4	2	-0.117	-0.044	-0.005	0.0000D 00	0.0000D 00	0.2346D 05
4	3	-0.117	-0.030	-0.005	0.0000D 00	0.0000D 00	0.2802D 05
4	4	-0.117	-0.015	-0.005	0.0000D 00	0.0000D 00	0.3031D 05
4	5	-0.117	0.000	-0.005	0.0000D 00	0.0000D 00	0.3113D 05
4	6	-0.117	0.015	-0.005	0.0000D 00	0.0000D 00	0.3031D 05
4	7	-0.117	0.030	-0.005	0.0000D 00	0.0000D 00	0.2802D 05
4	8	-0.117	0.044	-0.005	0.0000D 00	0.0000D 00	0.2346D 05
4	9	-0.117	0.059	-0.005	0.0000D 00	0.0000D 00	0.1354D 05
5	1	-0.090	-0.065	-0.003	0.0000D 00	0.0000D 00	0.1909D 05
5	2	-0.090	-0.032	-0.003	0.0000D 00	0.0000D 00	0.3319D 05
5	3	-0.090	0.000	-0.003	0.0000D 00	0.0000D 00	0.3618D 05
5	4	-0.090	0.032	-0.003	0.0000D 00	0.0000D 00	0.3319D 05
5	5	-0.090	0.065	-0.003	0.0000D 00	0.0000D 00	0.1909D 05
6	1	-0.052	-0.073	-0.001	0.0000D 00	0.0000D 00	0.2128D 05
6	2	-0.052	-0.037	-0.001	0.0000D 00	0.0000D 00	0.3715D 05
6	3	-0.052	0.000	-0.001	0.0000D 00	0.0000D 00	0.4041D 05
6	4	-0.052	0.037	-0.001	0.0000D 00	0.0000D 00	0.3715D 05
6	5	-0.052	0.073	-0.001	0.0000D 00	0.0000D 00	0.2128D 05
7	1	-0.015	-0.077	-0.000	0.0000D 00	0.0000D 00	0.2243D 05
7	2	-0.015	-0.039	-0.000	0.0000D 00	0.0000D 00	0.3913D 05
7	3	-0.015	0.000	-0.000	0.0000D 00	0.0000D 00	0.4265D 05
7	4	-0.015	0.039	-0.000	0.0000D 00	0.0000D 00	0.3913D 05
7	5	-0.015	0.077	-0.000	0.0000D 00	0.0000D 00	0.2243D 05
8	1	0.023	-0.079	-0.001	0.0000D 00	0.0000D 00	0.2207D 05
8	2	0.023	-0.040	-0.001	0.0000D 00	0.0000D 00	0.3852D 05
8	3	0.023	0.000	-0.001	0.0000D 00	0.0000D 00	0.4196D 05
8	4	0.023	0.040	-0.001	0.0000D 00	0.0000D 00	0.3852D 05
8	5	0.023	0.079	-0.001	0.0000D 00	0.0000D 00	0.2207D 05
9	1	0.061	-0.078	-0.005	0.0000D 00	0.0000D 00	0.2019D 05
9	2	0.061	-0.039	-0.005	0.0000D 00	0.0000D 00	0.3513D 05
9	3	0.061	0.000	-0.005	0.0000D 00	0.0000D 00	0.3830D 05
9	4	0.061	0.039	-0.005	0.0000D 00	0.0000D 00	0.3513D 05
9	5	0.061	0.078	-0.005	0.0000D 00	0.0000D 00	0.2019D 05
10	1	0.088	-0.077	-0.010	0.0000D 00	0.0000D 00	0.1500D 05
10	2	0.088	-0.058	-0.010	0.0000D 00	0.0000D 00	0.2586D 05
10	3	0.088	-0.039	-0.010	0.0000D 00	0.0000D 00	0.2966D 05
10	4	0.088	-0.019	-0.010	0.0000D 00	0.0000D 00	0.3280D 05
10	5	0.088	0.000	-0.010	0.0000D 00	0.0000D 00	0.3305D 05
10	6	0.088	0.019	-0.010	0.0000D 00	0.0000D 00	0.3280D 05
10	7	0.088	0.039	-0.010	0.0000D 00	0.0000D 00	0.2966D 05
10	8	0.088	0.058	-0.010	0.0000D 00	0.0000D 00	0.2586D 05
10	9	0.088	0.077	-0.010	0.0000D 00	0.0000D 00	0.1500D 05
11	1	0.104	-0.071	-0.015	0.0000D 00	0.0000D 00	0.1270D 05
11	2	0.104	-0.054	-0.015	0.0000D 00	0.0000D 00	0.2185D 05
11	3	0.104	-0.036	-0.015	0.0000D 00	0.0000D 00	0.2603D 05
11	4	0.104	-0.019	-0.015	0.0000D 00	0.0000D 00	0.2821D 05
11	5	0.104	0.000	-0.015	0.0000D 00	0.0000D 00	0.2870D 05
11	6	0.104	0.018	-0.015	0.0000D 00	0.0000D 00	0.2821D 05
11	7	0.104	0.036	-0.015	0.0000D 00	0.0000D 00	0.2603D 05
11	8	0.104	0.054	-0.015	0.0000D 00	0.0000D 00	0.2185D 05
11	9	0.104	0.071	-0.015	0.0000D 00	0.0000D 00	0.1270D 05
12	1	0.119	-0.060	-0.020	0.0000D 00	0.0000D 00	0.1043D 05
12	2	0.119	-0.045	-0.020	0.0000D 00	0.0000D 00	0.1801D 05
12	3	0.119	-0.030	-0.020	0.0000D 00	0.0000D 00	0.2154D 05
12	4	0.119	-0.015	-0.020	0.0000D 00	0.0000D 00	0.2318D 05
12	5	0.119	0.000	-0.020	0.0000D 00	0.0000D 00	0.2376D 05
12	6	0.119	0.015	-0.020	0.0000D 00	0.0000D 00	0.2318D 05
12	7	0.119	0.030	-0.020	0.0000D 00	0.0000D 00	0.2154D 05
12	8	0.119	0.045	-0.020	0.0000D 00	0.0000D 00	0.1801D 05
12	9	0.119	0.060	-0.020	0.0000D 00	0.0000D 00	0.1043D 05
13	1	0.135	-0.036	-0.025	0.0000D 00	0.0000D 00	0.7396D 04
13	2	0.135	-0.018	-0.025	0.0000D 00	0.0000D 00	0.1225D 05
13	3	0.135	0.000	-0.025	0.0000D 00	0.0000D 00	0.1368D 05
13	4	0.135	0.018	-0.025	0.0000D 00	0.0000D 00	0.1225D 05
13	5	0.135	0.036	-0.025	0.0000D 00	0.0000D 00	0.7396D 04

Fig. 9(f). Part 6 of output giving the cell layout and surface tractions for the full contact patch

R E S U L T S

POINT INDICES I	J	K	FIELD POINT COORDINATES XF	YF	ZF	SUB SURFACE STRESS COMPONENTS S XX	S YY	S ZZ	S XY	S YZ	S ZX	EQUIVALENT STRESS S EQ
7	3	1	-0.015	0.000	-0.012	-0.291D 05	-0.262D 05	-0.418D 05	0.878D-03	-0.113D-01	-0.269D 03	0.144D 05
7	3	2	-0.015	0.000	-0.036	-0.187D 05	-0.135D 05	-0.385D 05	-0.578D-03	0.502D-03	-0.885D 03	0.229D 05
7	3	3	-0.015	0.000	-0.061	-0.115D 05	-0.642D 04	-0.338D 05	0.147D-04	-0.992D-03	-0.117D 04	0.253D 05
7	3	4	-0.015	0.000	-0.085	-0.695D 04	-0.286D 04	-0.286D 05	0.430D-04	0.230D-03	-0.118D 04	0.240D 05
7	3	5	-0.015	0.000	-0.109	-0.411D 04	-0.111D 04	-0.239D 05	-0.255D-05	-0.126D-04	-0.106D 04	0.215D 05
7	3	6	-0.015	0.000	-0.133	-0.240D 04	-0.274D 03	-0.198D 05	-0.143D-04	-0.240D-03	-0.862D 03	0.186D 05
7	3	7	-0.015	0.000	-0.157	-0.134D 04	0.137D 03	-0.166D 05	0.000D 00	0.000D 00	-0.720D 03	0.161D 05
7	3	8	-0.015	0.000	-0.182	-0.729D 03	0.308D 03	-0.138D 05	0.000D 00	0.000D 00	-0.571D 03	0.137D 05
7	3	9	-0.015	0.000	-0.206	-0.365D 03	0.371D 03	-0.116D 05	0.000D 00	0.000D 00	-0.451D 03	0.117D 05
7	3	10	-0.015	0.000	-0.230	-0.148D 03	0.383D 03	-0.986D 04	0.000D 00	0.000D 00	-0.358D 03	0.100D 05
7	3	11	-0.015	0.000	-0.254	-0.193D 02	0.370D 03	-0.843D 04	0.000D 00	0.000D 00	-0.286D 03	0.863D 04
7	3	12	-0.015	0.000	-0.279	0.567D 02	0.347D 03	-0.727D 04	0.000D 00	0.000D 00	-0.230D 03	0.748D 04
7	3	13	-0.015	0.000	-0.303	0.100D 03	0.320D 03	-0.632D 04	0.000D 00	0.000D 00	-0.187D 03	0.654D 04

Fig. 9(g). Part 7 of output giving the six components of the stress tensor and the value of the equivalent stress at various locations below the surface of the body

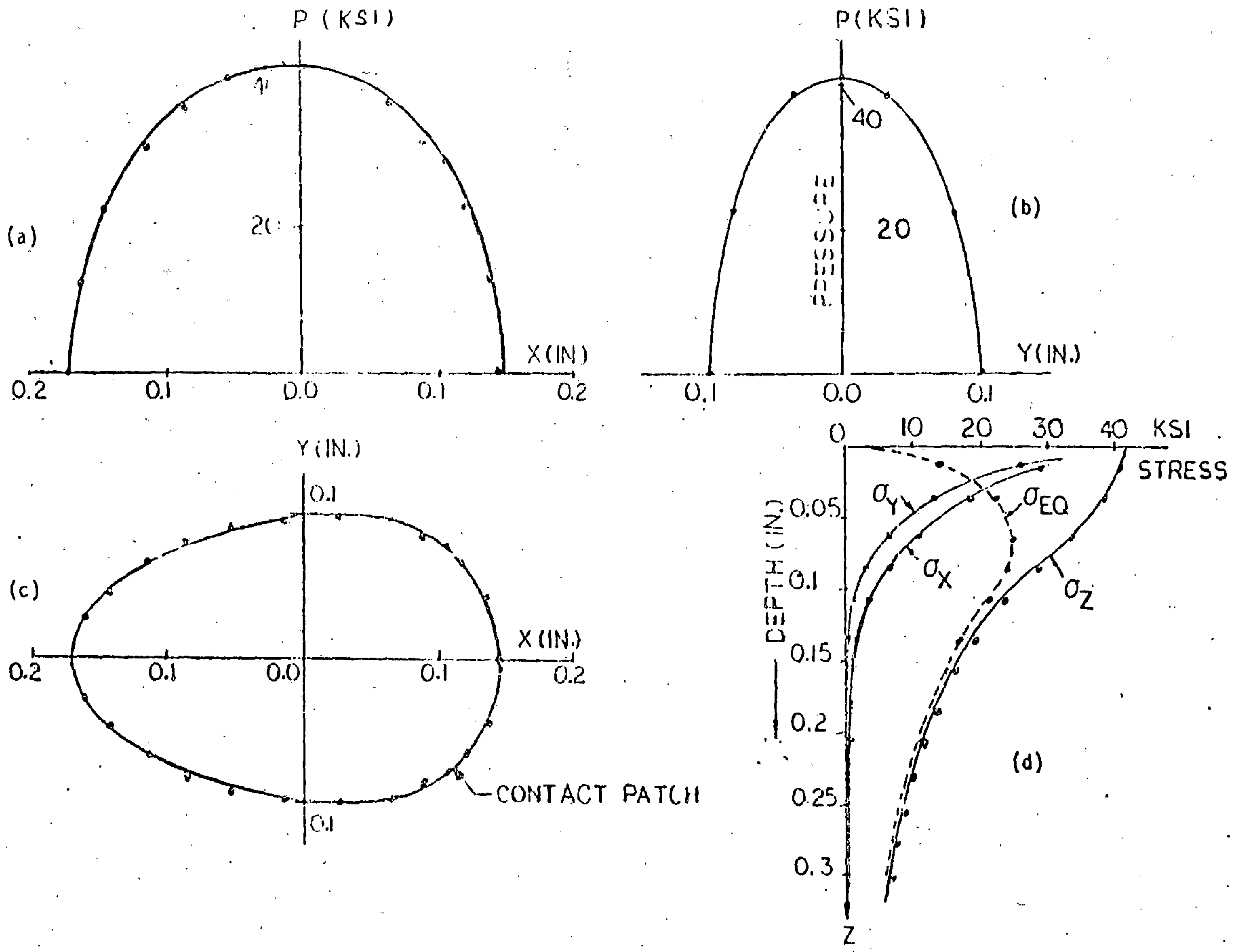

Fig. 9(h). Illustrating the graphical interpretation of results. (a) ξ pressure distribution (η=0 ζ=0); (b) η pressure distribution (ξ=0 ζ=0); (c) the contact patch; (d) subsurface stresses along depth (ξ=0 η=0)

```
EXAMPLE # 2. RAIL wHEEL CONTACT STRESS ANALYSIS (CONWHEEL)
     5       7      0      0      0      0      0               0.0
  0.003      1.719      1.25        18.0       30000000.  30000000. 0.3          0.3
  14.32809   20.0376    0.0        -1.43284   -1.05874    -0.518812  0.375       -1.25
 -0.6125    -1.271464   1.25       -0.7        0.0        -10.0      10.0         0.7
  0.6125    -1.271464   1.25        1.25       1.05875    -0.518812  0.375        1.433
 -14.32809   20.03716   0.0         1.50
  0.0        0.0        0.0         0.0        0.0        -12.341    12.341       1.171
  1.049545  -1.329907   1.28        1.719      1.521301   -0.561109  0.378        1.876
 -2.714595   4.662134   0.0         2.0
```

Fig. 10. Illustrating input data deck for example with approach $\delta = 0.003$

```
       EXAMPLE # 2. RAIL WHEEL CONTACT STRESS ANALYSIS (CONWHEEL)

       ITM=10  NC=13  MYOPT=0 MYMI=3  MYMA= 5  IBUG= 0  IOPT= 0
       E1= .3000000+008  ANU1= .300  E2= .3000000+008  ANU2= .300
       NSEG= 3     XEL= -.5270      XBR=  .2592
       NX(I) ARE;
           4     5     4
       THE FOLLOWING IS RAT(I)
        .200      .600      .200
            DELTA=  .30000-002            EPS= .10000-001

 THE FOLLOWING IS WHEEL DATA
     RADIUS,RW=  .18000+002      NO. OF SEGMENTS, NW= 5
     XWC=   .1719000+001  ZWC=  -.2389304+000  THETAW=  -.5503807+000

      AW              BW              CW                     AW              BW              CW
   .0000000        .0000000        .0000000              .0000000     -.1234100+002    .1234100+002
   .1049545+001   -.1329907+001    .1280000+001          .1521301+001 -.5611090+000    .3780000+000
  -.2714595+001    .4662134+001    .0000000

      XCW             ZCW               XCW            ZCW                XCW             ZCW
   .0000000        .0000000          .1171000+001  -.5568193-001       .1719000+001  -.2389304+000
   .1876000+001   -.4304468+000

 THE FOLLWING IS RAIL DATA
     NO. OF SEGMENTS, NR= 7

    XCR             XCR             XCR             XCR             XCP             XCR
 -.1432640+001  -.1250000+001  -.7000000+000   .7000000+000   .1250000+001   .1433000+001

      AR              BR              CR                     AR              BR              CR
   .1432809+002    .2003760+002    .0000000             -.1058740+001  -.5188120+000    .3750000+000
  -.6125000+000   -.1271464+001    .1250000+001          .0000000      -.1000000+002    .1000000+002
   .6125000+000   -.1271464+001    .1250000+001          .1058750+001  -.5188120+000    .3750000+000
  -.1432809+002    .2003716+002    .0000000
```

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Fig. 11(a). Part 4 of the output giving the mesh layout parameters, and the rail wheel coordinates

[illegible]

BOUNDARY OF CONTACT REGION

XI	ETA	XI	ETA	XI	ETA
[illegible]	[illegible]	[illegible]	[illegible]	-0.3212712D 00	0.1309564D 00
[illegible]	[illegible]	[illegible]	[illegible]	-0.1577550D 00	0.2124104D 00
[illegible]	[illegible]	-0.3915570D-02	[illegible]	0.7301914D-0[illegible]	0.2567803D 00
[illegible]	[illegible]	0.1595707D 00	[illegible]	0.1916268D 00	0.2626912D 00
0.2236629D 00	[illegible]				

