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16. Abstract

Cost analysis is important in every transportation industry, to the firms or agencies which provide service, to regulatory bodies, and to public policy makers. In the past, railroad cost analyses have been of two types: 1) statistical analyses of aggregate cross-section data from a variety of firms, or 2) very detailed operations-oriented studies. The premise of the work reported here is that a "hybrid" approach, using both economic theory and statistical methods on the one hand, and engineering analysis of operations on the other, can produce superior results.

This report covers Phase II of the project, which focused on analysis of a major class I railroad. A short-run variable cost function was estimated econometrically, and used as a basis for deriving the associated long-run function. We also developed a simple, but relatively accurate, network model to estimate operating costs. This model may be used to estimate origin-destination specific marginal operating costs. Econometric analysis of the output from the model leads to a theoretically justifiable equation for predicting marginal operating costs, and their sensitivity to changes in flows and input prices.

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METRIC CONVERSION FACTORS

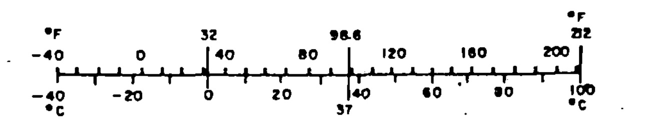
Approximate Conversions to Metric Measures

Symbol	When You Know	Multiply by	To Find	Symbol
		LENGTH		
in	inches	*2.5	centimeters	cm
ft	feet	30	centimeters	cm
yd	yards	0.9	meters	m
mi	miles	1.6	kilometers	km
		AREA		
in^2	square inches	6.5	square centimeters	cm^2
ft^2	square feet	0.09	square meters	m^2
yd^2	square yards	0.8	square meters	m^2
mi^2	square miles	2.6	square kilometers	km^2
	acres	0.4	hectares	ha
		MASS (weight)		
oz	ounces	28	grams	g
lb	pounds	0.45	kilograms	kg
	short tons (2000 lb)	0.9	tonnes	t
		VOLUME		
tsp	teaspoons	5	milliliters	ml
Tbsp	tablespoons	15	milliliters	ml
fl oz	fluid ounces	30	milliliters	ml
c	cups	0.24	liters	l
pt	pints	0.47	liters	l
qt	quarts	0.95	liters	l
gal	gallons	3.8	liters	l
ft^3	cubic feet	0.03	cubic meters	m^3
yd^3	cubic yards	0.76	cubic meters	m^3
		TEMPERATURE (exact)		
°F	Fahrenheit temperature	5/9 (after subtracting 32)	Celsius temperature	°C

*1 in = 2.54 (exactly). For other exact conversions and more detailed tables, see NBS Misc. Publ. 286, Units of Weights and Measures, Price $2.25, SD Catalog No. C13.10:286.

Approximate Conversions from Metric Measures

Symbol	When You Know	Multiply by	To Find	Symbol
		LENGTH		
mm	millimeters	0.04	inches	in
cm	centimeters	0.4	inches	in
m	meters	3.3	feet	ft
m	meters	1.1	yards	yd
km	kilometers	0.6	miles	mi
		AREA		
cm^2	square centimeters	0.16	square inches	in^2
m^2	square meters	1.2	square yards	yd^2
km^2	square kilometers	0.4	square miles	mi^2
ha	hectares (10,000 m^2)	2.5	acres	
		MASS (weight)		
g	grams	0.035	ounces	oz
kg	kilograms	2.2	pounds	lb
t	tonnes (1000 kg)	1.1	short tons	
		VOLUME		
ml	milliliters	0.03	fluid ounces	fl oz
l	liters	2.1	pints	pt
l	liters	1.06	quarts	qt
l	liters	0.26	gallons	gal
m^3	cubic meters	35	cubic feet	ft^3
m^3	cubic meters	1.3	cubic yards	yd^3
		TEMPERATURE (exact)		
°C	Celsius temperature	9/5 (then add 32)	Fahrenheit temperature	°F



DEVELOPMENT OF HYBRID COST FUNCTIONS FROM ENGINEERING AND STATISTICAL TECHNIQUES: THE CASE OF RAIL

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EXECUTIVE SUMMARY

Introduction

An understanding of the nature of costs of production is important in every regulated industry, both for individual firms and their regulators. At the most basic level a firm will require cost data for corporate planning. For example, a firm may wish to know what size plant to build, whether to upgrade the quality of plant or whether, at an existing tariff, the revenues for a service cover the incremental cost of providing the service.

Regulators and other policy makers also have many reasons to seek improved information about costs. When examined correctly, cost data can be used to determine whether there are in fact economies of scale in production, and whether regulation is a necessary tool of social control in a given industry. Regulators often ask whether a service is being subsidized by other service of a multiproduct firm, is subsidizing other services, and whether the provision of service by one mode will eliminate another mode over a given route.

Problem Studied

Previous railroad cost studies typically have examined a cross section of Class I railroads, using ICC data, and most have assumed a single product, usually total ton-miles. Several aspects of these studies have served to limit the inferences that can be drawn. They rely on data from the ICC accounts rather than on raw data from the firm. With few exceptions, they have specified a relatively simple functional form for costs, and assert that the form is appropriate without a test of that assertion. Few adjust for quality of service, and more importantly, many do not account for the multiproduct nature of virtually every rail firm. Finally, they do not attempt to adjust for the fact that some railroads operate with a more complicated network than others.

Our own research on railroad transport costs represents a very different approach to the problem. In an earlier report (Daughety and Turnquist, 1979) we developed a notion of "hybrid" analysis that reflected some crucial differences from the previous work.

1) Our analysis focused at the level of an individual firm, and used cost and production data obtained directly from the firm rather than from the ICC. This has a number of important advantages, including the avoidance of arbitrary cost allocations of the sort often found in the ICC accounts. We employed a time series analysis for a single firm rather than a cross-sectional analysis for a particular year.

2) The multi-product nature of the firm was incorporated into the analysis. Models were estimated with disaggregated volume (by commodity type) as well as with aggregate data. Output was characterized both by the volume of freight hauled and by the average speed of a shipment through the system. We explicitly recognized that speed of service is an important determinant of rail costs, and included this in our estimates.

3) We used information about the underlying technological production process, developed through engineering process functions, to improve both the specification of technology and the efficiency of our estimates.