NODE	XI	ETA	ZETA	P
1	-0.3854D 00	0.0000D 00	-0.6089D-01	0.3103D 05
2	-0.3854D 00	0.2656D-01	-0.6089D-01	0.2783D 05
3	-0.3854D 00	0.5312D-01	-0.6089D-01	0.1608D 05
4	-0.3533D 00	0.0000D 00	-0.5098D-01	0.5370D 05
5	-0.3533D 00	0.3006D-01	-0.5098D-01	0.5154D 05
6	-0.3533D 00	0.6012D-01	-0.5098D-01	0.4441D 05
7	-0.3533D 00	0.9018D-01	-0.5098D-01	0.2566D 05
8	-0.3213D 00	0.0000D 00	-0.4199D-01	0.6445D 05
9	-0.3213D 00	0.2910D-01	-0.4199D-01	0.6401D 05
10	-0.3213D 00	0.5820D-01	-0.4199D-01	0.5869D 05
11	-0.3213D 00	0.8730D-01	-0.4199D-01	0.4976D 05
12	-0.3213D 00	0.1164D 00	-0.4199D-01	0.2880D 05
13	-0.2892D 00	0.0000D 00	-0.3392D-01	0.7435D 05
14	-0.2892D 00	0.3359D-01	-0.3392D-01	0.7366D 05
15	-0.2892D 00	0.6717D-01	-0.3392D-01	0.6696D 05
16	-0.2892D 00	0.1007D 00	-0.3392D-01	0.5600D 05
17	-0.2892D 00	0.1343D 00	-0.3392D-01	0.3279D 05
18	-0.2347D 00	0.0000D 00	-0.2224D-01	0.8740D 05
19	-0.2347D 00	0.7358D-01	-0.2224D-01	0.8019D 05
20	-0.2347D 00	0.1472D 00	-0.2224D-01	0.4638D 05
21	-0.1578D 00	0.0000D 00	-0.9998D-02	0.9872D 05
22	-0.1578D 00	0.8496D-01	[illegible]	0.9047D 05
23	-0.1578D 00	0.1699D 00	-0.9998D-02	0.5253D 05
24	-0.6085D-01	0.0000D 00	-0.2617D-02	0.1050D 06
25	-0.6085D-01	0.6597D-01	-0.2617D-02	0.1008D 06
26	-0.6085D-01	0.1319D 00	-0.2617D-02	0.8605D 05
27	-0.6085D-01	0.1979D 00	-0.2617D-02	0.4525D 05
28	-0.3916D-02	0.0000D 00	-0.6133D-05	0.1103D 06
29	-0.3916D-02	0.6659D-01	-0.6133D-05	0.1053D 06
30	-0.3916D-02	0.1332D 00	-0.6133D-05	0.8966D 05
31	-0.3916D-02	0.1998D 00	-0.6133D-05	0.5295D 05
32	0.7302D-01	0.0000D 00	-0.7179D-02	0.1080D 06
33	0.7302D-01	0.7377D-01	-0.7179D-02	0.1036D 06
34	0.7302D-01	0.1467D 00	-0.7179D-02	0.8866D 05
35	0.7302D-01	0.2201D 00	-0.7179D-02	0.5236D 05
36	0.1275D 00	0.0000D 00	-0.2235D-01	0.9623D 05
37	0.1275D 00	0.5842D-01	-0.2235D-01	0.9382D 05
38	0.1275D 00	0.1168D 00	-0.2235D-01	0.8713D 05
39	0.1275D 00	0.1753D 00	-0.2235D-01	0.7275D 05
40	0.1275D 00	0.2337D 00	-0.2235D-01	0.4313D 05
41	0.1596D 00	0.0000D 00	-0.3565D-01	0.8339D 05
42	0.1596D 00	0.5956D-01	-0.3565D-01	0.8191D 05
43	0.1596D 00	0.1191D 00	-0.3565D-01	0.7484D 05
44	0.1596D 00	0.1787D 00	-0.3565D-01	0.6270D 05
45	0.1596D 00	0.2382D 00	-0.3565D-01	0.3609D 05
46	0.1916D 00	0.0000D 00	-0.5266D-01	0.6905D 05
47	0.1916D 00	0.5838D-01	-0.5266D-01	0.6724D 05
48	0.1916D 00	0.1168D 00	-0.5266D-01	0.6206D 05
49	0.1916D 00	0.1751D 00	-0.5266D-01	0.5204D 05
50	0.1916D 00	0.2335D 00	-0.5266D-01	0.3016D 05
51	0.2237D 00	0.0000D 00	-0.7402D-01	0.3990D 05
52	0.2237D 00	0.5075D-01	-0.7402D-01	0.3886D 05
53	0.2237D 00	0.1015D 00	-0.7402D-01	0.3536D 05
54	0.2237D 00	0.1522D 00	-0.7402D-01	0.2913D 05
55	0.2237D 00	0.2030D 00	-0.7402D-01	0.1687D 05

XI	ETA	XI	ETA	XI	ETA
-0.3853635D 00	0.4645741D-01	-0.3533274D 00	0.1052376D 00	-0.3212712D 00	0.1309574D 00
-0.2892151D 00	0.1511313D 00	-0.2347197D 00	0.1839611D 00	-0.1577850D 00	0.2124490D 00
-0.6085025D-01	0.2271370D 00	-0.3915570D-02	0.2334649D 00	0.7301914D-01	0.2567919D 00
0.1275146D 00	0.2640202D 00	0.1595707D 00	0.2677516D 00	0.1916268D 00	0.2627760D 00
0.2236629D 00	0.2284603D 00				

XI-FORCE= 1301.5 ETA-FORCE= 19007.6 ETA-MOMENT= -609.9

Fig. 11(b). Part of the output giving the iteration number, boundaries of the (ξ,η) coordinates of the final contact patch, the pressure distribution over the cells, the net ξ and ζ forces, and the η moment.

R E S U L T S

POINT INDICES			FIELD POINT COORDINATES			SUB SURFACE STRESS COMPONENTS						EQUIVALENT STRESS
I	J	K	XF	YF	ZF	S XX	S YY	S ZZ	S XY	S YZ	S ZX	S EQ
8	4	1	-0.004	0.000	[illegible]	[illegible]	[illegible]	-0.107D 06	-0.515D 03	-0.296D 03	0.815D 03	0.305D 05
8	4	2	-0.004	[illegible]	[illegible]	[illegible]	-0.423D 05	[illegible]	-0.310D 03	[illegible]	-0.751D 03	0.526D 05
8	4	3	[illegible]	[illegible]	-0.123	[illegible]	[illegible]	-0.914D 05	-0.449D 03	-0.193D 03	-0.105D 03	0.622D 05
8	4	4	[illegible]	0.000	-0.172	[illegible]	[illegible]	[illegible]	-0.496D 03	-0.297D 03	0.849D 03	0.629D 05
8	4	5	-0.004	0.000	-0.222	[illegible]	-0.672D 04	[illegible]	-0.446D 03	-0.365D 03	0.155D 04	[illegible]
8	4	6	-0.004	0.000	-0.271	[illegible]	-0.127D 04	-0.584D 05	-0.397D 03	-0.383D 03	0.194D 04	0.529D 05
8	4	7	-0.004	0.000	-0.321	-0.490D 04	-0.333D 04	-0.493D 05	-0.320D 03	-0.365D 03	0.194D 04	0.464D 05
8	4	8	-0.004	0.000	-0.370	[illegible]	-0.287D 03	-0.417D 05	-0.250D 03	-0.326D 03	0.178D 04	0.403D 05
8	4	9	-0.004	0.000	-0.419	-0.160D 04	0.271D 03	-0.354D 05	-0.192D 03	-0.282D 03	0.156D 04	0.348D 05
8	4	10	-0.004	0.000	[illegible]	-0.921D 03	0.557D 03	-0.302D 05	-0.148D 03	-0.239D 03	0.133D 04	0.302D 05
8	4	11	-0.004	0.000	-0.518	-0.342D 03	0.692D 03	-0.260D 05	-0.114D 03	-0.200D 03	0.113D 04	0.262D 05
8	4	12	-0.004	0.000	-0.567	-0.447D 02	0.741D 03	-0.225D 05	-0.885D 02	-0.168D 03	0.952D 03	0.229D 05
8	4	13	-0.004	0.000	-0.616	0.138D 03	0.743D 03	-0.196D 05	-0.694D 02	-0.140D 03	0.801D 03	0.201D 05

Fig. 11(c). Part 7 of output giving the six components of the equivalent stress at various locations below the surface of the body

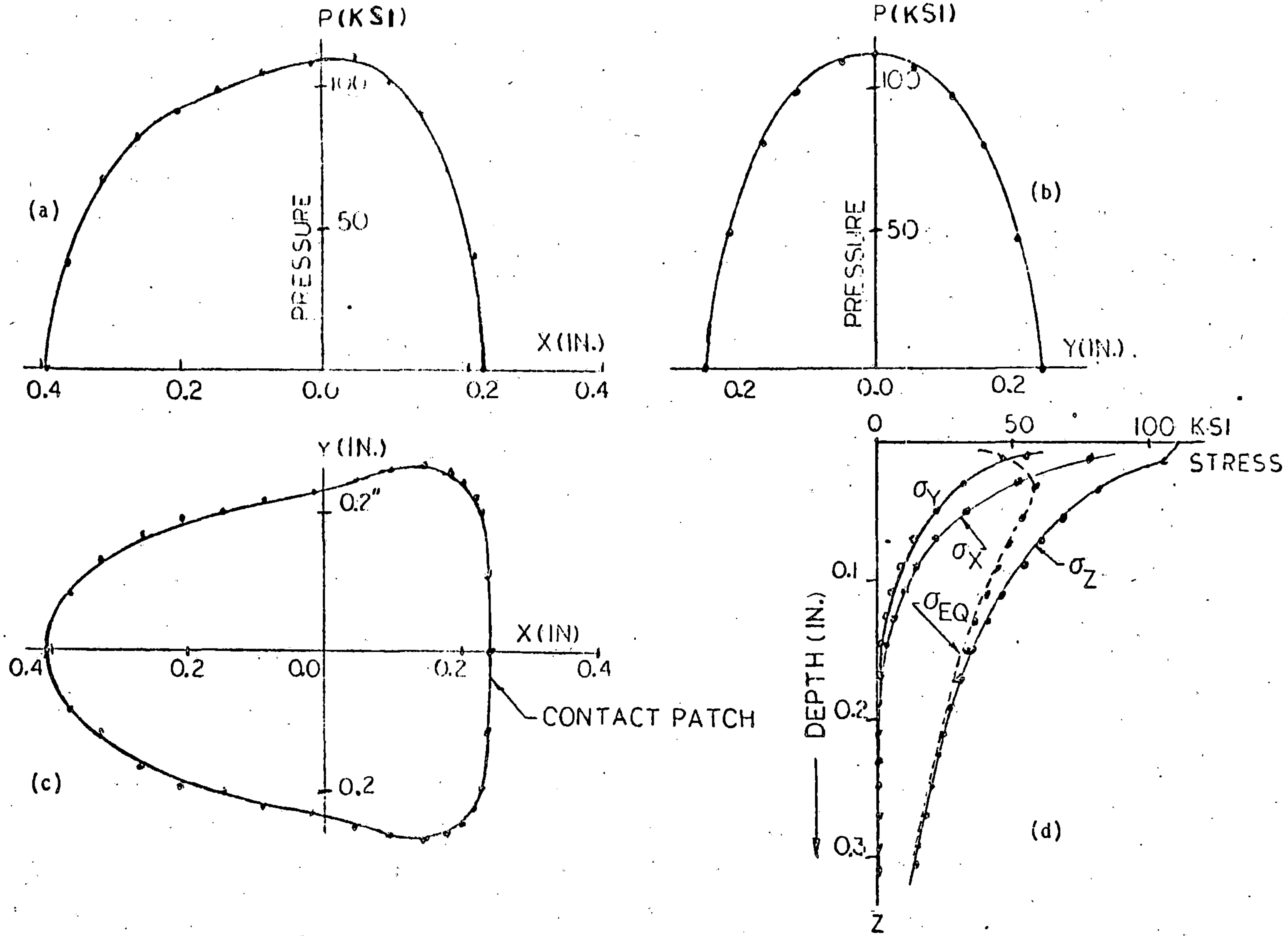

Fig. 11(d). Illustrating the graphical interpretation of results. (a) ξ pressure distribution (η=0 ζ=0) (b) η pressure distribution (ξ=0 ζ=0) (c) the contact patch (d) subsurface stresses along depth (ξ=0 η=0)

10. REFERENCES

1. Hashemi, J. and B. Paul,"Contact Stresses in Bodies with Arbitrary Geometry, Applications to Wheels and Rails," Technical Report No. 7, April 1979, FRA/ORD-79-23, Contract DOT-OS-60144, PB 299409/AS.

2. Paul, B. and J. Hashemi, "User's Manual for Program CONFORM (CONFORMal contact stress between wheels and rails)", Technical Report No. 5, June 1978, FRA/ORD-78-40, Contract DOT-OS-60144, PB 288927/AS.

3. Paul, B. and J. Hashemi, "Contact Geometry Associated with Arbitrary Wheel and Rail Profiles, in The General Problem of Rolling Contact, AMD-Vol. 40, Ed. by A. L. Browne and N. T. Tsai, American Society of Mechanical Engineers, NY, 1980, pp. 93-105.

4. Paul, B. and J. Hashemi, "Contact Pressures on Closely Conforming Elastic Bodies," in Solid Contact and Lubrication, AMD-Vol. 39, Ed. by H. S. Cheng and L. M. Keer, American Society of Mechanical Engineers, NY, 1980, pp. 67-78. Also see Trans. ASME Journ. Applied Mechanics, Vol. 48, 1981, pp. 543-548.

5. Paul, B. and S. Singh, "Calculating Subsurface Stresses due to Non-Hertzian Wheel-Rail Contact," Technical Report No. 10, March 1982, Contract DTFR 53-81-C-00227, Federal Railroad Administration.

11. LIST OF RELATED FRA REPORTS

A. FRA Technical Reports (Available from National Technical Information Service)

A1. Paul, B., "A Review of Rail-Wheel Contact Stress Problems," Technical Report No. 1, April 1975, FRA/ORD-76 141, PB 251238/AS, Contract DOT-OS-40093.

A2. Woodward, W., and Paul, B., "Contact Stresses for Closely Conforming Bodies - Application to Cylinders and Spheres," Technical Report No. 2, December 1976, DOT/TST/77-48, PB 271033/AS, Contract DOT-OS-40093.

A3. Paul, B., and Hashemi, J., "An Improved Numerical Method for Counterformal Contact Stress Problems," Technical Report No. 3, July 1977, FRA/ORD-78/26, Contract DOT-OS-60144, PB 286228/AS.

A4. Paul, B., and Hashemi, J., "User's Manual for Program COUNTACT COUNTer-formal contACT stress problems ", Technical Report No. 4, September 1977, FRA/ORD-78/27, Contract DOT-OS-60144. PB 286097/AS

A5. Paul, B., and Hashemi, J., "User's Manual for Program CONFORM (CONFORMal contact stresses between wheels and rails ", Technical Report No. 5, June 1978, FRA/ORD-78/40, Contract DOT-OS-60144, PB 288927/AS.

A6. Paul, B., and Hashemi, J., "Rail-Wheel Geometry Associated with Contact Stress Analysis," Technical Report No. 6, September 1979, FRA/ORD-78/41. Contract DOT-OS-60144.

A7. Hashemi, J. and Paul, B., "Contact Stresses in Bodies with Arbitrary Geometry, Applications to Wheels and Rails," Technical Report No. 7, April 1979, FRA/ORD/79-23, Contract DOT-OS-60144, PB 299409/AS.

A8. Paul, B., and Hashemi, J., "Numerical Determination of Contact Pressures Between Closely Conforming Wheels and Rails", Technical Report No. 8. July, 1979, FRA/ORD-79/41, Contract DOT-OS-60144, PB 80120462.

A9. Paul, B., "Fundamental Studies Related to Wheel-Rail Contact Stress," Final Report, Contract DOT-OS-60144, January 1981.

12. Program Listing

CONWHEEL

```
C*****************************************************************************
C                                 PROGRAM   CONWHEEL
C
C
C
C          BY B.PAUL AND S.SINGH
C          DEPARTMENT OF MECHANICAL  ENGINEERING
C          UNIVERSITY OF PENNSYLVANIA
C          PHILADELPHIA
C          P.A.  19104
C
C          MARCH 1982
C
C
C*********PURPOSE
C
C          TO ANALYZE CONTACT STRESSES IN RAILS AND WHEELS
C
C          1.THE BOUNDARIES OF THE CONTACT PATCH.
C          2.THE NORMAL CONTACT PRESSURE (P) DISTRIBUTION.
C          3.THE TOTAL FORCE AND MOMENT DUE TO P.
C          4.THE STATE OF STRESS BELOW THE SURFACE.
C          5,THE EQUIVALENT (J2) STRESS AT SUBSURFACE POINTS.
C
C
C*********REFERENCES
C
C          1.USER'S MANUAL FOR PROGRAM CONWHEEL.
C            B.PAUL AND S.SINGH,TECHNICAL REPORT NO.10,MARCH 1982.
C            CONTRACT DTFR 53-81-C-00227,F.R.A.
C          2.CONTACT STRESSES IN BODIES WITH ARBITRARY GEOMETRY,
C            APPLICATIONS TO WHEELS AND RAILS.
C            J. HASHEMI AND B.PAUL,TECHNICAL REPORT NO. 7,APRIL 1979
C            FRA/ORD-79-23,CONTRACT DOT-OS-60144,PB 299409/AS.
C          3.CONTACT GEOMETRY ASSOCIATED WITH ARBITRARY WHEEL AND
C            AND RAIL PROFILES.IN THE GENERAL PROBLEM OF ROLLING
C            CONTACTS,B.PAUL AND J.HASHEMI,AMD-VOL 40,ED.A.L.BROWNE
C            AND N.T.TSAI,AMERICAN SOCIETY OF MECHANICAL ENGINEERS,
C            NY,1980,PP 93-105
C          4.CONTACT PRESSURES  ON CLOSELY CONFORMING ELASTIC BODIES
C            B.PAUL AND J.HASHEMI, SOLID CONTACT AND LUBRICATION,
C            AMD-VOL.39,ED. H.S.CHENG AND L.M.KEER,AMERICAN SOCIETY
C            OF MECHANICAL ENGINEERS,NY,1980,PP 67-78.
C
C
C
C
C
C
C*********PRIMARY SUBROUTINES USED
C
C
C          1. MIDSEP
C          2. INSEP
C          3. CONFOM
C          4. SUBSIG
C
C
      CALL MIDSEP
      STOP
      END
      SUBROUTINE MIDSEP
C-----------------------------------------------------------------------------
C
C     PROGRAM  -MIDSEP-
C
C     PURPOSE.....
C        TO FIND INITIAL SEPARATION BETWEEN RAIL-WHEEL IN MIDPLANE ,
C     AND CALCULATE TRANSFORMED RAIL-WHEEL PARAMETERS.
C
C     METHOD.....
C        SEE "RAIL AND WHEEL GEOMETRY ASSOCIATED WITH CONTACT STREESS
C     ANALYSIS" BY B. PAUL AND J. HASHEMI
C
C     STANDARD SUBPROGRAMS.....
C           SUBROUTINE RAILO (XRO,ZRO,TETAR)
C           SUBROUTINE WHEELO (XWO,ZWO,TETAW)
C           SUBROUTINE RAIL (X,ZETAR)
C           SUBROUTINE WHEEL (X,ZETAW)
```

NOTE: The following Device Numbers have been assigned

NNR = 15 (Read Device No.)
NNW = 16 (Write Device No.)

```
C
C      INPUT VARIABLES.....
C         NW          NUMBER OF SEGMENTS IN WHEEL PROFILE
C         NR          NUMBER OF SEGMENTS IN RAIL PROFILE
C         XWC         X-COORD. OF WHEEL INITIAL CONTACT POINT
C         XRC         X-COORD. OF RAIL INITIAL CONTACT POINT
C         XIL,XIR     LEFT AND RIGHT XI BOUNDARIES
C         AR,BR,CR    COORDS. AND RADUIS OF RAIL ARC CENTERS
C         XCR         X-COORD. OF RAIL SEGMENT END POINT
C         AW,BW,CW    COORDS. AND RADUIS OF WHEEL ARC CENTERS
C         XCW         X-COORD. OF WHEEL SEGMENT END POINT
C
C      INPUT ARRANGEMENTS....
C     CARD ID. FORMAT              VARIABLES
C         A    (20A4)              TITLE
C         B    (4F12.0,4I5)        XWC,XRC,XIL,XIR,N.NW,NR
C         C    (6F12.0)            AR,BR,CR
C         D    (6F12.0)            XCR(1)
C         E    (6F12.0)            AW,BW,CW
C         F    (6F12.0)            XCW
C-----------------------------------------------------------------------
C
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION XOFFR(99),YOFFR(99),XOFFW(99),YOFFW(99)
      DIMENSION XI(200),DZ(200)
      COMMON/PROB01/TITLE(20),RAT(10),YB(20),NY(20),NX(10),NW,NR,NSEG
      COMMON/RAIL01/ATR(99),BTR(99),CR(99),XICR(99),NR1
      COMMON/RAIL02/AR(99),BR(99),XCR(99)
      COMMON/WHEE01/ATW(99),BTW(99),CW(99),XICW(99),NW1
      COMMON/WHEE02/AW(99),BW(99),XCW(99)
      COMMON/WHEE03/ZCW(99)
      COMMON/WHEE04/XWC,ZWC,TETAW,RW,XRC,DELTA,XBL,XBR,EPS,ITM,NXB,IBUG
      COMMON/BODY1/E1,ANU1,NNR,NNW
      COMMON/BODY2/E2,ANU2
      NNR=15
      NNW=16
    1  READ(NNR,101)(TITLE(I),I=1,20)
      READ(NNR,100)NW,NR,NOFF,NSEG,ITM,IOPT,IBUG,EPS
      READ(NNR,104)DELTA,XWC,XRC,RW,E1,E2,ANU1,ANU2
      IF(NSEG.EQ.0)GO TO 10
      READ(NNR,103)(NX(I),I=1,NSEG)
      READ(NNR,104)(RAT(I),I=1,NSEG)
      GO TO 20
 10   NSEG=3
      NX(1)=4
      NX(2)=5
      NX(3)=4
      RAT(1)=0.2
      RAT(2)=0.6
      RAT(3)=0.2
 20   CONTINUE
      IF(EPS.EQ.0.0)EPS=0.01
      IF(ITM.EQ.0)ITM=10
      CONTINUE
      IF (NOFF.EQ.0) GO TO 2
      READ(NNR,104)(XOFFR(I),YOFFR(I),I=1,NR)
      READ(NNR,104)(XOFFW(I),YOFFW(I),I=1,NW)
      CALL OFF(NR,XOFFR,YOFFR,AR,BR,CR,XCR,NR2)
      CALL OFF(NW,XOFFW,YOFFW,AW,BW,CW,XCW,NW2)
      NR=NR2-1
      NW=NW2-1
      NR1=NR-1
      NW1=NW-1
      GO TO 3
 2    CONTINUE
      NR1=NR-1
      NW1=NW-1
      READ(NNR,104) (AR(I),BR(I),CR(I),XCR(I),I=1,NR)
      READ(NNR,104) (AW(I),BW(I),CW(I),XCW(I),I=1,NW)
 3    CONTINUE
      CALL RAILO (XRC,ZRC,TETAR)
      CALL WHEELO (XWC,ZWC,TETAW)
      XIL=-RW/20.0
      XIR=-XIL/2.0
      NXB=10
      H=DABS(XIL/NXB)
      J=1
      XI(1)=XIL
   50 X=XI(J)
```

```
      CALL RAIL (X,ZETAR)
      CALL WHEEL1 (X,ZETAW)
      DZ(J)=ZETAW-ZETAR
      IF (DABS(X).GT..1D-12) GO TO 80
      H=XIR/NXB
   80 IF (X.GT.XIR) GO TO 90
      I=J+1
      XI(I)=XI(J)+H
      J=I
      GO TO 50
   90 CONTINUE
      IF (IBUG.EQ.0) GO TO 95
      WRITE(NNW,102)(TITLE(I),I=1,20)
      WRITE(NNW,107)
      WRITE(NNW,108) XWC,ZWC,TETAW
      WRITE(NNW,109)
      WRITE(NNW,110) (AW(I),BW(I),CW(I),I=1,NW)
      WRITE(NNW,111)
      WRITE(NNW,112) (ATW(I),BTW(I),I=1,NW)
      WRITE(NNW,113)
      WRITE(NNW,112) (XCW(I),ZCW(I),I=1,NW1)
      WRITE(NNW,114)
      WRITE(NNW,115) (XICW(I),I=1,NW1)
      WRITE(NNW,116)
      WRITE(NNW,117)
      WRITE(NNW,115) (XICR(I),I=1,NR1)
      WRITE(NNW,118)
      WRITE(NNW,110) (ATR(I),BTR(I),CR(I),I=1,NR)
      WRITE(NNW,123) XRC,ZRC,TETAR
      WRITE(NNW,121)
      WRITE(NNW,112) (AR(I),BR(I),I=1,NR)
      WRITE(NNW,122)
      WRITE(NNW,115) (XCR(I),I=1,NR1)
      WRITE(NNW,119) XIL,XIR,N,NW,NR
      WRITE(NNW,105)
      WRITE(NNW,106) (XI(I),DZ(I),I=1,J)
   95 CONTINUE
      WRITE(NNW,124)
       DO 220 IN=1,2
       IF (IN.EQ.1) XX=XIL
       IF (IN.EQ.2) XX=XIR
       DELX=XX*0.00001
 200   X1=XX
       CALL RAIL (X1,ZETAR)
       CALL WHEEL1(X1,ZETAW)
       Z1=(ZETAW-ZETAR)-DELTA
       X2=XX+DELX
       CALL RAIL (X2,ZETAR)
       CALL WHEEL1(X2,ZETAW)
       Z2=(ZETAW-ZETAR)-DELTA
       DEX=Z1*(X2-X1)/(Z2-Z1)
       IF (DABS(DEX).LE.DABS(10.*DELX))GO TO 210
       XX=X1-DEX
       GO TO 200
 210   IF (IN.EQ.1) XBL=XX
       IF (IN.EQ.2) XBR=XX
 220   CONTINUE
      CALL INTPEN
       RETURN
  100 FORMAT (8I5,F10.0)
  101 FORMAT (20A4)
  102 FORMAT (1H1,15X,20A4/)
  103 FORMAT (16I5)
  104 FORMAT (8F10.0)
  105 FORMAT(1H1,//,21X,'XI',12X,'DELTA ZETA',13X,'XI',12X,'DELTA ZETA')
  106 FORMAT (/(10X,2E18.7,3X,2E18.7),1H1)
  107 FORMAT (/,10X,'WHEEL PARAMETERS')
  108 FORMAT (/,15X,'XWC=',E15.7,2X,'ZWC=',E15.7,2X,'THETAW=',E15.7)
  109 FORMAT (/,17X,2('AW',12X,'BW',12X,'CW',18X))
  110 FORMAT (9X,3E15.7,3X,3E15.7)
  111 FORMAT (/,16X,3('ATW',12X,'BTW',15X))
  112 FORMAT (9X,2E15.7,3X,2E15.7,3X,2E15.7)
  113 FORMAT (/,16X,3('XCW',12X,'ZCW',15X))
  114 FORMAT (/,16X,6('XICW',11X))
  115 FORMAT (9X,6E15.7)
  116 FORMAT (/,10X,'RAIL PARAMETERS')
  117 FORMAT (/,16X,6('XICR',11X))
  118 FORMAT (/,16X,2('ATR',12X,'BTR',13X,'CR',15X))
  119 FORMAT (/,12X,'XIL=',E15.7,3X,'XIR=',E15.7,3X,'N=',I2,3X,'NW=',
```

```
     $I2,3X,'NR=',I2)
 121 FORMAT (/,18X,3('AR',13X,'BR',13X))
 122 FORMAT (/,15X,6('XCR',12X))
 123 FORMAT (/,15X,'XRC=',E15.7,2X,'ZRC=',E15.7,2X,'THETAR=',E15.7)
 124 FORMAT (1H1)
     END
     SUBROUTINE RAILO (X,Z,TETA)
C
C-----------------------------------------------------------------------
C
C    PURPOSE.....
C       TO FIND Z-COORD. AND SLOPE OF RAIL PROFILE AT INITIAL
C    CONTACT POINT.
C
C    METHOD.....
C       SEE "RAIL AND WHEEL GEOMETRY ASSOCIATED WITH CONTACT STREESS
C    ANALYSIS" BY B. PAUL AND J. HASHEMI
C
C-----------------------------------------------------------------------
C
     IMPLICIT REAL*8 (A-H,O-Z)
     DIMENSION ZC(99)
     COMMON/RAIL01/ATR(99),BTR(99),CR(99),XICR(99),NR1
     COMMON/RAIL02/A(99),B(99),XC(99)
     NR=NR1+1
     DO 5 I=1,NR1
     IF (X.LE.XC(I)) GO TO 6
   5 CONTINUE
     I=NR
   6 IF (CR(I).EQ.0.0) GO TO 7
     Z=B(I)+DSQRT(CR(I)**2-(X-A(I))**2)
     TETA=DATAN((A(I)-X)/(Z-B(I)))
     II=I
     GO TO 8
   7 II=0
     Z=A(I)*X+B(I)
     TETA=DATAN(A(I))
   8 CT=DCOS(TETA)
     ST=DSIN(TETA)
     DO 13 I=1,NR
     IF (CR(I).EQ.0.0) GO TO 12
     IF (I.EQ.NR) GO TO 10
     ZC(I)=B(I)+DSQRT(CR(I)**2-(XC(I)-A(I))**2)
  10 AX=A(I)-X
     BZ=B(I)-Z
     ATR(I)=AX*CT+BZ*ST
     BTR(I)=-AX*ST+BZ*CT
     GO TO 13
  12 ASC=A(I)*ST+CT
     ATR(I)=(A(I)*CT-ST)/ASC
     BTR(I)=(A(I)*X+B(I)-Z)/ASC
     IF (I.EQ.NR) GO TO 13
     ZC(I)=A(I)*XC(I)+B(I)
  13 CONTINUE
     IF (II.EQ.0) GO TO 15
     ATR(II)=0.0
     BTR(II)=-CR(II)
  15 DO 20 I=1,NR1
  20 XICR(I)=(XC(I)-X)*CT+(ZC(I)-Z)*ST
     RETURN
     END
     SUBROUTINE WHEELO (X,Z,TETA)
C
C-----------------------------------------------------------------------
C
C    PURPOSE.....
C    METHOD.....
C       SEE "RAIL AND WHEEL GEOMETRY ASSOCIATED WITH CONTACT STREESS
C    ANALYSIS" BY B. PAUL AND J. HASHEMI
C-----------------------------------------------------------------------
     IMPLICIT REAL*8 (A-H,O-Z)
     COMMON/WHEE01/ATW(99),BTW(99),CW(99),XICW(99),NW1
     COMMON/WHEE02/A(99),B(99),XC(99)
     COMMON/WHEE03/ZC(99)
     NW=NW1+1
     DO 5 I=1,NW1
     IF (X.LE.XC(I)) GO TO 6
   5 CONTINUE
     I=NW
```

```
    8 IF  (CW(I).EQ.0.0) GO TO 10
      Z=B(I)+DSQRT(CW(I)**2-(X-A(I))**2)
      TETA=DATAN((A(I)-X)/(Z-B(I)))
      II=I
      GO  TO 12
   10 Z=A(I)*X+B(I)
      TETA=DATAN(A(I))
      II=0
   12 CT=DCOS(TETA)
      ST=DSIN(TETA)
      DO  20  I=1,NW
      IF  (CW(I).EQ.0.0) GO TO 15
      IF  (I.EQ.NW) GO TO 13
      ZC(I)=B(I)+DSQRT(CW(I)**2-(XC(I)-A(I))**2)
   13 AX=A(I)-X
      BZ=B(I)-Z
      ATW(I)=AX*CT+BZ*ST
      BTW(I)=-AX*ST+BZ*CT
      GO  TO 20
   15 ASC=A(I)*ST+CT
      ATW(I)=(A(I)*CT-ST)/ASC
      BTW(I)=(A(I)*X+B(I)-Z)/ASC
      IF  (I.EQ.NW) GO TO 20
      ZC(I)=A(I)*X+B(I)
   20 CONTINUE
      IF  (II.EQ.0) GO TO 24
      ATW(II)=0.0
      BTW(II)=-CW(II)
   24 DO  25  I=1,NW1
   25 XICW(I)=(XC(I)-X)*CT+ST*(ZC(I)-Z)
      RETURN
      END
      SUBROUTINE  RAIL(XI,ZETA)
C
C-----------------------------------------------------------------------
C
C     PURPOSE.....
C         TO CALCULATE  THE  ZETA-COMPONENT  OF  THE  PROFILE  OF  RAIL
C     FOR ANY  GIVEN  XI
C
C     METHOD.....
C     SEE  "GEOMETRY  OF  RAIL  AND  WHEEL"  BY  B.  PAUL  AND  J.  HASHEMI.
C
C     DESCRIPTION  OF  ARGUMENTS.......
C         XI          X-COMPONENT  OF  THE  POINT  IN  QUESTION
C         ZETA        Z-COMPONENT  OF  THE  POINT  TO  BE  RETURNED  TO
C                     CALLING  PROGRAM
C-----------------------------------------------------------------------
C
      IMPLICIT  REAL*8  (A-H,O-Z)
      COMMON/RAIL01/ATR(99),BTR(99),CR(99),XICR(99),NR1
      DO  5  I=1,NR1
      IF  (XI.LE.XICR(I))  GO  TO  8
    5 CONTINUE
      I=NR1+1
    8 IF  (CR(I).EQ.0.0)  GO  TO  10
      ZETA=BTR(I)+DSQRT(CR(I)**2-(XI-ATR(I))**2)
      GO  TO  20
   10 ZETA=ATR(I)*XI+BTR(I)
   20 RETURN
      END
      SUBROUTINE  WHEEL1  (XI,ZETA)
C
C-----------------------------------------------------------------------
C
C     PURPOSE.....
C         TO  CALCULATE  THE  ZETA-COMPONENT  OF  THE  WHEEL  PROFILE
C     FOR  ANY  GIVEN  XI.
C
C     METHOD.....
C        SEE  "RAIL  AND  WHEEL  GEOMETRY  ASSOCIATED  WITH  CONTACT  STREESS
C     ANALYSIS"  BY  B.  PAUL  AND  J.  HASHEMI
C
C-----------------------------------------------------------------------
C
      IMPLICIT  REAL*8  (A-H,O-Z)
      COMMON/WHEE01/ATW(99),BTW(99),CW(99),XICW(99),NW1
      DO  5  I=1,NW1
      IF  (XI.LE.XICW(I))  GO  TO  8
    5 CONTINUE
```