In several respects the last point was particularly novel. Historically, most econometric estimates of cost functions have ignored valuable information on service-related variables which may be generated by engineering process functions. We have labeled our method a "hybrid" approach because it included such information.

This report builds on the first phase of the project in a number of important ways.

1) We have again focused our attention at the level of individual firm. This time, we have worked with data from a major class I railroad with a complex network; the Phase I effort purposely examined a small railroad with a simple network. Thus, we have developed techniques that address a wide range of existing firms. An important byproduct is that we can use the two case studies to examine the cross-section analyses discussed above.

2) Again we address the multi-product nature of the firm by including a quality variable (average speed of service) in the econometric model of the firm's costs. The econometric results include estimated short-run and long-run functions, thus allowing a direct comparison with results from the cross-section analyses discussed above.

3) We have expanded significantly the project's analysis of railroad operations. In our Phase I report engineering process functions were used to improve the econometric analysis. In this report we show how economic theory can be used to extend the operations/engineering analysis. Taken together, the two reports clearly show the advantages and potential of joint economic/engineering analysis of firm activities.

Results Achieved in Phase II

A short-run variable cost function was estimated using monthly data on 1) operating costs; 2) carloads moved; 3) average speed of service; 4) the prices of fuel, equipment, and labor; 5) a measure of track capital called "effective track." The long-run cost function was derived from the short-run function. Analysis of the estimation results indicated the following:

1) The firm faces significant economies of density; i.e. given the fixed configuration, at fixed speed-of-service increases in aggregate carloads moved will result in reductions in average costs per carload. Coupled with the Phase I results, this indicates that both large and small railroads can have significant density economies.

2) The major short-run factors of production (fuel, labor and equipment) are inelastic substitutes for one-another. Thus, each factor is a substitute for the others, but only to a small degree.

Comparison with the cross-section cost models indicates two sources of error in this literature:

1) Often such models do not control for systematic differences among firms, leading to biases in estimated coefficients. Moreover, cross-section analyses that do not control for firm differences cannot separate economies due to changes in firm size and configuration from economies due to more intensive configuration use (i.e. economies of density).

2) In general, cross-section studies have not used properly constructed quality-of-service measures. We find that eliminating the speed-of-service quality variable is not only a specification error in the model; such elimination tends to bias downward the estimate of returns-to-scale.

We also developed a simple, but accurate, model of rail operations that estimates system operating costs to within 15% of actual values. The model provides a rail firm with a convenient tool for operations cost analysis because it is easy to set up and inexpensive to solve. Moreover, we showed how to use the model to generate an origin-destination specific marginal operating cost prediction equation. This was another example of our hybrid analysis. Economic theory was used to formulate the estimation problem, and engineering analysis was used to provide the details on specific origin-destination movements. Together, the two methods produced a valid marginal cost function.

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CHAPTER 1

INTRODUCTION

An understanding of the nature of costs of production is important in every regulated industry, both for individual firms and their regulators. At the most basic level a firm will require cost data for corporate planning. For example, a firm may wish to know what size plant to build, whether to upgrade the quality of plant or whether, at an existing tariff, the revenues for a service cover the incremental cost of providing the service. Cost data may be used to justify tariff changes. A firm may want to know how a change in the level of output of one service affects the costs of providing another service, and it may rely in part on cost data to determine whether it would be profitable to discontinue a service, introduce a new service, or attempt to merge with another firm.

Regulators and other policy makers also have many reasons to seek improved cost information. When examined correctly, cost data can be used to determine whether there are in fact economies of scale in production, and whether regulation is a necessary tool of social control in a given industry. Evaluation of proposed tariffs requires accurate and appropriate cost information. Regulators often ask whether a service is being subsidized by other services of a multiproduct firm, is subsidizing other services, and whether the provision of service by one mode will eliminate another mode over a given route. Another example of current interest is the evaluation of seasonal or "peak-period" pricing policies. In general, regulators need cost information to determine how their policies will affect market structure and economic performance. These comments certainly apply to the railroad industry.

1.1 Other Railroad Cost Estimates

A number of studies have examined costs in the railroad industry. The early work in this area attempted to characterize the output of railroads as a single product, usually ton-miles. These studies examined a cross-section of Class I railroads, using ICC data, to test whether there were economies of scale in rail transport. The results were quite mixed. For example, Klein

(1947) used 1936 data to find economies of scale that were statistically significant, though modest. On the other hand, estimates by Borts (1960) and Griliches (1972) suggested that, while there may have been economies of scale for smaller railroads, scale economies were not prevalent for larger Class I railroads.

Several aspects of these studies limited the inferences that could be drawn. They relied on data from the ICC accounts rather than on raw data from the firms. They specified a relatively simple functional form for costs, and asserted that the form was appropriate without a test of that assertion. They did not adjust for quality of service, and more importantly, they did not account for the multiproduct nature of virtually every rail firm. Finally, they did not attempt to adjust for the fact that some railroads operate with a more complicated network than others.

Keeler (1974), Harris (1977) and Sammon (1978) have emphasized the differences between economies of firm size and economies of density. Returns to size are associated with a single firm serving a larger geographical area and more markets. Returns to density are associated with moving more traffic on a given network. This distinction has also been emphasized in the report by Daughety and Turnquist (1979), but the earlier econometric studies which used a very simple model form could not make this distinction.

Keeler (1974) and Hasenkamp (1976) used approaches grounded in production theory to examine multi-product aspects of railroad activities, distinguishing between freight and passenger activities. Using more sophisticated analysis, Brown, Caves and Christensen (1975) and Friedlaender and Spady (1979) have developed models that allow multiple outputs and relax several other assumptions of structural form. Caves, Christensen and Swanson (1980) have also used such techniques to examine productivity growth in U.S. railroads. In all these cases, cross-section data drawn from ICC reports or based on Klein's work (1947) have been used. Thus railroads with rates-of-return varying between -10% and +40%, facing different geography, having different mixes of equipment, customers and managerial perspectives were mixed together in the estimation process. Service variables such as speed could not be used, because such data are firm-specific and are not usually published. The above studies have represented important advances in the understanding of costs, but more work is needed, especially at the level of the individual firm.