CONWHEEL

```
      I=NW1+1
    8 IF  (CW(I).EQ.0.0) GO TO 10
      ZETA=BTW(I)+DSQRT(CW(I)**2-(XI-ATW(I))**2)
      GO TO 20
   10 ZETA=ATW(I)*XI+BTW(I)
   20 RETURN
      END
      SUBROUTINE INTPEN
C
C-----------------------------------------------------------------------
C        PROGRAM INTERPEN
C
C        PURPOSE......
C
C           TO FIND THE INTERPENETRATION CURVE FOR RAIL AND WHEEL CONTACT
C        STRESS ANALYSIS
C
C       METHOD.....
C         SEE "RAIL AND WHEEL GEOMETRY ASSOCIATED WITH CONTACT STREESS
C       ANALYSIS" BY B. PAUL AND J. HASHEMI
C
C       STANDARD SUBPROGRAMS.....
C          SUBROUTINE RAIL (X,ZR)
C          SUBFUNCTION YRW(X,ZW)
C
C       INPUT VARIABLES.....
C          NW           NUMBER OF SEGMENTS IN WHEEL PROFILE
C          NR           NUMBER OF SEGMENTS IN RAIL PROFILE
C          XWC          X-COORD. OF WHEEL INITIAL CONTACT POINT
C          ZWC          Z-COORD. OF WHEEL INITIAL CONTACT POINT
C          AW,BW,CW     COORDS. AND RADUIS OF WHEEL ARC CENTERS
C          XCW          X-COORD. OF WHEEL SEGMENT END POINT
C
C
C       INPUT ARRANGEMENTS....
C     CARD ID  FORMAT           VARIABLES
C          A    (20A4)          TITLE
C          D    (6F12.0)        AW,BW,CW
C          E    (6F12.0)        XCW
C-----------------------------------------------------------------------
C
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION X(200),Y(200)
      DIMENSION XB(100)
      DIMENSION HXB(10),XOO(10)
      COMMON/PROB01/TITLE(20),RAT(10),YB(20),NY(20),NX(10),NW,NR,NSEG
      COMMON/RAIL01/ATR(99),BTR(99),CR(99),XICR(99),NR1
      COMMON/RAIL02/AR(99),BR(99),XCR(99)
      COMMON/WHEE01/ATW(99),BTW(99),CW(99),XICW(99),NW1
      COMMON/WHEE02/AW(99),BW(99),XCW(99)
      COMMON/WHEE03/ZCW(99)
      COMMON/WHEE04/XWC,ZWC,TETAW,RW,XRC,DELTA,XBL,XBR,EPS,ITM,NXB,IBUG
      COMMON/BODY1/E1,ANU1,NNR,NNW
C
C.....INPUT DATA
      NW1=NW-1
      NR1=NR-1
C
C.....FIND LEFT EQUALLY SPACED INTERVALS
      H=DABS(XBL)/NXB
      I=1
      X(I)=XBL
      XX=XBL
C
C.....FIND ZETA OF RAIL
   50 CALL RAIL (XX,ZR)
      ZW=DELTA+ZR
C
C.....FIND ETA OF INTERPENETRATION CURVE
      Y(I)=YRW(XX,ZW)
      J=I+1
      X(J)=X(I)+H
      XX=X(J)
      I=J
      IF  (DABS(XX).GT.0.1D-12) GO TO 90
C
C.....FIND RIGHT EQUALLY SPACED INTERVALS
      H=XBR/NXB
   90 IF  (XX.LE.XBR) GO TO 50
```

```
      N=I-1
      IF  (IBUG.EQ.0)  GO TO 95
      WRITE(NNW,102)  TITLE
      WRITE(NNW,107)  DELTA,XBL,XBR
      WRITE(NNW,111)
      WRITE(NNW,108)  XWC,ZWC,TETAW
      WRITE(NNW,109)
      WRITE(NNW,110)  (AW(I),BW(I),CW(I),I=1,NW)
      WRITE(NNW,122)
      WRITE(NNW,115)  (XCW(I),I=1,NW1)
      WRITE(NNW,116)
      WRITE(NNW,117)
      WRITE(NNW,115)  (XICR(I),I=1,NR1)
      WRITE(NNW,118)
      WRITE(NNW,110)  (ATR(I),BTR(I),CR(I),I=1,NR)
      WRITE(NNW,119)  RW,NW,NR
      WRITE(NNW,106)
      WRITE(NNW,105)  (X(I),Y(I),I=1,N)
   95 CONTINUE
      WRITE(NNW,112)
       BB=XBR-XBL
       MX =0
       X00(1)=XBL
       DO  10  I=1,NSEG
       X00(I+1)=BB*RAT(I)+X00(I)
       HXB(I)=BB*RAT(I)/NX(I)
       NXI=NX(I)
       DO  10  J=1,NXI
       MX =MX+1
       XB(MX)=X00(I)+((J-1)+0.5)*HXB(I)
       XX =XB(MX)
       CALL  RAIL  (XX,ZR)
       ZW=DELTA+ZR
       YB(MX)=YRW(XX,ZW)
 10    CONTINUE
      CALL  CONFOM
      RETURN
C
C.....FORMAT  STATEMENTS
  101 FORMAT  (20A4)
  102 FORMAT  (1H1,//,15X,20A4/)
  103 FORMAT  (3F12.0,3I5)
  104 FORMAT  (6F12.0)
  105 FORMAT  (10X,2E16.7,3X,2E16.7)
  106 FORMAT  (/,19X,2('XI',12X,'ETA',18X))
  107 FORMAT  (/,10X,'DELTA=',E12.5,5X,'XBL=',E12.5,5X,'XBR=',E12.5)
  108 FORMAT  (/,20X,'XWC=',E15.7,2X,'ZWC=',E15.7,2X,'TETAW=',E15.7)
  109 FORMAT  (/,16X,2('AW',12X,'BW',12X,'CW',18X))
  110 FORMAT  (9X,3E15.7,3X,3E15.7)
  111 FORMAT  (/,10X,'WHEEL  PARAMETERS')
  112 FORMAT  (1H1)
  115 FORMAT  (9X,6E15.7)
  116 FORMAT  (/,10X,'RAIL  PARAMETERS')
  117 FORMAT  (/,16X,6('XICR',11X))
  118 FORMAT  (/,16X,2('ATR',12X,'BTR',13X,'CR',15X))
  119 FORMAT  (/,12X,'RW=',E15.7,3X,'NW=',I2,3X,'NR=',I2)
  122 FORMAT  (/,15X,6('XCW',12X))
      END
      DOUBLE  PRECISION  FUNCTION  YRW(X,Z)
C
C-----------------------------------------------------------------------
C     SUBPROGRAM  SUBFUNCTION  YRW  (X,Z)
C
C     PURPOSE.....
C         TO  FIND  ETA  COORD  OF  WHEEL  FOR  GIVEN  XI  AND  ZETA.
C
C     METHOD.....
C        SEE  "RAIL  AND  WHEEL  GEOMETRY  ASSOCIATED  WITH  CONTACT  STREESS
C     ANALYSIS"  BY  B.  PAUL  AND  J.  HASHEMI
C-----------------------------------------------------------------------
C
      IMPLICIT  REAL*8  (A-H,O-Z)
      COMMON/WHEE01/ATW(99),BTW(99),C(99),XICW(99),NW1
      COMMON/WHEE02/A(99),B(99),XC(99)
      COMMON/WHEE04/XW0,ZW0,TETAW,RW,XRC,D,XBL,XBR,EPS,ITM,NXB,IBUG
      DATA  IJ/0/
      IF  (IJ.EQ.1)  GO  TO  1
      ST=DSIN(TETAW)
      CT=DCOS(TETAW)
```

```
      IJ=1
C
C.....FIND WHEEL REFRENCE COORDS. OF POINT (XI,ZETA)
    1 XO=XW0+X*CT-Z*ST
      ZO=ZW0+X*ST+Z*CT
      DO 5 I=1,NW1
      IF (XO.LE.XC(I)) GO TO 8
    5 CONTINUE
      I=NW1+1
    8 IF (C(I).EQ.0.0) GO TO 10
      FOX=B(I)+DSQRT(C(I)**2-(XO-A(I))**2)
      GO TO 12
   10 FOX=A(I)*XO+B(I)
   12 RHO=RW-FOX
      DELTA=RHO**2-(ZO-RW)**2
      IF (DELTA.LT.0.0) GO TO 15
C
C.....FIND ETA OF THE WHEEL CORRESPONDING TO ZETA
      YRW=DSQRT(DELTA)
      GO TO 20
   15 YRW=0.0
   20 RETURN
      END
      SUBROUTINE CONFOM
C-----------------------------------------------------------------------
C
C     PURPOSE.....
C       TO CALCULATE THE CONTACT PATCH BOUNDARY ,THE PRESSURE
C     DISTRIBUTION OVER IT AND THE LOADING CONDITION , FOR GIVEN
C     RIGID BODY APPROACH DELTA.
C
C     METHOD.....
C       MODIFIED DISCRETIZATION METHOD IS USED TO SOLVE THE GOVERNING
C     INTEGRAL EQUATIONS. FOR MORE INFORMATION SEE "NUMERICAL
C     PROCEDURE FOR CONFORMAL CONTACT STRESS PROBLEMS" , BY
C     B. PAUL , AND J. HASHEMI.
C
C     DESCRIPTION OF MAJOR VARIABLES USED INTERNALLY.....
C
C     STANDARD SUBROUTINES.....
C         SUBROUTINE INSEP(X,Y,Z,FZ,XN,YN,ZN,I,M)
C         SUBROUTINE GDA(XF,YF,ZF,XNF,YNF,ZNF,XS,YS,ZS,XNS,YNS,ZNS
C                  ,HXS,HYS)
C         SUBROUTINE LEQT1F(B,1,N,IAI,F,IDGT,WKAREA,IER,)
C         SUBROUTINE PARAB(Y1,Y2,Y3,F1,F2,F3)
C
C     USER-SUPPLIED SUBPROGRAMS.....
C         USER MAY PROVIDE HIS OR HER OWN SUBROUTINE INSEP -----
C
C     ***NOTE.
C     THROUGHOUT THIS SUBROUTINE, THE SYMBOLE X,Y,Z REPRESENT THE
C     GLOBAL COORDINATES (XI,ETA,ZETA) WITH ORIGIN AT THE CONTACT
C     POINT C.***
C
C     DESCRIPTION OF INPUT VARIABLES.....
C         TITLE       ANY TITLE DESCRIBING PROBLEM (UP TO 80 CHARACTERS)
C
C     INPUTS FOR THE MAIN PROGRAM
C         ITM         ALLOWED MAXIMUM NO. OF ITERATIONS
C         NC          INDEX OF THE STRIP USED TO MONITOR CONVERGENCE
C         MYOPT       0 TO BYPASS THE INPUT FOR NY(I), 1 TO READ IN NY(I)
C         MYMI        MIN. NO. OF CELLS IN ANY STRIP
C         MYMA        MAX. NO. OF CELLS IN ANY STRIP
C         IDGT        NUMBER OF ACCURATE DIGITS WANTED IN SOLUTION. SET
C                     EQUAL TO ZERO TO BYPASS THIS ACCURACY TEST
C         E1,ANU1     ELASTIC MODULUS AND POISSON'S RATIO OF BODY 1
C         E2,ANU2     ELASTIC MODULUS AND POISSON'S RATIO OF BODY 2
C         NSEG        NO. OF STRIPS ALONG X-AXIS
C         XBL         LEFT X-INTERCEPT OF BOUNDARY CURVE
C         XBR         RIGHT X-INTERCEPT OF BOUNDARY CURVE
C         NX(I)       NO. OF STRIPS ALONG THE X-AXIS IN BAND I
C         NY(J)       NO. OF CELLS IN STRIP J LYING ON AND ABOVE X-AXIS
C         RAT(I)      WIDTH OF BAND I DIVIDED BY X-DIAMETER
C         YB(I)       HEIGHT ABOVE X-AXIS OF STRIP I
C         D           RIGID BODY APPROACH
C         EPS         TOLERANCE FOR CONVERGENCE CHECK (TYPICALLY 0.01)
C
C     INPUT DATA ARRANGEMENT.....
C
```

```
C              CARD ID. FORMAT              VARIABLES
C                   A    (20A4)             TITLE
C                   B    (6I5)              ITM,NC,MYOPT,MYMI,MYMA,IDGT
C                   C    (4F10.0)           E1,ANU1,E2,ANU2
C                   D    (I5,2F10.0)        NSEG,XBL,XBR
C                   E    (16I5)             NX(I) GROUP OF 16
C                   F    (16I5)             NY(J) GROUP OF 16 (OMIT IF MYOPT=0)
C                   G    (8F10.0)           RAT(I) GROUP OF 8
C                   H    (8F10.0)           YB(I) GROUP OF 8
C                   I    (2F10.0)           D,EPS
C
C       COMMENTS ON DIMENSION STATEMENTS....
C         THE MAXIMUM NO. OF FIELD POINTS IS USED AS DIMENSION FOR
C       B,F,WKAREA,X,Y,Z,XN,YN,ZN,WHICH IS CURRENTLY SET EQUAL TO 100.
C       TO CHANGE THIS ALL 100'S IN THE FIRST TWO DIMENSION STATEMENT
C       CARDS AND THE FIRST DATA STATEMENT CARD MUST BE CHANGED
C       TO DESIRED DIMENSION.THE MAXIMUM NUMBER OF FIELD POINTS
C       ALONG THE X-AXIS IS USED AS DIMENSION FOR P,XB,YB,HX,HY,XBN,YBN,
C       AR,NY,YBM WHICH IS CURRENTLY SET EQUAL TO 20. TO CHANGE THIS ALL
C       THESE 20'S MUST BE CHANGED,EXCEPT THE ONE FOR THE TITLE.
C       THE MAXIMUM NO. OF BANDS IS USED AS DIMENSIONS FOR RAT,
C       AND NX. TO CHANGE THIS ALL 10'S MUST BE CHANGED.
C-----------------------------------------------------------------------
C
        IMPLICIT REAL*8 (A-H,O-Z)
C
C.....DEFINE A FUNCTION USED FOR INTERPOLATION BETWEEN TWO POINTS.
        YFUN(X1,Y1,X2,Y2,XX)=(XX-X1)*(Y1-Y2)/(X1-X2)+Y1
        DIMENSION B(100,100)
        COMMON/PROB01/TITLE(20),RAT(10),YB(20),NY(20),NX(10),NW,NR,NSEG
        COMMON/WHEE04/XWC,ZWC,TETAW,RW,XRC,D,XBL,XBR,EPS,ITM,NXB,IBUG
        DIMENSION A(10000),F(100,3),WKAREA(100),X(100),Y(100),Z(100)
        DIMENSION P(20,5),XB(20),HX(20),HY(20),XBN(20),YBN(20)
        DIMENSION YBM(20),YBMM(20),AR(20)
        DIMENSION XN(100),YN(100),ZN(100),FXP(20,10),FYP(20,10),FZP(20,10)
        DIMENSION XSX(20),YSY(20,10),ZSZ(20,10),WX(20,10),WY(20,10)
        DIMENSION ZFF(20),XFM(20),YFM(20),ZFM(20),NSY(20)
        DIMENSION XNN(20,10),YNN(20,10),ZNN(20,10)
        DIMENSION AXX(20,10,20),AYY(20,10,20),AZZ(20,10,20)
        COMMON /BODY2/ E2,ANU2
        COMMON /BODY1/ E1,ANU1,NNR,NNW
        DATA IAI/100/
C
C
C.....READ INPUT DATA
        WRITE(NNW,227) TITLE
        MYOPT=0
        MYMI=3
        MYMA=5
C.....READ IN NO. OF SEGMENTS ALONG X-AXIS, AND X-INTERCEPTS(XBL,XBR)
C
C.....READ IN NO. OF COLUMNS IN STRIP I
C
C.....FIND TOTAL NO. OF COLUMNS ALONG THE X-AXIS
        MX=0
        DO 2 I=1,NSEG
      2 MX=MX+NX(I)
        IF (MYOPT .EQ. 0) GO TO 3
C
C.....READ IN THE NO. OF CELLS IN COLUMN I
        READ(NNR,225) (NY(I),I=1,MX)
C
C.....READ IN RATIO OF THE LENGTH OF SEGMENT I TO THE DISTANCE
C.....BETWEEN THE TWO X-INTERCEPTS
  3       CONTINUE
C
C.....READ IN THE Y-COORDINATE OF POINT K ON BOUNDARY IN COLUMN K
        NC=1
        YBMAX=YB(1)
        DO 6 I=1,MX
        IF (YB(I).LT.YBMAX) GO TO 7
        YBMAX=YB(I)
        NC=I+1
  7     CONTINUE
  6     CONTINUE
        DO 4 I=1,MX
      4 YBM(I)=YB(I)
C
C.....INITIAL VALUES FOR SOME OF THE VARIABLES
```

```
C
C.....PRINT THE INPUT DATA FOR CHECK OUT
      WRITE(NNW,219) ITM,NC,MYOPT,MYMI,MYMA,IBUG,IOPT
      WRITE(NNW,212) E1,ANU1,E2,ANU2
      WRITE(NNW,220) NSEG,XBL,XBR
      WRITE(NNW,229)
      WRITE(NNW,224) (NX(I),I=1,NSEG)
      WRITE(NNW,230)
      WRITE(NNW,228) (RAT(I),I=1,NSEG)
      WRITE(NNW,231) D,EPS
      IT=0
    5 K=1
      XBK=XBL
      BB=XBR-XBL
      DO 20 I=1,NSEG
      NXI=NX(I)
C
C.....FIND THE X-WIDTH OF CELLS IN COLUMN K
      HXX=BB*RAT(I)/NXI
      HX(K)=HXX
      XB(K)=XBK+HXX/2.
      XBK=XB(K)
      IF (NXI.LT.2) GO TO 16
      DO 15 J=2,NXI
      K=K+1
      HX(K)=HXX
C
C.....FIND THE X-COORDINATE OF CELLS IN ROW K
      XB(K)=XBK+HX(K)
      XBK=XB(K)
   15 CONTINUE
   16 XBK=XB(K)+HX(K)/2.
      K=K+1
   20 CONTINUE
      IF (IT.EQ.0) GO TO 32
C.....FIND BY INTERPOLATION THE YB(I) FOR XB(I)
      DO 29 I=1,MX
C
C.....LOCATE SURROUNDING POINTS 1 AND 2 FOR LINEAR INTERPOLATION
      DO 21 J=1,MX
      IF (XB(I).LE.XBN(J)) GO TO 22
   21 CONTINUE
C
C.....FOR POINTS NEAR THE RIGHT BOUND..BUT OUTSIDE THE OLD ONE
      X1=XBN(MX)
      Y1=YBN(MX)
      YM1=YBM(MX)
      X2=XBR
      Y2=0.0
      YM2=0.0
      GO TO 26
   22 IF (J.EQ.1) GO TO 23
      IF (I.EQ.1) GO TO 25
      IF (I.EQ.MX) GO TO 24
C
C.....FOR POINTS AWAY FROM LEFT OR RIGHT BOUND.
      J1=J-1
      X1=XBN(J1)
      Y1=YBN(J1)
      YM1=YBM(J1)
      X2=XBN(J)
      Y2=YBN(J)
      YM2=YBM(J)
      GO TO 26
C
C.....FOR POINTS NEER THE LEF, BUT OUTSIDE THE OLD BOUND.
   23 X1=XBL
      Y1=0.0
      YM1=0.0
      X2=XBN(1)
      Y2=YBN(1)
      YM2=YBM(1)
      GO TO 26
C
C.....FOR POINTS NEAR RIGHT BOUNDARY BUT INSIDE THE OLD ONE
   24 J1=J-1
      X1=XBN(J1)
      Y1=YBN(J1)
      YM1=YBM(J1)
```

```
      X2=XBN(J)
      Y2=0.0
      YM2=0.0
      IF (XBR.LT.X2) GO TO 26