1.2 Time Series Analysis at the Level of the Individual Firm

Our own research on railroad transport costs represents a very different approach to the problem. In an earlier report (Daughety and Turnquist, 1979), we developed a notion of "hybrid" analysis. We used information about the underlying technological production process, developed through engineering analysis, to improve the specification of technology and the efficiency of our statistical estimates of cost function coefficients. This approach reflects some crucial differences from earlier literature.

First, our analysis focused on the level of an individual firm, and used cost and production data obtained directly from the firm rather than from the ICC. This has a number of important advantages, including the avoidance of arbitrary cost allocations of the sort often found in the ICC accounts. (For a discussion of the kinds of problems arising from the use of ICC data, see, for example, Friedlaender (1969), Appendix A.) We employed a time series analysis for a single firm rather than a cross-sectional analysis for a particular year.

Second, the multi-product nature of the firm was incorporated into the analysis. Models were estimated with disaggregated volume (by commodity type), as well as with aggregate data. Output was characterized both by the volume of freight hauled and by the average speed of a shipment through the system. We explicitly recognized that speed of service is an important determinant of rail costs, and included this measure in our estimates.

This report builds on the first phase of the project in three important ways.

1) We have again focused our attention at the level of individual firm. This time, we have worked with data from a large railroad with a complex network; the Phase I effort purposely examined a small railroad with a simple network. Thus, we have developed techniques that address a wide range of existing firms. An important by-product is that we can use the two case studies to examine the cross-section analyses discussed above.

2) Again we address the multi-product nature of the firm by including a quality variable (average speed of service) in the econometric model of the firm's costs. The econometric results include estimated short-run and long-run functions, thus allowing a direct comparison with results from the cross-section literature.

3) We have expanded significantly the project's analysis of railroad operations with some very exciting results. The basic structure of the analysis is illustrated in Figure 1-1. Information on the network configuration, the traffic volume (demand), resources available and maintenance activities are used to support a network model to predict traffic flows on links in the network and associated operating costs. The information provided by this network model, together with the input data form the basis for statistical estimation of a function to predict marginal operating costs specific by origin and destination of traffic flows. In our Phase I report (Daughety and Turnquist, 1979), engineering process functions were used to improve the econometric analysis. In this report we show how economic theory can be used to extend the operations/engineering analysis. Taken together, the two reports clearly show the advantages and potential of joint economic/engineering analysis of firm activities.

The report proceeds in the following way. Chapter 2 presents the analysis and results of estimating a short-run variable cost function for the subject railroad. We also demonstrate how to construct the long-run function from the short-run function. We then examine the long-run results.

Chapter 3 develops a network-based model of rail firm operations, reflecting yard and linehaul activity. The development of the model, data requirements and the results of some sample runs are presented and discussed. Chapter 4 uses economic theory to extend the model in Chapter 3 to develop a function for predicting marginal operating costs for specific origin-destination pairs based on prices of inputs such as fuel, labor and equipment, and the quantity of goods being shipped. Finally, Chapter 5 summarizes the results of the research conducted in both phases.

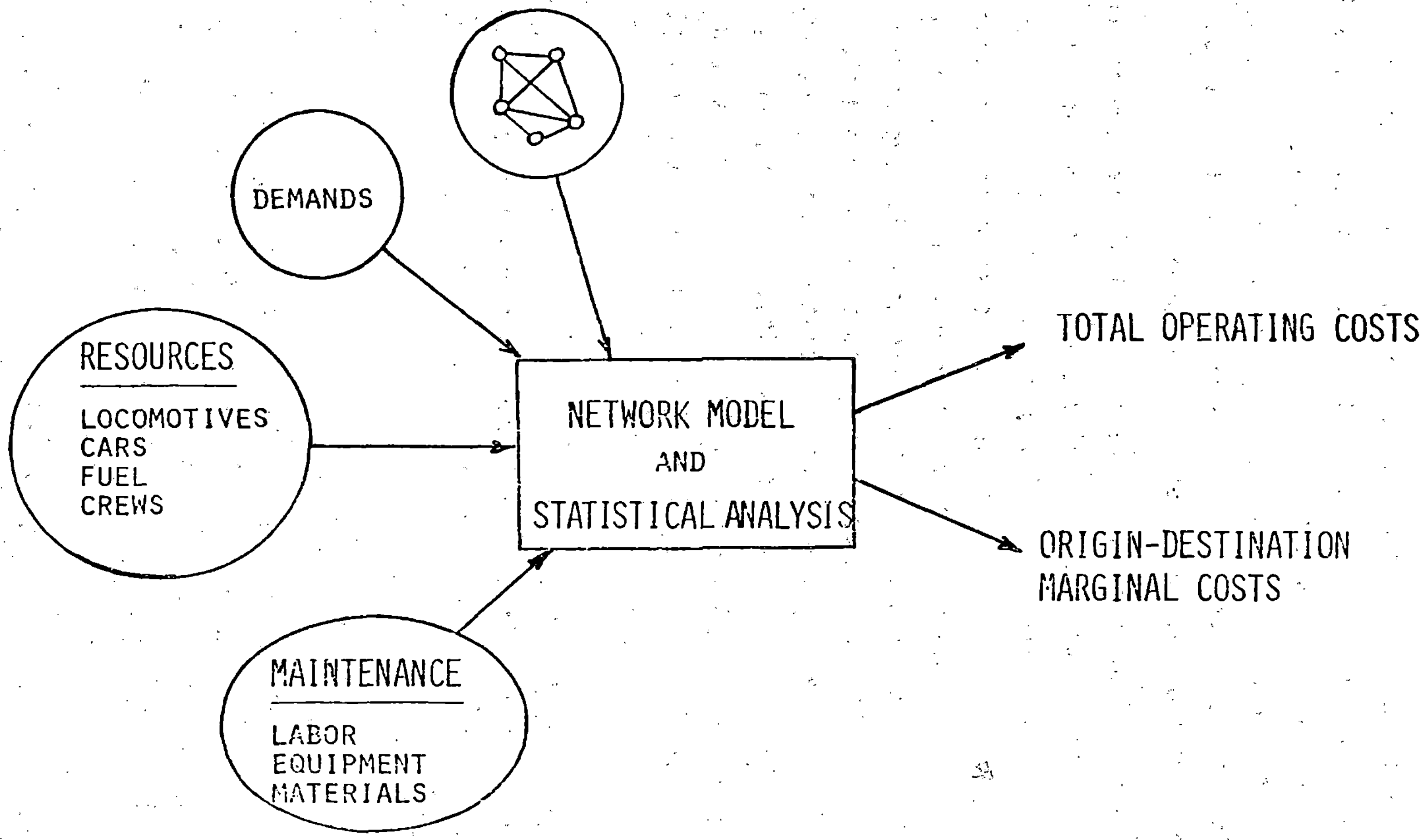

Figure 1-1. Structure of operating cost analysis.