      Y2=YBN(J)
      YM2=YBM(J)
      GO TO 26
C
C.....FOR POINTS NEAR THE LEFT BOUNDARY BUT INSIDE THE OLD ONE
   25 J1=J-1
      X1=XBN(J1)
      Y1=0.0
      YM1=0.0
      X2=XBN(J)
      Y2=YBN(J)
      YM2=YBM(J)
      IF (XBL.GT.X1) GO TO 26
      Y1=YBN(J1)
      YM1=YBM(J1)
C
C.....INTERPOLATE BETWEEN 1 AND 2 TO FIND YB(I)FOR GIVEN XB(I)
   26 XXX=XB(I)
      YB(I)=YFUN(X1,Y1,X2,Y2,XXX)
      YBMM(I)=YFUN(X1,YM1,X2,YM2,XXX)
   29 CONTINUE
      DO 31 L=1,MX
   31 YBM(L)=YBMM(L)
C
C.....BEGIN TO READ OR WRITE THE VALUES FOR NY(I)
   32 I=1
      N=0
      DO 55 K=1,MX
      YIN=0.0
C
C.....CHECK THE NO. OF POINTS ALONG THE Y-AXIS OPTION MYOPT
      IF (MYOPT.EQ.1) GO TO 40
C
C.....FIND NO. OF CELLS IN COLUMN K TO HAVE THE BEST ASPECT RATIO
      YNY=YB(K)/HX(K)+.5
      NYY=YNY
      NY(K)=NYY+2.*(YNY-NYY)
C
C.....SET THE NINIMUM VALUE FOR NY(K) MYMI
      IF (NY(K).LE.MYMI) GO TO 35
      IF (NY(K).GT.MYMA) NY(K)=MYMA
      GO TO 40
   35 NY(K)=MYMI
   40 MY=NY(K)
      X(I)=XB(K)
      Y(I)=YIN
C
C.....FIND THE Y-WIDTH OF CELLS IN COLUMN K
      HY(K)=2.*YB(K)/(2*NY(K)-1)
      DO 50 J=2,MY
      I=I+1
C
C.....FIND THE X AND Y COORDINATES OF CENTROID OF CELL I
      X(I)=XB(K)
      Y(I)=YIN+HY(K)
      YIN=Y(I)
   50 CONTINUE
      I=I+1
      N=N+MY
   55 CONTINUE
      K11=0
      IT=IT+1
      J=0
      DO 100 IS=1,MX
      XS=XB(IS)
      HYS=HY(IS)
      HXS=HX(IS)
      AR(IS)=HXS*HYS
      MYS=NY(IS)
      DO 100 JS=1,MYS
      J=J+1
      I=0
C
C     CALCULATE THE COEFFICIENT B(I,J) AND F(I,1) GIVEN BY EQ. (2) AND (3)
C
```

```
      DO 100 IFF=1,MX
      XF=XB(IFF)
      MYF=NY(IFF)
      DO 100 JF=1,MYF
      I=I+1
      K11=K11+1
      IF (J.GT.1) GO TO 60
C
C.....FIND THE INITIAL SEPARATION
      XX=X(I)
      YY=Y(I)
      CALL INSEP(XX,YY,ZZ,FZ,XNN1,YNN1,ZNN1)
      Z(I)=ZZ
      XN(I)=XNN1
      YN(I)=YNN1
      ZN(I)=ZNN1
C
C.....CALCULATE LEFT HAND SIDE OF THE EQS.
      F(I,1)=(D-FZ)*ZNN1
   60 IF (I.GT.1) GO TO 65
      YS=Y(J)
      ZS=Z(J)
      XNS=XN(J)
      YNS=YN(J)
      ZNS=ZN(J)
   65 YF=Y(I)
      XNF=XN(I)
      YNF=YN(I)
      ZNF=ZN(I)
      ZF=Z(I)
C
C.....LOCATE CELL ON OR AWAY FROM X-AXIS
      IF (JS.EQ.1) GO TO 80
C
C.....CALCULATE B(I,J) FOR POINTS AWAY FROM X-AXIS
      B(I,J)=GDA(XF,YF,ZF,XNF,YNF,ZNF,XS,YS,ZS,XNS,YNS,ZNS,HXS,HYS)+
     $GDA(XF,YF,ZF,XNF,YNF,ZNF,XS,-YS,ZS,XNS,-YNS,ZNS,HXS,HYS)
      GO TO 100
C
C.....CALCULATE B(I,J) FOR POINTS ON THE X-AXIS
   80 B(I,J)=GDA(XF,YF,ZF,XNF,YNF,ZNF,XS,YS,ZS,XNS,YNS,ZNS,HXS,HYS)
  100 CONTINUE
C
C.....SOLVE THE SYSTEM OF LINEAR EQUATIONS
      NTOTC=I
      DO 101 K=1,NTOTC
      DO 101 L=1,NTOTC
      K22=K+(L-1)*NTOTC
      A(K22)=B(K,L)
 101  CONTINUE
 7777 FORMAT(2X,10E13.5)
      CALL DGELG (F,A,NTOTC,1,0.1D-14,IER,DET)
      IF (IER.EQ.0) GO TO 550
 520  WRITE(NNW,525)
 525  FORMAT(/10X,'***** MATRIX B IS ALGORITHMICALLY SINGULAR *****')
      WRITE(NNW,526)IER
 526  FORMAT(/10X,'***** IER= ',I3,'*****')
      IF (IER.EQ.-1)GO TO 540
      GO TO 999
 540  WRITE(NNW,541)
 541  FORMAT(1H0,'FATAL SINGULARITY IN SUBR DGELG')
      GO TO 999
 550  CONTINUE
C
      IF (IBUG.EQ.0) GO TO 900
      IF ((ITM-IT).GT.IBUG)GO TO 900
C   PRINT THE BOUNDARY OF THE GIVEN ITERATION
      WRITE(NNW,223) IT
      WRITE(NNW,213)
      WRITE(NNW,214)
      WRITE(NNW,222) (XB(I),YB(I),I=1,MX)
C
C.....PRINT THE SOLUTION (PRESSURE DISTRIBUTION).
      WRITE(NNW,211)
      WRITE(NNW,215) (I,X(I),Y(I),Z(I),F(I,1),I=1,N)
 900  CONTINUE
C.....INITIALIZE SOME OF THE VARIABLES
      RY=YB(NC)
      IFP=0
```

```
      IL=0
      IJ=0
      MYY=0
      DO 180 I=1,MX
      MY=NY(I)
      DO 150 J=1,MY
      IJ=IJ+1
      P(I,J)=F(IJ,1)
C
C.....CHECK FOR PRESSURES TO BE ALL POSSITIVE AND FIND NEW BOUNDARY OF CONTACT
      IF (P(I,J).LT.0.0) GO TO 160
C
C.....CHECK FOR BEING THE FIRST CELL
      IF (IL.NE.1) GO TO 150
C
C.....THE PRESSURE IS CHANGING SIGN AT THE LEFT X-BOUND.
      I1=I-1
      X1=XB(I1)
      P1=P(I1,J)
      X2=XB(I)
      P2=P(I,J)
      XXX=0.0
      XBL=YFUN(P1,X1,P2,X2,XXX)
      XBLM=XB(I)
      IL=2
  150 CONTINUE
      IJ1=IJ-1
 7575 FORMAT(2I10,2F20.6)
      SECA=(DSQRT(1.+(YN(IJ)/ZN(IJ))**2)+DSQRT(1.+(YN(IJ1)/ZN(IJ1))**2))
     $/2.0
      Y1=Y(IJ)*SECA
      F1=F(IJ,1)
      Y2=Y(IJ1)*SECA
      F2=F(IJ1,1)
      IF (F2.LE.F1) GO TO 155
      YBN(I)=PARAB (Y1,Y2,F1,F2)/SECA
      IF (YBN(I).LE.YB(I)) GO TO 156
      IF (YBN(I).LE.YBM(I)) GO TO 180
  155 YBN(I)=(YB(I)+YBM(I))/2.
      GO TO 180
  156 YBM(I)=YB(I)
      GO TO 180
  160 IF (J.GT.1) GO TO 170
      IF (XB(I).GT.0.0) GO TO 200
C
C.....THE PRESSURE ALONG THE LEFT X-BOUND. IS STILL NEGATIVE
      IL=1
      GO TO 179
  170 Y1=Y(IJ)
      P1=F(IJ,1)
      IJ1=IJ-1
      Y2=Y(IJ1)
      P2=F(IJ1,1)
C
C.....FIND THE NEW YB WHEN P CHANGES FROM -VE TO +VE
      XXX=0.0
      YBN(I)=YFUN(P1,Y1,P2,Y2,XXX)
      YBM(I)=Y1
  179 IJ=MYY+MY
      IFP=1
  180 MYY=IJ
      IF (IL.EQ.2) GO TO 183
C
C.....LOCATE THE POINTS FOR PARABOLIC EXTERAPOLATION
      KI=1+NY(1)
      ZNA=(ZN(1)+ZN(KI))/2.
      F1=P(1,1)
      F2=P(2,1)
      IF (F2.LE.F1) GO TO 181
      X1=XB(1)/ZNA
      X2=XB(2)/ZNA
      XBLN=PARAB(X1,X2,F1,F2)*ZNA
      IF (XBLN.GE.XBL) GO TO 182
      IF (XBLN.LT.XBLM) GO TO 181
      XBL=XBLN
      GO TO 183
  181 XBL=(XBL+XBLM)/2.
      GO TO 183
  182 XBLM=XBL
```

```
      XBL=XBLN
  183 I1=MX-1
      F1=P(MX,1)
      F2=P(I1,1)
      IF (F2.LE.F1) GO TO 190
C
C.....THE BOUND. TO BE SHORTENED
      IN=N-NY(MX)
      ZNA=(ZN(IN)+ZN(N))/2.
      X1=XB(MX)/ZNA
      X2=XB(I1)/ZNA
      XBRN=PARAB(X1,X2,F1,F2)*ZNA
      IF (XBRN.LE.XBR) GO TO 186
      IF (XBRN.GT.XBRM) GO TO 190
      XBR=XBRN
      GO TO 192
  186 XBRM=XBR
      XBR=XBRN
      GO TO 192
C
C.....THE BOUNDARY TO BE EXTENDED
  190 XBR=(XBR+XBRM)/2.
  192 IF (IFP.EQ.1) GO TO 340
C
C.....INITIALIZE FORCE AND MOMENT VARIABLES.
      FTX=0.0
      FTZ=0.0
      TORK=0.0
      IJ=0
C
C.....CALCULATE FORCES AND MOMENTS APPLIED.
      DO 199 I=1,MX
      MY=NY(I)
      DO 199 J=1,MY
      IJ=IJ+1
      IF (J.EQ.1) GO TO 196
      C=1.
      GO TO 198
  196 C=.5
  198 FTT=P(I,J)*AR(I)*C
      FTZ=FTZ+FTT
      FTF=FTT*XN(IJ)/ZN(IJ)
      FTX=FTX+FTF
       TORK=TORK+FTT*X(IJ)-FTF*Z(IJ)
  199 CONTINUE
      FTX=2.*FTX
      FTZ=2.*FTZ
      TORK=2.*TORK
C
C.....WRITE OUT THE LOADING CONDITION
      IF (IBUG.EQ.0)GO TO 902
      IF ((ITM-IT).GT.IBUG)GO TO 902
      WRITE(NNW,216) FTX,FTZ,TORK
 902  CONTINUE
      GO TO 340
  200 I1=I-1
C
C.....FIND THE MAX X-INTERCEPT BY INTERPOLATION
      IN=N-NY(MX)
      ZNA=(ZN(IN)+ZN(N))/2.
      X1=XB(I1)/ZNA
      P1=P(I1,J)
      X2=XB(I)/ZNA
      P2=P(I,J)
      XXX=0.0
      XBR=YFUN(P1,X1,P2,X2,XXX)*ZNA
      XBRM=XB(I)
      IF (IL.EQ.2) GO TO 340
C
C.....LOCATE THE POINTS FOR EXTRAPOLATION
      KI=1+NY(1)
      ZNA=(ZN(1)+ZN(KI))/2.
      F1=P(1,1)
      F2=P(2,1)
      IF (F2.LE.F1) GO TO 260
      X1=XB(1)/ZNA
      X2=XB(2)/ZNA
C
C.....FIND THE NEW LEFT X-BOUNDARY BY EXTRAPOLATION
```

```
      XBLN=PARAB(X1,X2,F1,F2)*ZNA
      IF  (XBLN.GE.XBL) GO TO 255
      IF  (XBLN.LT.XBLM) GO TO 260
,11450
      GO  TO 340
  255 XBLM=XBL
      XBL=XBLN
      GO  TO 340
  260 XBL=(XBL+XBLM)/2.
C
C.....WRITE OUT THE X-INTERCEPTS OF CONTACT PATCH
 340  CONTINUE
      IF  (IBUG.EQ.0)GO TO 901
      IF  ((ITM-IT).GT.IBUG)GO TO 901
      WRITE(NNW,221) XBL,XBR
 901  CONTINUE
C
C.....CHECK THE TOLERANCE ON YB(NC)
      IF  (DABS(1.-RY/YBN(NC)).LE.EPS) GO TO 450
C
C.....MAXIMUM NO. OF ITERATIONS REACHED ?
      IF  (IT.EQ.ITM) GO TO 450
C
C.....STORE THE NEW YB(I) AND OLD XB(I) IN A NEW ARRAYS
      DO 410 I=1,MX
      XBN(I)=XB(I)
  410 CONTINUE
C
C.....REPEAT THIS PROCEDURE , AS MANY TIMES AS REQIRED
      GO  TO 5
C
 450  CONTINUE
      WRITE(NNW,223) IT
      WRITE(NNW,213)
      WRITE(NNW,214)
      WRITE(NNW,222) (XB(I),YB(I),I=1,MX)
C
C.....PRINT THE SOLUTION (PRESSURE DISTRIBUTION).
      WRITE(NNW,211)
      WRITE(NNW,215) (I,X(I),Y(I),Z(I),F(I,1),I=1,N)
C.....WRITE OUT THE FINAL BOUNDARY OF CONTACT PATCH
      WRITE (NNW,221) XBL,XBR
      WRITE(NNW,216) FTX,FTZ,TORK
C
C
 232  FORMAT (1H1,2I5,7F10.3)
 233  FORMAT (1X,8F7.3,1F13.3,4I8)
C
C
C     CALL SUBROUTINE SUBSIG TO CALCULATE SUBSURFACE STRESSES
C
C
      CALL CREPAG(F,MX,NY,AMUX,AMUY)
      CALL SUBSIG(MX,NY,XBL,XBR,X,Y,Z,HX,HY,XN,YN,ZN,F,ANU2,
     $AMUX,AMUY,IOPT,IBUG,NNW,NNR)
  999 WRITE(NNW,1000)
 1000 FORMAT (1H1)
      RETURN
C
C.....FORMAT STATMENTS
  211 FORMAT (//(22X,'NODE',8X,'XI',12X,'ETA',12X,'ZETA',13X,'P'))
  212 FORMAT(20X,'E1=',E13.7,2X,'ANU1=',F5.3,2X,'E2=',E13.7,2X,'ANU2=',
     $F5.3)
  213 FORMAT (//(40X,'BOUNDARY OF CONTACT REGION'))
  214 FORMAT (/(18X,'XI',12X,'ETA',12X,'XI',13X,'ETA',12X,'XI',14X,'ETA'
     $))
  215 FORMAT (/(20X,I5,4E15.4))
  216 FORMAT(//(23X,'XI-FORCE=',F8.1,3X,'ETA-FORCE=',F8.1,3X,'ETA-MOMENT
     $=',F8.1))
  217 FORMAT (I15,2F10.0)
  218 FORMAT (8F10.0)
  219 FORMAT (20X,'ITM=',I2,2X,'NC=',I2,2X,'MYOPT=',I1,1X,'MYMI=',I1,
     $2X,'MYMA=',I2,2X,'IBUG=',I2,2X,'IOPT=',I2)
  220 FORMAT (20X,'NSEG=',I2,5X,'XBL=',F7.4,5X,'XBR=',F7.4)
  221 FORMAT (/(24X,'LEFT XI-BOUNDARY=',F10.5,4X,'RIGHT XI-BOUNDARY=',F1
     $0.5))
  222 FORMAT (/(10X,6E15.7))
  223 FORMAT (///(46X,'ITERATION =',I2))
  224 FORMAT (20X,12I5)
```

```
  225 FORMAT(16I5)
  226 FORMAT (20A4)
  227 FORMAT (1H1,//20X,20A4/)
  228 FORMAT (20X,5(F5.3,5X))
  229 FORMAT (20X,'NX(I) ARE;')
  230 FORMAT (20X,'THE FOLLOWING IS RAT(I)')
  231 FORMAT (25X,'DELTA=',E12.5,10X,'EPS=',E12.5)
      END
      SUBROUTINE INSEP(X,Y,Z,FZ,XN,YN,ZN)
C
C-----------------------------------------------------------------------
C       PURPOSE.....
C          TO CALCULATE THE INITIAL SEPARATION BETWEEN RAIL AND
C       WHEEL
C
C       METHOD......
C       SEE "GEOMETRY OF RAIL AND WHEEL" BY B. PAUL AND J. HASHEMI.
C
C       STANDARD SUBPROGRAMS.....
C          SUBROUTINE RAIL (X,Z,XN,ZN)
C          SUBROUTINE MIDWEL (X,Z,ZIW)
C          SUBFUNCTION WHEEL (X,Y,ZI)
C
C       DESCRIPTION OF ARGUMENTS......
C          X,Y          COORDINATES OF A GIVEN POINT
C          Z            Z-COORDINATE OF THE POINT (X,Y) TO BE RETURNED
C                       TO CALLING PROGRAM
C          FZ           INITIAL SEPARATION AT POINT (X,Y)RETURNED TO
C                       CALLING PROGRAM.
C          XN,YN,ZN     COMPONENTS OF NORMAL (UNIT) TO BE RETURNED
C                       TO CALLING PROGRAM
C
C       DESCRIPTION OF INPUT.....
C       WHEEL PROFILE INFORMATION (REFERRED TO WHEEL COORDINATE (X,Z)
C       SYSTEM)
C          NW           NUMBER OF SEGMENTS IN WHEEL PROFILE
C          AW(I)        X-COMP. OF CENTER OF CIRCULAR ARC OR SLOPE OF LINEAR
C                       SEGMENT I
C          BW(I)        Z-COMP. OF CENTER OF CIRCULAR ARC OR Z-INTERCEPT OF
C                       LINEAR SEGMENT I
C          CW(I)         RADIUS OF ARC  SEGMENT .(ZERO FOR LINEAR SEGMENT)
C          RW           NOMINAL RADIUS OF WHEEL (AT ORIGIN OF X,Z SYSTEM)
C          XWC,ZWC      COORDINATES OF THE INITIAL POINT OF CONTACT ON
C                       WHEEL PROFILE
C          THETAW       THE SLOPE OF TANGENT (DZ/DX) AT CONTACT POINT C
C          ATW(I)       TRANSFORMATION OF A(I)
C          BTW(I)       TRANSFORMATION OF B(I)
C          XICW(I)      TRANSFORMATION OF XCW(I)
C          XCW(I)       X-COORDINATE OF TRANSITION POINT
C          ZCW(I)       Z-COORDINATE OF TRANSITION POINT
C       TRANSFORMED RAIL PROFILE PARAMETERS (IN XI,ETA COORD. SYSTEM)
C          NR           NUMBER OF SEGMENTS IN RAIL PROFILE
C          ATR(I)       XI-COMP.OF CENTER OF CIRCULAR ARC OR SLOPE OF
C                       LINEAR SEGMENT I
C          BTR(I)       ZETA-COMP. OF CENTER OF CIRCULAR ARC (OR ZETA-
C                       INTERCEPT OF LINEAR SEGMENT I)
C          CR(I)        RADIUS OF CIRCULAR SEGMENT I (OR ZERO FOR LINEAR SEGMENT)
C          XICR(I)      XI-COORDINATE OF POINT WHERE JUMP IN CURVATURE OCCURS
C
C       INPUT ARRANGEMENTS......
C          CARD ID FORMAT      VARIABLES
C             B     (2I5)      NW,NR
C             C     (4F12.0)   RW,XWC,ZWC,TETAW
C             D     (6F12.0)   AW(I),BW(I),CW(I)
C             E     (6F12.0)   XCW(I),ZCW(I)
C             F     (6F12.0)   ATW(I),BTW(I)
C             G     (6F12.0)   XICW(I)
C
C-----------------------------------------------------------------------
      IMPLICIT REAL*8 (A-H,O-Z)
C
C.....THE DIMENSIONS ARE SET FOR MAXIMUM NUMBER OF SEGMENTS IN
C       RAIL AND WHEEL EQUAL TO 10.
      COMMON/RAIL01/ATR(99),BTR(99),CR(99),XICR(99),NR1
      COMMON/RAIL02/AR(99),BR(99),XCR(99)
      COMMON/WHEE01/ATW(99),BTW(99),CW(99),XICW(99),NW1
      COMMON/WHEE02/AW(99),BW(99),XCW(99)
      COMMON/WHEE03/ZCW(99)
      COMMON/WHEE04/XWC,ZWC,TETAW,RW,XRC,DELTA,XBL,XBR,EPS,ITM,NXB,IBUG
```

CONWHEEL

```
      COMMON/BODY1/E1,ANU1,NNR,NNW
      DATA IJ/0/
      IF (IJ.GT.0) GO TO 2
      EPS1=0.1 D-12
      NW=NW1+1
      NR=NR1+1
      NW1=NW-1
      NR1=NR-1
C
C.....PRINT THE INPUT OUT
      WRITE(NNW,45)
      WRITE(NNW,39)  RW,NW
      WRITE(NNW,40)  XWC,ZWC,TETAW
      WRITE(NNW,46)
      WRITE(NNW,41)  (AW(L),BW(L),CW(L),L=1,NW)
      WRITE(NNW,48)
      WRITE(NNW,42)  (XCW(L),ZCW(L),L=1,NW1)
      WRITE(NNW,50)
      WRITE(NNW,44)  NR
      WRITE(NNW,52)
      WRITE(NNW,43)  (XCR(L),L=1,NR1)
      WRITE(NNW,51)
      WRITE(NNW,41)  (AR(L),BR(L),CR(L),L=1,NR)
      WRITE(NNW,39)
      IJ=1
    2 IF (Y.NE.0.0) GO TO 10
      IF (DABS(X).LE.EPS1) GO TO 23
    5 CALL RAIL1(X,Z,XN,ZN)
      CALL MIDWEL (X,ZIW)
      ZI=ZIW
      GO TO 20
   10 ZIW=WHEEL(X,Y,ZI)
      ZI=ZIW
   20 FZ=ZIW-Z
      GO TO 30
   23 FZ=0.0
      ZI=0.0
      Z=0.0
      XN=0.0
      ZN=1.0
   30 YN=0.0
      RETURN
   36 FORMAT (6F12.0)
   37 FORMAT (16I5)
   38 FORMAT (15X,'RADIUS, RW=',E12.5,5X,'NO. OF SEGMENTS, NW=',I2)
   39 FORMAT (1H1)
   40 FORMAT (15X,'XWC=',E15.7,2X,'ZWC=',E15.7,2X,'THETAW=',E15.7)
   41 FORMAT(11X,3E15.7,5X,3E15.7)
   42 FORMAT (11X,2E15.7,2X,2E15.7,2X,2E15.7)
   43 FORMAT (10X,6E15.7)
   44 FORMAT (15X,'NO. OF SEGMENTS, NR=',I2)
   45 FORMAT (/,10X,'THE FOLLOWING IS WHEEL DATA')
   46 FORMAT (/,18X,'AW',13X,'BW',13X,'CW',18X,'AW',13X,'BW',13X,'CW')
   47 FORMAT (/,17X,3('ATW',12X,'BTW',14X))
   48 FORMAT (/,17X,3('XCW',12X,'ZCW',14X))
   49 FORMAT (/,15X,6('XICW',11X))
   50 FORMAT (/,10X,'THE FOLLWING IS RAIL DATA')
   51 FORMAT (/,17X,2('AR',12X,'BR',13X,'CR',17X))
   52 FORMAT (/,15X,6('XCR ',11X))
      END
      SUBROUTINE RAIL1(XI,ZETA,XN,ZN)
C
C-----------------------------------------------------------------------
C     PURPOSE.....
C         TO CALCULATE THE ZETA-COMPONENT OF THE PROFILE OF RAIL
C     FOR ANY GIVEN XI
C
C     METHOD......
C     SEE "GEOMETRY OF RAIL AND WHEEL" BY B. PAUL AND J. HASHEMI.
C
C     DESCRIPTION OF ARGUMENTS......
C         XI          X-COMPONENT OF THE POINT IN QUESTION
C         ZETA        Z-COMPONENT OF THE POINT TO BE RETURNED TO
C                     CALLING PROGRAM
C         XN,ZN       COMPONENTS OF UNIT NORMAL TO RAIL  SURFACE
C                     TO BE RETURNED TO CALLING PROGRAM
C-----------------------------------------------------------------------
C
      IMPLICIT REAL*8 (A-H,O-Z)
```