CHAPTER 2

ESTIMATING SHORT-RUN AND LONG-RUN COST FUNCTIONS FOR A RAILROAD FIRM

In this chapter we discuss the formulation and estimation of short-run and long-run cost functions for a railroad firm.[1] The procedure involves: 1) estimating the short-run variable cost function as a function of outputs, variable factor prices and a fixed factor; 2) adding a short-run fixed cost; and 3) optimizing over the fixed factor to derive the long-run function. The estimated short-run and long-run functions are described and discussed.

Before entering into a discussion of the technical detail involved in constructing a cost model, it is important to clarify the type of model we will construct. One may divide statistically estimated models into two types: forecasting and explanatory. Models of the first type are constructed to provide estimates of costs without attention to the precise role of any particular variable in the model; the purpose of the model is to predict well. Explanatory models are more concerned with the linkages among various variables and the causes of cost generation. The objective is not forecasting, but insight into the nature of the cost generation process and the sensitivity of that process to specific input variables. The model we have constructed is of the second type, since our focus is on trying to understand the production technology of a rail system.

2.1 Basis for the Procedure Used

From economic theory we know that the long-run costs of a firm are a function of the output levels the firm produces and the prices it pays for the factors of production:

$$c = c(z, p)$$

where c is cost, z is a vector of outputs and p is a vector of input prices. If one obtained from a firm monthly observations of costs incurred and levels of output produced, one would probably see something like Figure 2-1 (here we assume one output). This would reflect the fact that while the firm would

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[1] Daughety and Turnquist (1979) discuss long-run and short-run cost functions, and their relation to one another. This discussion is in Appendix B of that report, especially on pp. B-19 through B-26.

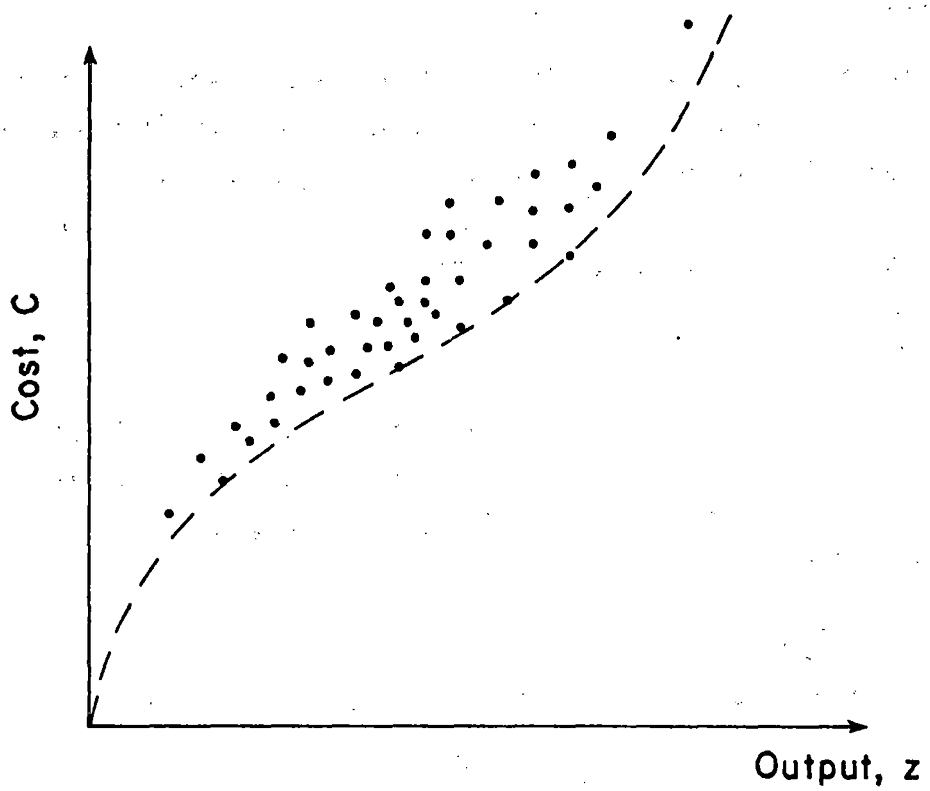

Figure 2-1. Hypothetical cost data and the long-run cost function.

prefer to be on its long-run cost curve (the dashed line), changes in output level can cause the firm to incur short-run costs in excess of long-run costs simply due to its inability to adjust all the factors of production instantaneously. This is especially true in the case of a railroad, because changes in its fixed plant (track, etc.) cannot be made rapidly. In other words, the points above the long-run curve represent points on the family of short-run curves whose envelope is the long-run curve.

Stated mathematically, let x be the vector of inputs used by the firm to produce the vector of outputs z. Assume that some of the inputs are variable (the vector x^v) and some are not as variable (i.e. fixed: x^f) with $x = (x^v, x^f)$. The input price vector p is partitioned in a similar manner: $p = (p^v, p^f)$. The short-run variable cost function is:

$$c^v(z, p^v; x^f)$$

i.e. short-run variable costs (c^v) are a function of the vector of outputs (z), the vector of prices associated with the variable factors (p^v) and the levels of the fixed factors (x^f). Short-run total costs are simply short-run variable costs plus short-run fixed costs:

$$c^v(z, p^v; x^f) + p^f x^f.$$

Long-run costs are found by optimizing over the fixed factors:

$$c(z, p) = \min_{x^f}[c^v(z, p^v; x^f) + p^f x^f].$$

This suggests the following procedure for estimating a long-run cost function:

1) Estimate the family of short-run variable cost functions, $c^v, p^v; x^f)$.

2) Compute a price for the fixed factors and use it to construct short-run fixed costs, $p^f x^f$.

3) Combine the two short-run functions and find the level of x^f which minimizes total short-run cost; i.e. solve:

$$\min_{x^f}[c^v(z, p^v; x^f) + p^f x^f].$$

This yields the optimal level of the fixed factor x^{f*}:

$$x^{f*} = x^f(z, p^v, p^f).$$

4) Plug this into the short-run cost functions to obtain a long-run cost function:

$$c(z, p) = c^{v}[z, p^{v}; x^{f}(z, p^{v}, p^{f})] + p^{f} x^{f}(z, p^{v}, p^{f}).$$

Obviously, this is a lot of work. Why not just estimate the long-run function directly? Figure 2-1 shows that if the firm cannot adjust all its factors of production in each time interval (e.g. within the month) then the resulting observations will be on or above (never below) the long-run cost function. If all we see is a scatter of points, passing a line through these points will overestimate the location of the long-run cost function, biasing the estimation results. The procedure outlined in steps 1-4 above was first proposed by Keeler (1967, 1974) and Eads, Nerlove and Raduchel (1969) and avoids the problem of overestimation.

2.2 Formulation of the Short-Run Cost Model

Our approach has been to use the same general cost modelling approach as developed in the first phase of this work. Specifically, short-run variable costs were modelled as

$$c^{v}(y, s, p_1, p_2, p_3, p_4, p_5; k) \qquad (2\text{-}1)$$

where y is quantity of goods moved (in carloads), s is the average speed of service through the system, p_i is the price of the i^{th} variable input factor (fuel, crew labor, non-crew labor, locomotives and cars) and k is the fixed factor. Each of these will be discussed below. For reasons to be explained later, this model was reduced to the following one:

$$c^{v}(y, s, p_F, p_{LA}, p_{LO}; k) \qquad (2\text{-}2)$$

where p_F is the price of fuel, p_{LA} is the price of labor and p_{LO} is the price of locomotives. As we will explain in Section 2.3, (2-2) contains all the information that would have appeared in (2-1).

In order to estimate (2-2) a functional form was assumed. We have used the translog form, developed by Christensen, Jorgenson and Lau (1973). The translog model has a number of important advantages.

1) The form is sufficiently general so as not to restrict the results of the estimation process. The translog can be viewed as an approximation to a general cost function.

2) The translog is the form used by a number of other studies (see Daughety and Turnquist, 1979), thus allowing direct comparisons.

3) The logarithmic form allows easy computation of important results of the analysis, as we will see below.

The translog representation of (2-2) will be the following[2]:

$$C = \sum_{i=1}^{6} \alpha_{i0} X_i + \frac{1}{2} \sum_{i=1}^{6} \sum_{j=1}^{6} \alpha_{ij} X_i X_j \qquad (2\text{-}3)$$

where

C = ℓn (short-run variable cost/average of short-run variable costs)
X_i = ℓn (variable i/average of observations for variable i)

with the following correspondences:

i	X_i
1	Flow (in carloads/month)
2	Speed (in miles/day/car)
3	Fuel Price (in dollars/gallon)
4	Wages (in dollars/hour)
5	Locomotive Rental Price (in dollars/month)
6	Fixed Factor (miles of effective track)[3]

Thus, X_1 and X_2 represent the outputs, X_3 to X_5 the input prices, and X_6 the fixed factor. The X_i are formed by taking an appropriate observation, dividing by the sample mean for the variable, and then taking the logarithm of the resulting ratio. This accomplishes the following.

1) It centers the estimation at the "point-of-means" thereby placing the tightest part of the model's prediction confidence interval in the middle of the data base.

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2 Upper case variables used in connection with the estimation will represent the logarithic, standardized variable from the original model.

3 Additional discussion of this factor is presented in Section 2.3.

2) It protects the proprietary nature of the data; by not providing the sample mean we can still provide a complete analysis of the cost function without revealing proprietary information on absolute levels of cost.

We also estimate factor share equations simultaneously to improve the efficiency of the estimation (see Christensen and Greene (1976)):

$$m_i = \alpha_{i0} + \sum_{j=1}^{6} \alpha_{ij} X_j \qquad i = 3,4 \qquad (2\text{-}4)$$

where

m_i = the share of cost associated with variable factor i (i = 3,4).

Note that we use only two such equations, since total factor shares sum to one and thus only two of the three are needed (the third is redundant). The factor share equations are derived using Shephard's Lemma (see Daughety and Turnquist, 1979; p. B-22) which, for our problem, is:

$$m_i = \frac{\partial C}{\partial X_i} \qquad i = 3,4 \qquad (2\text{-}5)$$

(In non-logarithmic terms, $m_i = (\frac{\partial c^v}{\partial p_i})(\frac{p_i}{c^v})$, which becomes (2-5).)

Economic theory dictates that a cost function should have the following properties (see Varian, 1978).

1) It should be monotonically non-decreasing in output: $\frac{\partial c}{\partial y} > 0$, $\frac{\partial c}{\partial s} > 0$.
2) It should be concave in prices.
3) It should be linearly homogeneous in prices; i.e., if we multiply all prices by a constant, cost should be multiplied by the same constant.

The third requirement is the most straightforward to satisfy. To maintain price-homogeneity, we restrict the parameters to satisfy the following conditions:

$$\sum_{i=3}^{5} \alpha_{i0} = 1$$

$$\sum_{j=3}^{5} \alpha_{ij} = 0 \qquad i = 1,\ldots,6. \qquad (2\text{-}6)$$

Moreover, since the cost function and the marginal cost function are assumed to be continuous functions, the cross-partials should obey symmetry; i.e., $\frac{\partial^2 C}{\partial X_i \partial X_j} = \frac{\partial^2 C}{\partial X_j \partial X_i}$. Thus, we will restrict the problem to satisfy symmetry:

$$\alpha_{ij} = \alpha_{ji} \qquad i,j = 1,\ldots,6. \tag{2-7}$$

The first two requirements (output monotonicity and price concavity) are not easily enforced. The first requirement, monotonicity, is an inequality condition which places nonlinear restrictions on the parameters. The second condition is presently unenforceable in any meaningful manner. Lau (1978) has provided a non-linear method for restricting the α_{ij} so that the translog cost function is concave in prices; unfortunately, this does not really restrict the underlying cost function to be concave in prices.