CONWHEEL

```
      COMMON/RAIL01/ATR(99),BTR(99),CR(99),XICR(99),NR1
      DO 5 I=1,NR1
      IF (XI.LE.XICR(I)) GO TO 8
    5 CONTINUE
      I=NR1+1
    8 IF (CR(I).EQ.0.0) GO TO 10
      ZETA=BTR(I)+DSQRT(CR(I)**2-(XI-ATR(I))**2)
      XN=(XI-ATR(I))/CR(I)
      ZN=DABS((ZETA-BTR(I))/CR(I))
      GO TO 20
   10 ZETA=ATR(I)*XI+BTR(I)
      ZN=1./DSQRT(1.+ATR(I)**2)
      XN=-ATR(I)*ZN
   20 RETURN
      END
      SUBROUTINE MIDWEL (XI,ZETA)
C
C-------------------------------------------------------------------------
C     PURPOSE.....
C         TO CALCULATE ZETA-COMPONENT OF WHEEL PROFILE IN
C     MIDPLANE FOR ANY GIVEN XI
C
C     METHOD......
C     SEE "GEOMETRY OF RAIL AND WHEEL" BY B. PAUL AND J. HASHEMI.
C
C     DESCRIPTION OF ARGUMENTS......
C         XI          X-COMPONENT OF THE POINT IN QUESTION
C         ZETA        Z-COMPONENT OF THE POINT TO BE RETURNED TO
C                     CALLING PROGRAM
C-------------------------------------------------------------------------
C
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON/WHEE01/ATW(99),BTW(99),CW(99),XICW(99),NW1
      COMMON/WHEE02/AW(99),BW(99),XC(99)
      DO 5 I=1,NW1
      IF (XI.LE.XICW(I)) GO TO 8
    5 CONTINUE
      I=NW1+1
    8 IF (CW(I).EQ.0.0) GO TO 10
      ZETA=BTW(I)+DSQRT(CW(I)**2-(XI-ATW(I))**2)
      GO TO 20
   10 ZETA=ATW(I)*XI+BTW(I)
   20 RETURN
      END
      DOUBLE PRECISION FUNCTION WHEEL (X,Y,ZI)
C
C-------------------------------------------------------------------------
C     PURPOSE.....
C         TO CALCULATE ZETA-COMPONENT OF WHEEL AT ANY GIVEN POINT
C     XI AND ETA
C
C     METHOD......
C     SEE "GEOMETRY OF RAIL AND WHEEL" BY B. PAUL AND J. HASHEMI.
C
C     DESCRIPTION OF ARGUMENTS......
C         X,Y         COORDINATES OF A POINT
C         ZI          INITIAL GUESS FOR Z- COMPOMENT OF THE POINT
C         WHEEL       THE VALUE FOR Z TO BE RETURNED TO CALLING PROG.
C-------------------------------------------------------------------------
C
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON/WHEE04/XO,ZO,TETA,RW,XRC,DELTA,XBL,XBR,EPS,ITM,NXB,IBUG
      DATA JJ/0/
      IF (JJ.EQ.1) GO TO 5
      ST=DSIN(TETA)
      CT=DCOS(TETA)
      EPS=.1D-12
      JJ=1
    5 IJ=1
      Z=ZI
      XX=XO+X*CT
      ZZ=ZO+X*ST
    7 XOX=XX-Z*ST
      ZOZ=ZZ+Z*CT
      CALL WHEELZ (XOX,ZM,DZM)
      F1=(RW-ZM)**2-(RW-ZOZ)**2
      DFZ=2.*((RW-ZM)*ST*DZM+CT*(RW-ZOZ))
      DZ=(Y**2-F1)/DFZ
      IF (DABS(Z).LT.EPS) GO TO 10
```

```
      IF  (DABS(DZ/Z).LT.(.1D-07)) GO TO 20
      GO  TO 10
    6 IF  (DABS(DZ).LE.EPS) GO TO 20
   10 IF  (IJ.GT.10) GO TO 50
      Z=Z+DZ
      IJ=IJ+1
      GO  TO 7
   20 WHEEL=Z
      GO  TO 60
   50 WHEEL=Z
      WRITE(16,55) X,Z,DZ
   55 FORMAT (20X,'ERROR ',10X,'X=',E12.5,5X,'Z=',E12.5,5X,'DZ=',E12.5)
   60 RETURN
      END
      SUBROUTINE WHEEL2 (X,Z,DZ)
C
C------------------------------------------------------------------------
C     PURPOSE.....
C         TO CALCULATE THE Z-COMPONENY ,DZ/DX OF THE WHEEL POOFILE
C     IN MIDPLANE IN THE LOCAL SYSTEM OF COORDINATES X-Y-Z
C
C     METHOD......
C     SEE "GEOMETRY OF RAIL AND WHEEL" BY B. PAUL AND J. HASHEMI.
C
C     DESCRIPTION OF ARGUMENTS......
C         X          X-COMPONENT OF THE POINT
C         Z          Z-COMPONENT OF THE POINT TO BE RETURNED
C         DZ         DERIVATIVE OF Z WITH RESPECT TO X TO BE RETURNED
C------------------------------------------------------------------------
C
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON/WHEE02/ A(99),B(99),XC(99)
      COMMON/WHEE01/ATW(99),BTW(99),CW(99),XICW(99),NW1
      DO 5 I=1,NW1
      IF  (X.LE.XC(I))  GO TO 8
    5 CONTINUE
      I=NW1+1
    8 IF  (CW(I).EQ.0.0) GO TO 10
      Z=B(I)+DSQRT(CW(I)**2-(X-A(I))**2)
      DZ=(A(I)-X)/(Z-B(I))
      GO  TO 20
   10 Z=A(I)*X+B(I)
      DZ=A(I)
   20 RETURN
      END
      DOUBLE PRECISION FUNCTION GDA(XF,YF,ZF,XNF,YNF,ZNF,XS,YS,ZS,XNS,YN
     1S,ZNS,HXS,HYS)
C
C------------------------------------------------------------------------
C     GDA  (XF,YF,ZF,XNF,YNF,ZNF,XS,YS,ZS,XNS,YNS,ZNS,HXS,HYS)
C
C     PURPOSE.....
C         TO EVALUATE THE INTEGRAL OF THE GREEN FUNCTION G OVER THE
C     AREA DA
C
C     METHOD......
C
C     STANDARD SUBPROGRAMS.....
C         FUNCTION BIF(H1,H2,H3,H4,C)
C
C     DESCRIPTION OF ARGUMENTS......
C------------------------------------------------------------------------
C
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON /BODY1/ E1,ANU1,NNR,NNW
      COMMON /BODY2/ E2,ANU2
      PI=3.141592654
      EPS=1.E-10
      C=1.
      XXSF=XS-XF
      YYSF=YS-YF
      ZZSF=ZS-ZF
      DSF=DSQRT(XXSF**2+ZZSF**2)
      R=DSQRT(DSF**2+YYSF**2)
      CAC=1./DSQRT(1.+(XNS/ZNS)**2)
      CBC=1./DSQRT(1.+(YNS/ZNS)**2)
      HXN=HXS/CAC
      HYN=HYS/CBC
      IF  (HXN.GT.HYN) GO TO 2
```

```
      H=HYN
      GO TO 3
    2 H=HXN
    3 IF (R.LE.(1.5*H)) GO TO 6
      GDA=HXS*HYS*(GR1(XF,YF,ZF,XS,YS,ZS,XNF,YNF,ZNF,XNS,YNS,ZNS,R)+GR2
     1(XF,YF,ZF,XS,YS,ZS,XNF,YNF,ZNF,XNS,YNS,ZNS,R))/ZNS
      GO TO 50
    6 H1=(YYSF+.5*HYS)/CBC
      IF (DABS(H1).LE.EPS) GO TO 10
      H4=H1-HYN
      IF (DABS(H4).GT.EPS) GO TO 20
   10 C=.5
      H1=HYN
      H4=-HYN
   20 IF (DABS(XXSF).GT.EPS) GO TO 30
   25 H2=.5*HXN
      GO TO 35
   30 H2=DSF*XXSF/DABS(XXSF)+.5*HXN
   35 H3=H2-HXN
      CK=(1.-ANU1**2)/(PI*E1)+(1.-ANU2**2)/(PI*E2)
      GDA=CK*BIF(H1,H2,H3,H4,C)
   50 RETURN
      END
      DOUBLE PRECISION FUNCTION BIF (H1,H2,H3,H4,C)
C
C-----------------------------------------------------------------------
C     BIF (H1,H2,H3,H4,C)
C
C     PURPOSE.....
C         TO EVALUATE THE INTEGRAL OF DA/R WHEN IT IS SINGULAR
C     USING THE LUKE'S FORMULA
C
C     METHOD......
C
C     DESCRIPTION OF ARGUMENTS......
C-----------------------------------------------------------------------
      IMPLICIT REAL *8 (A-H, O-Z)
      PI=3.14159254
      T1=DATAN(H2/H1)
      B1=DATAN(H3/H1)
      T2=DATAN(H1/H2)
      B2=DATAN(H4/H2)
      T3=DATAN(H1/H3)
      B3=DATAN(H4/H3)
      T4=DATAN(H2/H4)
      B4=DATAN(H3/H4)
      AT1=DABS(T1)
      AT2=DABS(T2)
      AT3=DABS(T3)
      AT4=DABS(T4)
      AB1=DABS(B1)
      AB2=DABS(B2)
      AB3=DABS(B3)
      AB4=DABS(B4)
      C11=DLOG(DTAN(PI/4.+AT1/2.))
      C12=DLOG(DTAN(PI/4.+AB1/2.))
      C21=DLOG(DTAN(PI/4.+AT2/2.))
      C22=DLOG(DTAN(PI/4.+AB2/2.))
      C31=DLOG(DTAN(PI/4.+AT3/2.))
      C32=DLOG(DTAN(PI/4.+AB3/2.))
      C41=DLOG(DTAN(PI/4.+AT4/2.))
      C42=DLOG(DTAN(PI/4.+AB4/2.))
      C1=T1/AT1*C11-B1/AB1*C12
      C2=T2/AT2*C21-B2/AB2*C22
      C3=T3/AT3*C31-B3/AB3*C32
      C4=T4/AT4*C41-B4/AB4*C42
      BIF=DABS(DABS(H1)*C1+DABS(H2)*C2-DABS(H3)*C3-DABS(H4)*C4)*C
      RETURN
      END
      DOUBLE PRECISION FUNCTION GR1(XF,YF,ZF,XS,YS,ZS,XNF,YNF,ZNF,XNS,YN
     1S,ZNS,R)
C-----------------------------------------------------------------------
C
C     PURPOSE.....
C         TO EVALUATE THE GREEN FUNCTION FOR BODY 1(BOUSSINESQ
C     INFLUENCE FUNCTION IS USED)
C
C     METHOD......
C
```

```
CONWHEEL
C       DESCRIPTION OF ARGUMENTS......
C-------------------------------------------------------------------------------
C
        IMPLICIT REAL *8 (A-H,O-Z)
        COMMON /BODY1/ E1,ANU1,NNR,NNW
        PI=3.141592654
        GR1=(1.-ANU1**2)/(PI*E1*R)
        RETURN
        END
        DOUBLE PRECISION FUNCTION GR2(XF,YF,ZF,XS,YS,ZS,XNF,YNF,ZNF,XNS,YN
       1S,ZNS,R)
C
C-------------------------------------------------------------------------------
C       PURPOSE.....
C           TO EVALUATE THE GREEN FUNCTION FOR BODY 2(BOUSSINESQ
C       INFLUENCE FUNCTIOM IS USED)
C
C       METHOD.......
C
C       DESCRIPTION OF ARGUMENTS......
C-------------------------------------------------------------------------------
C
        IMPLICIT REAL *8 (A-H,O-Z)
        COMMON /BODY2/ E2,ANU2
        PI=3.141592654
        GR2=(1.-ANU2**2)/(PI*E2*R)
        RETURN
        END
        DOUBLE PRECISION FUNCTION PARAB(SM,SL,PM,PL)
C-------------------------------------------------------------------------------
C       PARAB(SM,SL,PM,PL)
C
C       PURPOSE......
C           TO EXTRAPOLATE BETWEEN TWO POINTS AND FIND ORDINATE
C       WHEN ABSESSIA IS ZERO
C
C       METHOD.......
C           PARABOLIC EXTRAPOLATION BETWEEN THE TWO POINTS AND
C       PREPENDICULAR TO ORDINATE IS USED.
C
C       DESCRIPTION OF ARGUMENTS.......
C           (SM,PM)    COORDINATES OF POINT M
C           (SL,PL)    COORDINATES OF POINT L
C           PARAB      VALUE OF THE ORDINATE TO BE RETURNED TO THE
C                      CALLING PROGRAM.
C-------------------------------------------------------------------------------
C
        IMPLICIT REAL*8 (A-H,O-Z)
        PARAB=(PL**2*SM-PM**2*SL)/(PL**2-PM**2)
        RETURN
        END
        SUBROUTINE SUBSIG(MX,NY,XBL,XBR,X,Y,Z,HX,HY,XN,YN,ZN,F,ANU,
       $AMUX,AMUY,IOPT,IBUG,NNW,NNR)
C
C
C       BY B.PAUL AND S.SINGH
C
C       11TH APRIL 1980
C
C
C*****PURPOSE
C
C       THIS SUBROUTINE CALCULATES THE NORMAL AND SHEAR STRESSES BELOW
C       THE SURFACE OF THE BODY OF AN ARBITRARY PROFILE WHEN THE SURFACE
C       OF THE BODY IS LOADED WITH ARBITRARY LOADS,BOTH NORMAL AND
C       TANGENTIAL
C
C
C*****METHOD
C
C       THE CALCULATIONS ARE DONE IN THE FOLLOWING STAGES
C
C
C         1.THE OUTPUT INFORMATION OF PROGRAM 'CONFORM'IS TRANSFORMED
C           SO THAT IT IS USABLE BY SUBSIG
C
C         2.ESTABLISH A LOCAL COORDINATE SYSTEM SUCH THAT THE Z-AXIS
C           IS NORMAL TO THE SURFACE
C
```

```
C         3.CALCULATE ALL VECTOR QUANTATIES E.G. POSITION OF SOURCE
C           POINTS,FIELD POINTS,CELL WIDTHS,CELL LENGTHS,NORMAL AND
C           TANGENTIAL LOADS WITH REFERENCE TO THE LOCAL COORDINATE
C           SYSTEM.
C
C         4.CALCULATE THE STRESSES DUE TO THE NORMAL AND TANGENTIAL
C           LOADS USING BOUSSINESQE'S SOLUTION FOR NORMAL LOADS
C           CERRUTI'S SOLUTION FOR THE TANGENTIAL LOADS IN THE TWO
C           TANGENTIAL DIRECTIONS,ASSUMING A SEMI INFINITE BODY
C           WITH SINGLE CONCENTRATED LOAD AT THE ORIGIN
C
C         5.TRANSFORMING THE STRESS COMPONENTS FROM THE LOCAL
C           COORDINATE SYSTEM TO THE GLOBAL COORDINATE SYSTEM
C
C         6.SUPERIMPOSING ALL STRESS COMPONENTS IN THE GLOBAL
C           COORDINATE SYSTEM DUE TO ALL SURFACE LOADS
C
C         7.WRITING OUT THE RESULTS VIZ. THE SIX STRESS COMPONENTS
C           AND THE EQUIVALENT STRESS.
C
C
C*****OPTIONS
C
C         THE USER CAN EXERCISE THE FOLLOWING OPTIONS
C
C
C             IOPT=0    THE PROGRAM SCANS THE LOADED REGION OF THE SURFACE
C                       OF THE BODY AND DETERMINES THE LOCATION OF THE
C                       POINT WITH MAXIMUM NORMAL LOAD AND CALCULATES
C                       THE STRESSES WITHIN THE BODY UP TO A DEPTH EQUAL
C                       TO THE LENGTH OF THE  LOADED
C                       REGION AT AS MANY POINTS ALONG THE DEPTH AS THE
C                       NUMBER OF CELLS ALONG THE LENGTH AXIS.
C
C             IOPT=1    THE USER MAY SPECIFY THE THE X AND THE Y LOCATION
C                       OF THE POINT OF INTEREST,THE MAXIMUMDEPTH TO BE
C                       SCANNED,AND THE NUMBER OF POINTS ALONG THE DEPTH
C                       AT WHICH THE STRESSES ARE TO BE CALCULATED.
C
C             IOPT=2    THE USER MAY SPECIFY THE X , Y AND THE Z LOCATION
C                       OF EACH OF THE POINTS OF INTEREST,AND THE TOTAL
C                       NUMBER OF POINTS OF INTEREST.
C
C
C
C*****DESCRIPTIONS OF VARIABLES
C
C             MX        NUMBER OF CELLS ALONG X-AXIS.
C             NY        NUMBER OF CELLS ALONG Y-AXIS AT EACH X-LOCATION
C             XBL       LEFT BOUNDARY OF   THE LOADED
C                       REGION
C             XBR       RIGHT BOUNDARY OF THE   LOADED
C                       REGION.
C             X         X-COORDINATE OF THE CELL CENTROID (SOURCE POINT)
C             Y         Y-COORDINATE OF THE CELL CENTROID (      --     )
C             Z         Z-COORDINATE OF THE CELL CENTROID (      --     )
C             HX        LENGTH OF CELL ALONG X-DIRECTION
C             HY        WIDTH OF CELL ALONG Y-DIRECTION
C             XN        X-COMPONENT OF THE UNIT NORMAL TO THE SURFACE
C             YN        Y-COMPONENT OF THE UNIT NORMAL TO THE SURFACE
C             ZN        Z-COMPONENT OF THE UNIT NORMAL TO THE SURFACE
C             F(I,1)    NORMAL LOAD ON THE SURFACE (Z-DIRECTION)
C             F(I,2)    TANGENTIAL LOAD ON THE SURFACE (X-DIRECTION)
C             F(I,3)    TANGENTIAL LOAD ON THE SURFACE (Y-DIRECTION)
C             ANU       POISSON'S RATIO
C             SSXX      NORMAL STRESS IN THE X-DIRECTION
C             SSYY      NORMAL STRESS IN THE Y-DIRECTION
C             SSZZ      NORMAL STRESS IN THE Z-DIRECTION
C             SSXY      SHEAR STRESS IN THE XY-DIRECTION
C             SSYZ      SHEAR STRESS IN THE YZ-DIRECTION
C             SSZX      SHEAR STRESS IN THE ZX-DIRECTION
C             SCRIT     EQUIVALENT STRESSDEFINED AS SQUARE ROOT OF THREE TIMES J2
C             J2        SECOND INVARIANT OF STRESS DEVIATION TENSOR
C
C
      IMPLICIT REAL*8(A-H,O-Z)
       REAL J2,JJ2
      DIMENSION B(100,100),F(100,3),WKAREA(100),X(100),Y(100),Z(100)
      DIMENSION P(20,5),XB(20),YB(20),HX(20),HY(20),XBN(20),YBN(20)
```