An important difference between this study and our Phase I study is the assumption underlying the speed variable s (X_2 in the translog representation). In the Phase I study we dealt with a medium-to-small railroad that was a bridge-line between two carriers. In that case it made sense to assume that the average speed-of-service, s, was exogenous.

In the present case, the railroad studied is a major railroad which presumably sets the speed of service so as to maximize profits. Thus, s should be an endogenous variable set by the firm so as to equate the marginal revenue with respect to speed (MR_s) to the marginal cost with respect to speed ($MC_s = \frac{\partial c}{\partial s}$). This restriction is not linear in the parameters of (2-3). However a slight manipulation leads to a linear restriction. We note that:

$$MC_s \frac{s}{c} = \frac{\partial C}{\partial X_2}$$

$$= \alpha_{20} + \sum_{j=1}^{6} \alpha_{2j} X_j. \tag{2-8}$$

Thus, if there is evidence that the firm endogenously sets the speed of service s, we will append equation (2-8) to the system to be estimated.

Therefore, our system of equations to be estimated is (2-3), (2-4) and (2-8) subject to (2-6) and (2-7). Before passing on to a discussion of the data to be used we need to account for one other problem: autocorrelation. We will be using monthly data, and thus observations in any given month may reflect some of the same environmental aspects as affected the previous month's observations. Let us pose our system to be estimated with error terms (ε_i) as follows (t represents observation in month t):

$$C^t = \sum_{i=1}^{6} \alpha_{i0} X_i^t + \frac{1}{2} \sum_{i=1}^{6} \sum_{j=1}^{6} \alpha_{ij} X_i^t X_j^t + \varepsilon_1^t$$

$$MC_s^t \frac{s^t}{c^t} = \alpha_{20} + \sum_{j=1}^{6} \alpha_{2j} X_j^t + \varepsilon_2^t \qquad (2\text{-}9)$$

$$m_i^t = \alpha_{i0} + \sum_{j=1}^{6} \alpha_{ij} X_j^t + \varepsilon_i^t \qquad i = 3,4.$$

We will assume that the error terms are <u>first-order autocorrelated</u>; i.e. this month's error term is affected by last month's. A representation of this is the following:

$$\varepsilon_i^t = \sum_{j=1}^{4} \rho_{ij} \varepsilon_j^{t-1} + u_i^t \qquad i = 1,\ldots,4 \qquad (2\text{-}10)$$

where the u_i^t are uncorrelated (and, we will assume jointly normally distributed) and the ρ_{ij} are called the autocorrelation coefficients (see Theil, 1971). This is a standard autocorrelation assumption made in cases where autocorrelation is handled explicitly.

Thus, our statistical problem is to estimate the system (2-9) subject to the constraints (2-6) and (2-7) and the assumption on the error process (2-10). Once this system is estimated, we can then apply standard techniques of numerical analysis to find k^*, the optimal level of the fixed factor, and derive the long-run cost function.

2.3 Firm Selection and Data Development

Several criteria were used in considering potential railroad candidates for this study. We desired a large Class I railroad with a management willing to work with us in providing the data required for analysis. While the firm in question completed some merger activity during the period studied, the companies were already owned and operated as subsidiaries, so that data would be added across the firms to maintain a consistent reporting base.

One obvious requirement for the analysis was that we obtain a complete set of data, reported on a consistent basis, for all of the variables discussed in the theoretical section. We were able to obtain such a set of data on a monthly basis for the 35 months between January, 1976 and November, 1978.

One of the additional points we considered in our development was the possibility of including explicitly a variable to represent technological change in the system. If technology were adjusting during the time interval of data collection, we would have needed to specify a time-varying cost model. However, because the time horizon was only 35 months, the issue of technological change was not a major point to be included in the model.

The following sections discuss the sources and nature of data used in the analysis.

Operating Costs and Revenues

We obtained operating cost data directly from the company's records. Since these data do not include implicit capital costs on cars and locomotives, an estimate of these additional costs was made using the car and locomotive prices and the numbers of cars and locomotives. These additional costs plus operating costs yield the short-run variable cost data required for our estimation. Thus, the short-run variable cost includes maintenance, fuel, labor, cars, locomotives, staff and supplies.

Operating revenues, reported on a monthly basis, were also obtained directly from the firm's records.

Labor

At the beginning of our study, we intended to include two kinds of labor in the analysis, crew labor and noncrew labor. We computed a factor price for noncrew labor by dividing the total number of noncrew hours actually worked into the sum of the noncrew wage bill, payroll taxes, and medical insurance paid by the firm. We also computed a crew labor price by dividing the straight time actually worked for crews into the sum of total wage payments to crews, payroll taxes for crew labor, and medical insurance payments.

We found that the prices for crew and noncrew labor had a correlation coefficient in excess of 0.95. Therefore, we merged the information to get a single price of labor, calculated by taking the total wage bill, payroll tax and insurance payments for both crew and noncrew labor, and then dividing that sum by the total hours worked in the two categories. We also generated a labor share by dividing the total wage, payroll tax and medical insurance payments by the total short-run variable cost of the firm.

We investigated whether labor should be treated as a variable or fixed factor, and found overwhelming institutional evidence that it should be treated as variable. The basis for this is the existence of "extra boards" of workers who have no fixed assignment, nor fixed salary. These people may be assigned to any job for which they are qualified, or not assigned at all if they are not needed. This allows the firm to redistribute labor where needed in the firm. This spatial and temporal redistribution mechanism leads to great flexibility in the use of labor. Further, if the labor requirements of the firm are reduced to such a level that redistribution of labor by extra boards cannot fully employ the labor, then the firm can (and often does) furlough unneeded laborers.

Locomotives and Cars

For each time period, we obtained data on the number of cars owned and leased, by type of car. We also found the number and types of locomotives, by type, both owned and leased. We obtained data on the prices of cars and locomotives from Economic ABZ's of the Railroad Industry (1980), Welty (1978) and from ICC Transport Statistics. These prices for physical units of capital were converted to equivalent rental prices using the interest rate on equipment obligations of the firm and depreciation rates from Swanson (1968).