CONWHEEL

```
      DIMENSION YBM(20),YBMM(20),AR(20),TITLE(20),RAT(10),NY(20),NX(10)
      DIMENSION XN(100),YN(100),ZN(100),FXP(20,10),FYP(20,10),FZP(20,10)
      DIMENSION XSX(20),YSY(20,10),ZSZ(20,10),WX(20,10),WY(20,10)
      DIMENSION ZFF(20),XFM(20),YFM(20),ZFM(20),NSY(20)
      DIMENSION XNN(20,10),YNN(20,10),ZNN(20,10)
      DIMENSION AXX(20,10,20),AYY(20,10,20),AZZ(20,10,20)
     $ ,AXY(20,10,20),AYZ(20,10,20),AZX(20,10,20),ACT(20,10,20)
C
C
C.....TRANSFORM THE OUTPUT OF CONFORM SO THAT IT IS ACCEPTABLE
C.....FOR USE BY SUBROUTINE 'SUBSIG'
C
C     1.DEFINITION OF FULL
C     2.TRANSFORMATION OF COORDINATES OF SOURCE POINTS TO DOUBLE SUBSCRIPTED
C       VARIABLES
C
      IF (IOPT.EQ.3) GO TO 60
      IJ=0
      DO 5 I=1,MX
      NSSY=NY(I)
      NSY(I)=2*NY(I)-1
      DO 5 J=1,NSSY
      IJ=IJ+1
      K1=NSSY+(J-1)
      K2=NSSY-(J-1)
      XSX(I)=X(IJ)
      YSY(I,K1)=Y(IJ)
      YSY(I,K2)=-Y(IJ)
      ZSZ(I,K1)=Z(IJ)
      ZSZ(I,K2)=Z(IJ)
      XNN(I,K1)=XN(IJ)
      XNN(I,K2)=XN(IJ)
      YNN(I,K1)=YN(IJ)
      YNN(I,K2)=-YN(IJ)
      ZNN(I,K1)=ZN(IJ)
      ZNN(I,K2)=ZN(IJ)
      FXP(I,K1)=F(IJ,2)
      FXP(I,K2)=F(IJ,2)
      FYP(I,K1)=F(IJ,3)
      FYP(I,K2)=-F(IJ,3)
      FZP(I,K1)=F(IJ,1)
      FZP(I,K2)=F(IJ,1)
C.....WIDTHS OF INDIVIDUAL CELLS
      WX(I,K1)=HX(I)
      WX(I,K2)=HX(I)
      WY(I,K1)=HY(I)
      WY(I,K2)=HY(I)
C
 5    CONTINUE
C.....BEGENING OF SUBSIG PROPER
C
C
      IF (IBUG.EQ.0) GO TO 101
      WRITE(NNW,900)
 900  FORMAT(1H1,24X,'I N P U T     D A T A' / 24X,
     1'-------------------')
      WRITE(NNW,901)
 901  FORMAT(2X,'POINT',6X,'SOURCE  POINT'10X,'S U R F A C E'
     14X,'T R A C T I O N S',/,1X,'INDICES',4X,'GLOBAL  COORDINATES'
     2,14X,'LOCAL  COORDINATES')
      WRITE(NNW,902)
 902  FORMAT(1X,'--------',2X,'--------------------------',3X,
     1'-----------------------------------')
      WRITE(NNW,903)
 903  FORMAT(2X,'I',3X,'J',6X,'X S',6X,'Y S',6X,'Z S',7X,
     1'TX',10X,'TY',10X,'TZ')
      WRITE(NNW,902)
      DO 101 I=1,MX
      LMY=NSY(I)
      DO 101 J=1,LMY
      WRITE(NNW,904)I,J,XSX(I),YSY(I,J),ZSZ(I,J),FXP(I,J),FYP(I,J),
     $FZP(I,J)
 101  CONTINUE
 904  FORMAT(I3,1X,I3,1X,3F9.3,2X,E11.4,1X,E11.4,1X,E11.4)
      NR=NNR
      IF (IOPT.EQ.0) GO TO 11
      IF (IOPT.EQ.1) GO TO 12
      IF (IOPT.EQ.2) GO TO 13
C
```

```
C       (IOT=2)USER SPECIFIES X,Y,Z WHERE STRESSES ARE WANTED
 13     CONTINUE
        READ (NR,800)NP
        DO 21 N=1,NP
        READ(NR,801)XFM(N),YFM(N),ZFF(N)
 21     CONTINUE
         L=1
         M=1
        GO TO 19
C
C       (IOPT=1)USER SPECIFIES LOCATION FOR DEPTH PROBE
 12     READ(NR,800)NP
        READ(NR,801)XFM(1),YFM(1),ZFM(1)
        DEPH=ZFM(1)/NP
        AN=0.0
        DO 15 IN=1,NP
        ZFF(IN)=(AN-0.5)*DEPH
        AN=AN-1.0
 15     CONTINUE
        DO 22 N=1,NP
        XFM(N)=XFM(1)
        YFM(N)=YFM(1)
 22     CONTINUE
         L=1
         M=1
        GO TO 19
C
C       (IOPT=0)PROGRAM SCANS FOR MAX. PRESSURE
 11     DEPH=-XBL+XBR
        DD=DEPH/MX
        AN=0.0
        DO 16 IN=1,MX
        ZFF(IN)=(AN-0.5)*DD
        AN=AN-1.0
 16     CONTINUE
        PMAX=0.0
        IJ=0
        DO 17 I=1,MX
        LMY=NSY(I)
        DO 17 J=1,LMY
        IJ=IJ+1
 802    FORMAT(7I5,2F20.4)
        IF (FZP(I,J).LT.PMAX) GO TO 17
        PMAX=FZP(I,J)
        IMAX=I
        JMAX=J
 17     CONTINUE
        NP=MX
        DO 23 N=1,NP
        XFM(N)=XSX(IMAX)
        YFM(N)=YSY(IMAX,JMAX)
 23     CONTINUE
        L=IMAX
        M=JMAX
        GO TO 19
 19     CONTINUE
 800    FORMAT(1I10)
 801    FORMAT(8F10.4)
C
        PI=3.141592654
C
C
        DO 20 N=1,NP
C
C
C       SELECTION OF FIELD POINTS
        XFO=XFM(N)
        YFO=YFM(N)
        ZFO=ZFF(N)
C
C      INITIALISATION OF RESULTANT STRESS MATRIX
        SIGXX=0.0
        SIGYY=0.0
        SIGZZ=0.0
        SIGXY=0.0
        SIGYZ=0.0
        SIGZX=0.0
C
C       SELECTION OF SOURCE POINTS
```

```
      NC1=20
 998   FORMAT(1X,10I5)
      DO 30 I=1,MX
      NC2=30
      LMY=NSY(I)
      DO 30 J=1,LMY
      NC3=31
C
C
C     COORDINATES OF SOURCE POINTS--GLOBAL--
      XS0=XSX(I)
      YS0=YSY(I,J)
      ZS0=ZSZ(I,J)
C
C.....WIDTH OF SOURCE CELLS
      WXX=WX(I,J)
      WYY=WY(I,J)
C
C.....DIRECTION COSINES OF UNIT NORMAL AT SOURCE POINT
      XXN=XNN(I,J)
      YYN=YNN(I,J)
      ZZN=ZNN(I,J)
C
C.....ELEMENTS OF STRESS TENSOR TRANSFORMATION MATRIX
      CALL TRANS1(XXN,YYN,ZZN,A11,A12,A13,A21,A22,A23,A31,A32,A33)
C
C
C.....TRANSFORMATION OF FIELD POINT TO LOCAL COORDINATES
      XF=XF0*A11+YF0*A12+ZF0*A13
      YF=XF0*A21+YF0*A22+ZF0*A23
      ZF=XF0*A31+YF0*A32+ZF0*A33
C
C.....TRANSFORMATION OF SOURCE POINT COORDINATES
      XS=XS0*A11+YS0*A12+ZS0*A13
      YS=XS0*A21+YS0*A22+ZS0*A23
      ZS=XS0*A31+YS0*A32+ZS0*A33
C
C.....TRANSFORMATION OF WIDTHS
      WWX=WXX*A11+WYY*A12
      WWY=WXX*A21+WYY*A22
C
C
C      READING IN OF LOADS
      PPX=FXP(I,J)*WWX*WWY
      PPY=FYP(I,J)*WWX*WWY
      PPZ=FZP(I,J)*WWX*WWY
C
C      DIVISION OF INDIVIDUAL CELLS INTO SUBCELLS FOR NECESSARY ACCURACY
       IN=1
 500   CONTINUE
       MI=IN
       MJ=IN
C
C      INITIALIZING STRESS TENSOR TO ZERO FOR INDIVIDUAL SUBCELL LOADS
       SXX=0.0
       SYY=0.0
       SZZ=0.0
       SXY=0.0
       SYZ=0.0
       SZX=0.0
C
C      DETERMINATION OF STRESSES DUE TO LOADS IN INDIVIDUAL SUB CELLS
       NC=0
       DO 510 II=1,MI
      NC4=510
       DO 510 JJ=1,MJ
       NC5=511
C
C      DETERMINATION OF COORDINATES OF LOADS IN EACH SUBCELL
       XXS=XS-(WWX/2.0)*(MI-1.0-(II-1.0)*2)/MI
       YYS=YS-(WWY/2.0)*(MJ-1.0-(JJ-1.0)*2)/MJ
       ZZS=ZS
C
C      DETERMINATION OF LOADS ON EACH SUBCELL
       PX=PPX/(MI*MJ)
       PY=PPY/(MI*MJ)
       PZ=PPZ/(MI*MJ)
C
C      TRANSFORMATION OF COORDINATES WITH RESPECT TO LOADS ON SUBCELLS
```

```
      XX =XF-XXS
      YY =YF-YYS
      ZZ =ZF-ZZS
C
C
C     CALCULATION OF STRESS DUE TO EACH LOAD
      CALL SUBFLT(XX,YY,ZZ,PX,PY,PZ,ANU,SXX,SYY,SZZ,SXY,SYZ,SZX)
C
C
 510  CONTINUE
C.......TRANSFORMATIONOF STRESS TENSOR ELEMENTS TO GLOBAL
C       COORDINATES
C
      CALL TRANS2(A11,A12,A13,A21,A22,A23,A31,A32,A33,SXX,SYY,SZZ,
     $SXY,SYZ,SZX,SSXX,SSYY,SSZZ,SSXY,SSYZ,SSZX)
C
C     CALCULATION OF SECOND INVARIANT AND CRITICAL STRESS
      JJ2=((SSXX-SSYY)**2+(SSYY-SSZZ)**2+(SSZZ-SSXX)**2)/6.0
      SSCT=(3.0*JJ2)**0.5
      WWXY=DMIN1(WWX,WWY)
      IF ((ZZ/WWXY).GT.4.0)GO TO 530
      IF (IN.GT.1) GO TO 520
      SSCT1=SSCT
      IN=IN+1
      GO TO 500
 520  SSCT2=SSCT
      DSSCT=DABS(SSCT2-SSCT1)
      IF (SSCT1.NE.0.000)DSSCT=DSSCT/SSCT1
      IF (DSSCT.LE.0.05)GO TO 530
      SSCT1=SSCT2
      IN=IN+1
      IF (IN.LE.5) GO TO 500
 530  CONTINUE
C
C
C     SUMMATION OF STRESSES DUE TO ALL SOURCE POINTS
      SIGXX=SIGXX+SSXX
      SIGYY=SIGYY+SSYY
      SIGZZ=SIGZZ+SSZZ
      SIGXY=SIGXY+SSXY
      SIGYZ=SIGYZ+SSYZ
      SIGZX=SIGZX+SSZX
  30  CONTINUE
C
C
C     CALCULATION OF SECOND INVARIANT OF STRESS ND VON MISSES
C     CRITICAL STESS
      J2=((SIGXX-SIGYY)**2+(SIGYY-SIGZZ)**2+(SIGZZ-SIGXX)**2)/6.0
     $+(SIGXY**2+SIGYZ**2+SIGZX**2)
C
C
      SCRIT=(3.0*J2)**0.5
C
C
      AXX(L,M,N)=SIGXX
      AYY(L,M,N)=SIGYY
      AZZ(L,M,N)=SIGZZ
      AXY(L,M,N)=SIGXY
      AYZ(L,M,N)=SIGYZ
      AZX(L,M,N)=SIGZX
C
      ACT(L,M,N)=SCRIT
  20  CONTINUE
C
C
C     OUTPUT OF RESULTS
  40  CONTINUE
      WRITE(NNW,905)
 905  FORMAT (1H1,55X,'R  E  S  U  L  T  S',/,55X,'----------------------
     1--',//)
      WRITE(NNW,906)
 906  FORMAT (3X,'POINT',16X,'FIELD POINT',18X,'S U B   ',
     1'S U R F A C E',3X,'S T R E S S   C O M P O N E N T S'
     2,12X,'EQUIVALENT',/,2X,'INDICES',15X,'COORDINATES',89X,'STRESS')
      WRITE(NNW,907)
 907  FORMAT(1X,'---------',5X,'------------------------------',4X,
     1'----------------------------------------------------'
     2'--------------------',2X,'----------')
      WRITE(NNW,908)
```

```
 908   FORMAT(2X,'I   J   K',10X,'XF',8X,'YF',8X,'ZF',12X,'S XX',
      18X,'S YY',8X,'S ZZ',8X,'S XY',8X,'S YZ',8X,'S ZX',8X,'S EQ')
       WRITE(NNW,907)
       DO 50 N=1,NF
       WRITE(NNW,909)L,M,N,XFM(N),YFM(N),ZFF(N),AXX(L,M,N),AYY(L,M,N),
      1AZZ(L,M,N),AXY(L,M,N),AYZ(L,M,N),AZX(L,M,N),ACT(L,M,N)
 50    CONTINUE
 909   FORMAT(1X,I2,2I3,3X,F10.3,2X,2F10.3,3X,6E12.3,3X,E10.3)
 60    CONTINUE
       RETURN
       END
       SUBROUTINE CREPAG(F,MX,NY,AMUX,AMUY)
C
C
C*****PURPOSE
C
C         THIS IS A DUMMY SUBROUTINE TO SUBSTITUTE FOR 'CREEPAGE L1'
C      TO GIVE THE TANGENTIIAL FORCES DEVELOPED AT THE CONTACT
C      INTERFACE DUE TO NORMAL LOADS.HERE CONDITIONS OF PURE SLIP
C      ARE ASSUMED GENERATING TANGENTIAL LOADS PROPORTIONAL TO
C      THE NORMAL LOADS , THE CONSTANT OF PROPORTIONALITY BEING
C      THE COEFFECIENT OF FRICTION.
C
C
C
       IMPLICIT REAL*8(A-H,O-Z)
       DIMENSION F(100,3),NY(20)
       IJ=0
       DO 10 I=1,MX
       NSSY=NY(I)
       DO 10 J=1,NSSY
       IJ=IJ+1
       F(IJ,2)=F(IJ,1)*AMUX
       F(IJ,3)=F(IJ,1)*AMUY
 10    CONTINUE
       RETURN
       END
       SUBROUTINE SUBFLT(XX,YY,ZZ,PX,PY,PZ,ANU,SXX,SYY,SZZ,SXY,SYZ,SZX)
C
C
C      BY B.PAUL AND S.SINGH
C
C      10TH APRIL 1980
C
C
C*****PURPOSE
C
C         THIS SUBROUTINE CALCULATES THE STRESS COMPONENTS WITHIN THE
C      BODY AT A GIVEN FIELD POINT DUE TO CONCENTRATED NORMAL AND
C      TANGENTIAL LOADS AT THE ORIGIN ASSUMING A SEMI INFINITE BODY.
C
C
C*****METHOD
C
C         THE PROGRAM UTILIZES THEBOUSSINESQE'S SOLUTION FOR
C      NORMAL LOADS AND THE CERRUTIS'S SOLUTION FOR THE TANGENTIAL
C      LOADS
C
C
C*****DESCRIPTION OF INPUT AND OUTPUT VARIABLES
C
C
C            XX        X-COORDINATE OF THE FIELD POINT
C            YY        Y-COORDINATE OF THE FIELD POINT
C            ZZ        Z-COORDINATE OF THE FIELD POINT
C            PX        X-COMPONENT OF THE LOAD AT ORIGIN
C            PY        Y-COMPONENT OF THE LOAD AT ORIGIN
C            PZ        Z-COMPONENT OF THE LOAD AT ORIGIN
C            ANU       POISSON'S RATIO
C            SXX       NORMAL STRESS IN X DIRECTION
C            SYY       NORMAL STRESS IN Y DIRECTION
C            SZZ       NORMAL STRESS IN Z DIRECTION
C            SXY       SHEAR STRESS IN THE XY DIRECTION
C            SYZ       SHEAR STRESS IN THE YZ-DIRECTION
C            SZX       SHEAR STRESS IN THE ZX DIRECTION
C
C
C
C
```

```
C
      IF (PY.EQ.0.00) GO TO 502
      SIGXXY=PY*YY*F1*(F3*(3.0*R2-YY2-2.0*R*YY2/RZ)-3.0*XX2/R2)
      SIGYYY=PY*YY*F1*(F3*(R2-XX2-2.0*R*XX2/RZ)-3.0*YY2/R2)
      SIGZZY=-3.0*PY*YY*ZZ2*F2
      SIGXYY=PY*XX*F1*(F3*(-R2+YY2+2.0*R*YY2/RZ)-3.0*YY2/R2)
      SIGYZY=-3.0*PY*YY*YY*ZZ*F2
      SIGZXY=-3.0*PY*XX*YY*ZZ*F2
  502 CONTINUE
C
C
C     CALCULATION OF STRESSES DUE TO NORMAL LOADS -QZ
C
C
      IF (PZ.EQ.0.00) GO TO 503
      SIGXXZ=PZ*F1*(F3*(ZZ*RZ2+XX2*(2.0*R+ZZ)-R2*RZ)-3.0*XX2*ZZ/R2)
      SIGYYZ=PZ*F1*(F3*(ZZ*RZ2+YY2*(2.0*R+ZZ)-R2*RZ)-3.0*YY2*ZZ/R2)
      SIGZZZ=-3.0*PZ*ZZ*ZZ*ZZ*F2
      SIGXYZ=PZ*XX*YY*F1*(F3*(2.0*R+ZZ)-3.0*ZZ/R2)
      SIGYZZ=-3.0*PZ*YY*ZZ*ZZ*F2
      SIGZXZ=-3.0*PZ*XX*ZZ*ZZ*F2
  503 CONTINUE
C
C     SUMMATION OF STRESSES DUE TO INDIVIDUAL LOADS ON SUBCELLS
C
       SXX=SXX+SIGXXX+SIGXXY+SIGXXZ
       SYY=SYY+SIGYYX+SIGYYY+SIGYYZ
       SZZ=SZZ+SIGZZX+SIGZZY+SIGZZZ
       SXY=SXY+SIGXYX+SIGXYY+SIGXYZ
       SYZ=SYZ+SIGYZX+SIGYZY+SIGYZZ
       SZX=SZX+SIGZXX+SIGZXY+SIGZXZ
       RETURN
       END
      SUBROUTINE TRANS1(XXN,YYN,ZZN,A11,A12,A13,A21,A22,A23,A31,A32,
     $A33)
C
C
C
C*****PURPOSE
C
C         THIS SUBROUTINE CALCULATES THE ELEMENTS OF THE COORDINATE
C     TRANSFORMATION MATRIX
C
C
```

```
CONWHEEL
C       CALCULATION OF SOME AUXILIARY FACTORS
C
        IMPLICIT REAL*8(A-H,O-Z)
        PI=3.141592654
        R=(XX*XX+YY*YY+ZZ*ZZ)**0.5
        R2=R*R
        RZZ=R+ZZ
        RZ2=RZZ*RZZ
        RZ=RZZ
C
        F1=1.0/(2.0*PI*R*R*R)
        F2=F1/R2
 9999   FORMAT(8F10.4)
        F3=(1.0-2.0*ANU)/RZ2
C
        XX2=XX*XX
        YY2=YY*YY
        ZZ2=ZZ*ZZ
C
C
C       CALCULATION OF STRESSE DUE TO TANGENTIAL LOADS -PX
C
C
        IF (PX.EQ.0.00) GO TO 501
        SIGXXX=PX*XX*F1*(F3*(R2-YY2-2.0*R*YY2/RZ)-3.0*XX2/R2)
        SIGYYX=PX*XX*F1*(F3*(3.0*R2-XX2-2.0*R*XX2/RZ)-3.0*YY2/R2)
        SIGZZX=-3.0*PX*XX*ZZ2*F2
        SIGXYX=PX*YY*F1*(F3*(-R2+XX2+2.0*R*XX2/RZ)-3.0*XX2/R2)
        SIGYZX=-3.0*PX*XX*YY*ZZ*F2
        SIGZXX=-3.0*PX*XX2*ZZ*F2
 501    CONTINUE
C
C
C       CALCULATION OF STRESSES DUE TO TANGENTIAL LOADS -PY
C
```

```
CONWHEEL
      IMPLICIT REAL*8(A-H,O-Z)
      A11=0.0
      A12= ZZN/(1.0-XXN**2)**0.5
      A13=-YYN/(1.0-XXN**2)**0.5
      A21= (1.0-XXN**2)**0.5
      A22=-XXN*YYN/(1.0-XXN**2)**0.5
      A23=-XXN*ZZN/(1.0-XXN**2)**0.5
      A31=-XXN
      A32=-YYN
      A33=-ZZN
      RETURN
      END
      SUBROUTINE TRANS2(A11,A12,A13,A21,A22,A23,A31,A32,A33,
     $SXX,SYY,SZZ,SXY,SYZ,SZX,SSXX,SSYY,SSZZ,SSX¥,SSYZ,SSZX)
C
C
C
C*****PURPOSE
C
C          THIS SUBROUTINE TRANSFORMS THE ELEMENTS OF THE STRESS TENSOR
C     FROM LOCAL COORDINATE SYSTEM TO THE GLOBAL COORDINATE SYSTEM.
C
      IMPLICIT REAL*8(A-H,O-Z)
       SSXX=A11*A11*SXX+A21*A21*SYY+A31*A31*SZZ+
     $(A11*A21+A11*A21)*SXY+(A21*A31+A21*A31)*SYZ+(A31*A11+A31*A11)*SZX
       SSYY=A12*A12*SXX+A22*A22*SYY+A32*A32*SZZ+
     $(A12*A22+A12*A22)*SXY+(A22*A32+A22*A32)*SYZ+(A32*A12+A32*A12)*SZX
       SSZZ=A13*A13*SXX+A23*A23*SYY+A33*A33*SZZ+
     $(A13*A23+A13*A23)*SXY+(A23*A33+A23*A33)*SYZ+(A33*A13+A33*A13)*SZX
       SSXY=A12*A11*SXX+A22*A21*SYY+A32*A31*SZZ+
     $(A12*A21+A22*A11)*SXY+(A22*A31+A32*A21)*SYZ+(A12*A31+A32*A11)*SZX
       SSYZ=A13*A12*SXX+A23*A32*SYY+A33*A32*SZZ+
     $(A13*A22+A23*A12)*SXY+(A23*A32+A33*A22)*SYZ+(A13*A32+A33*A12)*SZX
       SSZX=A13*A11*SXX+A23*A21*SYY+A33*A31*SZZ+
     $(A13*A21+A23*A11)*SXY+(A23*A31+A33*A21)*SYZ+(A13*A31+A33*A11)*SZX
      RETURN
      END
C        SUBROUTINE DGELG
C
C        PURPOSE
C           TO SOLVE A GENERAL SYSTEM OF SIMULTANEOUS LINEAR EQUATIONS.
C
C        USAGE
C           CALL DGELG (R,A,M,N,EPS,IER,DET)
C
C        DESCRIPTION OF PARAMETERS
C           R      - DOUBLE PRECISION M BY N RIGHT HAND SIDE MATRIX
C                    (DESTROYED). ON RETURN R CONTAINS THE SOLUTIONS
C                    OF THE EQUATIONS.
C           A      - DOUBLE PRECISION M BY M COEFFICIENT MATRIX
C                    (DESTROYED).
C           M      - THE NUMBER OF EQUATIONS IN THE SYSTEM.
C           N      - THE NUMBER OF RIGHT HAND SIDE VECTORS.
C           EPS    - SINGLE PRECISION INPUT CONSTANT WHICH IS USED AS
C                    RELATIVE TOLERANCE FOR TEST ON LOSS OF
C                    SIGNIFICANCE.
C           IER    - RESULTING ERROR PARAMETER CODED AS FOLLOWS
C                    IER=0  - NO ERROR,
C                    IER=-1 - NO RESULT BECAUSE OF M LESS THAN 1 OR
C                             PIVOT ELEMENT AT ANY ELIMINATION STEP
C                             EQUAL TO 0,
C                    IER=K  - WARNING DUE TO POSSIBLE LOSS OF SIGNIFI-
C                             CANCE INDICATED AT ELIMINATION STEP K+1,
C                             WHERE PIVOT ELEMENT WAS LESS THAN OR
C                             EQUAL TO THE INTERNAL TOLERANCE EPS TIMES
C                             ABSOLUTELY GREATEST ELEMENT OF MATRIX A.
C           DET    - DETERMINANT OF MATRIX A.
C
C        REMARKS
C           INPUT MATRICES R AND A ARE ASSUMED TO BE STORED COLUMNWISE
C           IN M*N RESP. M*M SUCCESSIVE STORAGE LOCATIONS. ON RETURN
C           SOLUTION MATRIX R IS STORED COLUMNWISE TOO.
C           THE PROCEDURE GIVES RESULTS IF THE NUMBER OF EQUATIONS M IS
C           GREATER THAN 0 AND PIVOT ELEMENTS AT ALL ELIMINATION STEPS
C           ARE DIFFERENT FROM 0. HOWEVER WARNING IER=K - IF GIVEN -
C           INDICATES POSSIBLE LOSS OF SIGNIFICANCE. IN CASE OF A WELL
C           SCALED MATRIX A AND APPROPRIATE TOLERANCE EPS, IER=K MAY BE
C           INTERPRETED THAT MATRIX A HAS THE RANK K. NO WARNING IS
C           GIVEN IN CASE M=1.
```

```
C
C        SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
C           NONE
C
C        METHOD
C           SOLUTION IS DONE BY MEANS OF GAUSS-ELIMINATION WITH
C           COMPLETE PIVOTING.
C
C     ..................................................................
C
      SUBROUTINE DGELG (R,A,M,N,EPS,IER,DET)
C
C
      DOUBLE PRECISION R,A,PIV,TB,TOL,PIVI,DABS,EPS,DET
      DIMENSION A(10000),R(100)
      IF(M)23,23,1
C
C     SEARCH FOR GREATEST ELEMENT IN MATRIX A
    1 IER=0
      DET=1.0D0
      PIV=0.D0
      MM=M*M
      NM=N*M
      DO 3 L=1,MM
      TB=DABS(A(L))
      IF(TB-PIV)3,3,2
    2 PIV=TB
      I=L
    3 CONTINUE
      TOL=EPS*PIV
C     A(I) IS PIVOT ELEMENT. PIV CONTAINS THE ABSOLUTE VALUE OF A(I).
C
C
C     START ELIMINATION LOOP
      LST=1
      DO 17 K=1,M
      DET=DET*PIV
C
C     TEST ON SINGULARITY
      IF(PIV)23,23,4
    4 IF(IER)7,5,7
    5 IF(PIV-TOL)6,6,7
    6 IER=K-1
    7 PIVI=1.D0/A(I)
      J=(I-1)/M
      I=I-J*M-K
      J=J+1-K
C     I+K IS ROW-INDEX, J+K COLUMN-INDEX OF PIVOT ELEMENT
C
C     PIVOT ROW REDUCTION AND ROW INTERCHANGE IN RIGHT HAND SIDE R
      DO 8 L=K,NM,M
      LL=L+I
      TB=PIVI*R(LL)
      R(LL)=R(L)
    8 R(L)=TB
C
C     IS ELIMINATION TERMINATED
      IF(K-M)9,18,18
C
C     COLUMN INTERCHANGE IN MATRIX A
    9 LEND=LST+M-K
      IF(J)12,12,10
   10 II=J*M
      DO 11 L=LST,LEND
      TB=A(L)
      LL=L+II
      A(L)=A(LL)
   11 A(LL)=TB
C
C     ROW INTERCHANGE AND PIVOT ROW REDUCTION IN MATRIX A
   12 DO 13 L=LST,MM,M
      LL=L+I
      TB=PIVI*A(LL)
      A(LL)=A(L)
   13 A(L)=TB
C
C     SAVE COLUMN INTERCHANGE INFORMATION
      A(LST)=J
C
```

```
C       ELEMENT REDUCTION AND NEXT PIVOT SEARCH
        PIV=0.D0
        LST=LST+1
        J=0
        DO 16 II=LST,LEND
        PIVI=-A(II)
        IST=II+M
        J=J+1
        DO 15 L=IST,MM,M
        LL=L-J
        A(L)=A(L)+PIVI*A(LL)
        TB=DABS(A(L))
        IF(TB-PIV)15,15,14
   14   PIV=TB
        I=L
   15   CONTINUE
        DO 16 L=K,NM,M
        LL=L+J
   16   R(LL)=R(LL)+PIVI*R(L)
   17   LST=LST+M
C       END OF ELIMINATION LOOP
C
C
C       BACK SUBSTITUTION AND BACK INTERCHANGE
   18   IF(M-1)23,22,19
   19   IST=MM+M
        LST=M+1
        DO 21 I=2,M
        II=LST-I
        IST=IST-LST
        L=IST-M
        L=A (L)+.5D0
        DO 21 J=II,NM,M
        TB=R(J)
        LL=J
        DO 20 K=IST,MM,M
        LL=LL+1
   20   TB=TB-A(K)*R(LL)
        K=J+L
        R(J)=R(K)
   21   R(K)=TB
   22   RETURN
C
C
C       ERROR RETURN
   23   IER=-1
        RETURN
        END
        SUBROUTINE OFF(NT,XOF,YOF,AX,BX,CX,XCX,NS)
         IMPLICIT REAL*8(A-H,O-Z)
        DIMENSION XOF(100),YOF(100),AX(100),BX(100),CX(100),XCX(100)
        DIMENSION PP(3,3),QQ(10,3),RR(3,10),D(10),H(3)
        NC=1
        I=1
        X1=XOF(I)
        Y1=YOF(I)
        I=I+1
        X2=XOF(I)
        Y2=YOF(I)
        NS=1
        IF ((X2-X1).EQ.0.0) AX(NS)=1.0D50
        AX(NS)=(Y2-Y1)/(X2-X1)
        BX(NS)=Y2-AX(NS)*X2
        CX(NS)=0.0
        XCX(NS)=X2
 25     CONTINUE
        IF ((X1-X1).EQ.0.0) AM1=1.0D50
        AM1=(Y2-Y1)/(X2-X1)
        I=I+1
        X3=XOF(I)
        Y3=YOF(I)
        IF ((X3-X2).EQ.0.0)GO TO 30
        AM2=(Y3-Y2)/(X3-X2)
        GO TO 40
 30     AM2=1.0D50
 40     CONTINUE
        DM=AM2-AM1
        IF (DABS(DM).GT.0.001) GO TO 50
        NOFF=1
```

```
      CALL LINLIN(X1,Y1,X2,Y2,X3,Y3,A,B,C,XC)
      NC=1
      GO TO 90
50    IF (NC.NE.1) GO TO 60
      CALL LINCIR (X1,Y1,X2,Y2,X3,Y3,A,B,C,XC)
      NOFF=2
      X0=A
      Y0=B
      NC=2
      NS=NS+1
      GO TO 90
60     IF((X3-X2).EQ.0.0) AM2=1.0 D 50
       AM2=(Y3-Y2)/(X3-X2)
61     IF ((Y2-Y0).EQ.0.0) AN1=-1.0D50
       AN1=-(X2-X0)/(Y2-Y0)
       IF (DABS(AM2-AN1).LT.0.002) GO TO 70
       CALL CIRCIR(X0,Y0,X2,Y2,X3,Y3,A,B,C,XC)
       NOFF=3
       NC=2
       DR=((X0-A)**2+(Y0-B)**2)**0.5
       IF (DABS(DR/C).GT.0.05) NS=NS+1
62     X0=A
       Y0=B
       GO TO 90
70     CALL CIRLIN (X0,Y0,X2,Y2,X3,Y3,A,B,C,XC)
       NOFF=4
       NC=1
       NS=NS+1
       GO TO 90
90     AX (NS)=A
       BX (NS)=B
       CX (NS)=C
       XCX(NS)=XC
       IF (I.EQ.NT) GO TO 100
       X1=X2
       Y1=Y2
       X2=X3
       Y2=Y3
       GO TO 25
100    RETURN
       END
       SUBROUTINE LINLIN (X1,Y1,X2,Y2,X3,Y3,A,B,C,XC)
       IMPLICIT REAL*8(A-H,O-Z)
       IF ((X3-X2).EQ.0.0) GO TO 10
       A=(Y3-Y2)/(X3 -X2)
       B=Y3-A*X3
       GO TO 20
10     A=1.0D50
       B=Y3-A*X2
20     CONTINUE
       C=0.0
       XC=X3
       RETURN
       END
       SUBROUTINE LINCIR (X1,Y1,X2,Y2,X3,Y3,A,B,C,XC)
       IMPLICIT REAL*8(A-H,O-Z)
       SQ=(Y3**2-Y2**2)+(X3**2-X2**2)
       Y32=Y3-Y2
       Y21=Y2-Y1
       X32=X3-X2
       X21=X2-X1
       IF (Y21.EQ.0.0) GO TO 10
       IF (Y32.EQ.0.0) GO TO 20
       A=((0.5*SQ/Y32)-(Y2+X2*X21/Y21))/(X32/Y32+X21/Y21)
       B=-A*X21/Y21+Y2+X2*X21/Y21
       GO TO 30
10     A=X2
       B=-(A*X32/Y32)+(0.5*SQ/Y32)
       GO TO 30
20     A=(X3+X2)/2.0
       B=-(X21/Y21)*A+(Y2+X2*X21/Y21)
       GO TO 30
30     C=((X3-A)**2+(Y3-B)**2)**0.5
       XC=X3
       RETURN
       END
       SUBROUTINE CIRCIR(X0,Y0,X2,Y2,X3,Y3,A,B,C,XC)
       IMPLICIT REAL*8(A-H,O-Z)
       SQ=(Y3**2-Y2**2)+(X3**2-X2**2)
```

```
      Y32=Y3-Y2
      Y20=Y2-Y0
      X32=X3-X2
      X20=X2-X0
      IF (X20.EQ.0.0) GO TO 10
      IF (Y32.EQ.0.0) GO TO 20
      A=((0.5*SQ/Y32)-(Y2-X2*Y20/X20))/(X32/Y32+Y20/X20)
      B=A*Y20/X20+Y2-X2*Y20/X20
      GO TO 30
10    A=X2
      B=-A*X32/Y32+0.5*SQ/Y32
      GO TO 30
20    A=(X3+X2)/2.0
      B=A*Y20/X20+Y2-X2*Y20/X20
30    C=((X3-A)**2+(Y3-B)**2)**0.5
      XC=X3
      RETURN
      END
      SUBROUTINE CIRLIN (X0,Y0,X2,Y2,X3,Y3,A,B,C,XC)
      IMPLICIT REAL*8(A-H,O-Z)
      IF ((Y2-Y0).EQ.0.0) GO TO 10
      A=-(X2-X0)/(Y2-Y0)
      B=Y2-X2*A
      GO TO 20
10    A=1.0D50
      B=Y2-X2*1.0D50
20    C=0.0
      XC=X3
      RETURN
      END
```