We then found that the factor prices for locomotives and cars over the 35 month period had a correlation coefficient in excess of 0.95. Therefore, we used the locomotive factor price to represent the two categories. Also, the omitted share equation (as discussed earlier) was the one for locomotives and cars; thus we did not compute a factor share for these inputs. This means that the share attributed to the locomotive price is the "equipment share."

Speed

Data enabling us to calculate average speed on the system were obtained directly from the railroad's operating records. We gathered data on the total loaded car-miles during each month for the whole system, and then divided this by the total loaded car-days on-line for that month. This calculation yielded a monthly average velocity, in terms of miles per day for cars on the system. This approach provides a convenient way to estimate average speed-of-service.

It might seem surprising to some readers that we are not including terms reflecting variability in the speed of service as a reliability measure. There is a subtle, but important point here. Firms can employ inputs to attempt to control the quality of output (e.g. control for the purpose of maintaining a selected speed of service with low variability). Such decisions are reflected in the firm's use of inputs and thus the choice of degree of reliability is endogeneous to the firm and is incorporated in the setting of the expected speed. In other words, there is an optimal level of reliability that is adopted by an expected profit-maximizing firm, which is a function of the chosen expected speed of service. This is especially fortuitous since data on reliability are difficult to accumulate.

Flow

From company data we obtained information on all movements in the system, by origin-destination and by two-digit Standard Transportation Commodity Code, on a monthly basis. The output is measured in total carloads moved for each month.

Fixed Factor

It is not an easy task to characterize the fixed factors of a system as complex as a railroad. The fixed factors would include track, switches, land and buildings, to name the most obvious of the elements. We have employed a measure of miles of track to represent the level of the fixed factor in our analysis. We have done so because track represents the largest component of the system that can be regarded as fixed within a monthly horizon but which can be varied (at least to some extent) over a 35 month horizon. Moreover, investment in track appeared to be the main component of a general vector of fixed factors that was adjusting during the period studied.

In our study we observed that the total track-miles in place did not vary significantly over the 35 month period. However, the firm was investing in track in amounts significantly greater than would be required to maintain a constant quality of track in the face of normal depreciation. Thus, it was apparent that the firm was varying not the quantity of track in place over the three year period, but rather was improving track quality through track investment.

Thus, we constructed a measure of effective track in the following way. Consider the disposition of investment in track during period t. The amount of gross investment is I_t. I_t is considered to have three possible uses: (1) expansion or contraction of the system (generally small in our case), (2) coverage of depreciation of existing track, and (3) quality improvements in the existing system. Thus, we define:

T_t = the number of miles of track at the end of period t

δ = depreciation rate of existing track

Δk_t = improvements in track during period t.

Then the uses of I_t (gross investment in track-miles) are summarized as:

$$I_t = (T_t - T_{t-1}) + \delta T_{t-1} + \Delta k_t. \qquad (2\text{-}11)$$

The first term on the right hand side of the equation indicates the amount by which actual track-miles change during period t. The second term shows how much investment (in track-miles) would be required to offset normal depreciation (wear and tear) on existing track. The last term represents improvement in track quality above normal requirements to cover depreciation during period t.

We observe that the above equation can be rewritten, using the fact that $k_t - k_{t-1} = \Delta k_t$, where k_t denotes the effective track at time t. Then the following relationship can be stated:

$$k_t = k_{t-1} + I_t - (T_t - T_{t-1}) - \delta T_{t-1} \qquad (2\text{-}12)$$

(We reemphasize that the third term on the right hand side is essentially negligible in our actual case study.) Because this is a difference equation one of the k_t must be chosen arbitrarily. The equation then defines the remaining values relative to this one. This was done by letting $k_t = T_t$ at the end of 1976.

The following calculation illustrates the procedure more clearly. The numbers employed are purely illustrative, and bear no particular relationship to the data obtained from the actual firm studied.

<u>Given</u>:

(1) actual track-miles at the beginning of the year = 8000 miles;

(2) annual depreciation rate = 0.03;

(3) number of track-miles laid during each of the twelve months of the year in order:

(100, 150, 200, 50, 50, 100, 50, 50, 150, 50, 50, 200)

reflecting a total of 1200 miles laid during the year;

(4) number of actual miles in place at the end of the year = 8000 miles.

<u>Calculation</u>:

The improvements in track over the year can be determined using (2-12):

$$\Delta k_{year} = 1200 - 0.03(8000) = 960$$

Then the improvement in the quality of track from the first month of the year would be

$$\Delta k_{month\ 1} = \left(\frac{100}{1200}\right)(960) = 80 \text{ miles.}$$

For the second month, we would have

$$\Delta k_{month\ 2} = \left(\frac{150}{1200}\right)(960) = 120 \text{ miles.}$$

The sequence is repeated for each month. If we initialize actual and effective track to be 8000 miles at the beginning of the year, then by the end of the year effective track will be 8960 miles, although actual track is only 8000 miles. The difference of 960 miles represents a real improvement in the quality of track in the system. Obviously, this number will change as a function of which k_t value is specified <u>a priori</u>. However, relative performance is preserved.

The Price of the Fixed Factor (Track)

An extensive study of track costs was undertaken by Danzig, et al (1976). Table 2-1 displays some of the cost data. Thus, the net cost per mile of track is in the range of about $55,000-$60,000 per mile, assuming 25% tie replacement and some ballast replacement. We note that the ballast cost includes labor expenditures which we would in principle exclude because we desire capital costs, exclusive of labor. However, since total ballast costs are minimal, we view this as a minor problem, especially since we have used a broad range of track costs in our derivations of long-run cost functions from the estimated short-run cost function.

Maintenance Costs Implicitly Included

The method used to derive the factor prices for locomotives, cars and the fixed factor implicitly includes maintenance costs. This follows from the fact that the monthly "rental-price-equivalent" that we form is a price for the services of a new item each month, not for an unmaintained item. Thus, there is no need to separately include such maintenance costs.

2.4 Implicit Exogeniety of Speed-of-Service

Contrary to our expectations, our statistical evidence suggests that average speed-of-service is exogenously determined in the short-run. This was discovered when we estimated the marginal revenue with respect to speed-of-service and discovered that it was often negative (in many cases substantially negative).

Our procedure for imputing the marginal revenue with respect to speed-of-service was to assume that total revenue (TR) is a function of speed (s) and flow (y). Differentiating totally we have that:

$$dTR = MR_s ds + MR_y dy$$

where MR_s is the marginal revenue with respect to speed, MR_y is the marginal revenue with respect to flow and ds and dy are the infinitesmals of speed and flow. Solving for MR_s we have:

Table 2-1

Operating Expenses for Material and Retirement Accounts to Relay One Main Line Track Mile with New Rail of Same Weight and Welded Lengths of Rail Replaced

Item	136 pounds per yard Continuous Welded Rail	119 pounds per yard Continuous Welded Rail
New Rail Only	$62,235	$54,455
New Rail plus Plates, Angle Bars, Anchors, Spikes, Coating, etc.	82,510	73,415
Salvage Value of Old Rail	41,970	37,305
Net Cost per Mile	$40,540	$36,110
Tie Replacement Rate		
20%	≈ $12,000	≈ $12,000
25%	≈ 16,000	≈ 16,000
30%	≈ 20,000	≈ 20,000
Ballast (Resurface 3 inches ballast, single line track, including labor cost)	≈ 3,000	≈ 3,000

Source: J. Danzig, J. Rugg, J. Williams and W. Hay, Procedures for Analyzing the Economic Costs of Railroad Roadway for Pricing Purposes, U.S. Department of Transportation, Washington, D.C., 1976.

$$MR_s = \frac{dTR - MR_y dy}{ds} .$$

Let dTR be approximated by the change in operating revenues from month to month:

$$dTR^t = OR^t - OR^{t-1}$$

where OR^t is the operating revenue for the firm in month t. Furthermore, let $dy^t = y^t - y^{t-1}$ and $ds^t = s^t - s^{t-1}$. Finally, since the study period (January 1976-November 1978) predates liberalization of ICC rules on contract rate-making, marginal and average revenue with respect to y are the same; i.e. $MR_y^t = AR_y^t$. Let us approximate AR_y^t as:

$$AR_y^t = \frac{OR^t}{y^t} .$$

Thus, our approximation for MR_s^t becomes:

$$MR_s^t = \frac{(OR^t - OR^{t-1}) - \frac{OR^t}{y^t}(y^t - y^{t-1})}{s^t - s^{t-1}} . \qquad (2\text{-}13)$$

MR_s^t was computed via (2-13) using the monthly data described above. Sixty-five percent of the computed values were negative. If s were endogenous to the firm this would not happen; marginal revenue should exceed zero since otherwise reducing s would contribute to revenues and presumably reduce costs, thereby increasing profits.

The implication of this is that these data will not support treating speed as an endogenous variable. Thus, our model assumes that speed is exogenous to the firm in the short-run. The exogeneity of speed-of-service implies that the speed equation (2-8) should be dropped from the system (2-9). The resulting system to be estimated has three equations and twenty coefficients.

2.5 Estimation Results for the Short-Run Variable Cost Function

The system of equations (2-9), minus (2-8), was estimated subject to the error structure assumption (2-10), homogeneity in prices (2-6) and symmetry (2-7) via full information maximum likelihood using the WYMER program, on Northwestern's CDC/Cyber computer. The homogeneity restrictions were satisfied by normalizing the cost variable and all prices to the price of locomotives. Thus, values of coefficients associated with the price of locomotives are implied by the regression and the standard errors of these coefficients are computed by approximation.[4] Table 2-2 provides the estimated coefficients for the cost function and the standard errors. Mnemonics for the prices have been used (i.e. PFUEL is P_F, etc.).

The first-order coefficient estimates for flow, capital and the three price terms are significant (at the .001 level) and of the correct sign. Thus, the cost function is non-decreasing in outputs as required, and is increasing in prices; it is homogeneous in price since this restriction was enforced during the estimation. The function is not concave since the own second-order coefficients are significant and positive (i.e. α_{33}, α_{44}, and α_{55}).

The first-order price terms are the elasticities of cost with respect to price at the point of means. Thus, for example, a one percent change in the wage rate will increase costs by slightly over .5 percent. The coefficient on the price of locomotives represents the impact of both locomotives and cars.

Increases in the amount of effective track (K) reduce short-run variable costs at the point of means; a one percent increase in effective track miles implies a 0.2771 percent reduction in short-run variable costs. The negative sign is expected because the cost of the improvement is not included in short-run variable costs.

The first-order speed-of-service parameter (α_{20}) is insignificant but a test[5] of the hypothesis that the speed variables should be dropped is soundly rejected at the .005 level. Thus, even though the first-order speed term is insignificant, the speed variable itself is very important for proper model specification.

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4 Assume $\alpha \sim N(\alpha, \Sigma)$ and let $h(\alpha)$ be some function of the coefficients. If we expand $h(\alpha)$ around a point α_0, in a first-order approximation, then $\mathrm{Var}(h(\alpha)) = (\nabla h)'\Sigma\nabla h$ provides the squared standard error for the function $h(\cdot)$.

5 This is a joint test that $\alpha_{20} = \alpha_{12} = \alpha_{22} = \alpha_{23} = \alpha_{24} = \alpha_{25} = \alpha_{26} = 0$, performed as discussed in Daughety and Turnquist (1979), p. 65. The resulting χ^2 value was 37.005 with five degrees of freedom.

Table 2-2

Short-Run Variable Cost Function Estimates

Variable	Coefficient	Estimate	Standard Error
Y (flow)	α_{10}	0.3984	0.0694
S (speed)	α_{20}	-0.0659	0.0746
PFUEL	α_{30}	0.1902	0.0600
PLABOR	α_{40}	0.5253	0.0547
PLOCO	α_{50}	0.2845	0.0248
K (effective track-miles)	α_{60}	-0.2771	0.0887
Y•Y	α_{11}	4.1260	1.5776
Y•S	α_{12}	-2.6510	1.3848
Y•PFUEL	α_{13}	-0.0167	0.0069
Y•PLABOR	α_{14}	0.0090	0.0258
Y•PLOCO	α_{15}	0.0077	0.0264
Y•K	α_{16}	-2.8813	1.0598
S•S	α_{22}	-0.3681	1.8906
S•PFUEL	α_{23}	0.0404	0.0069
S•PLABOR	α_{24}	0.0067	0.0298
S•PLOCO	α_{25}	-0.0471	0.0306
S•K	α_{26}	2.9855	1.2109
PFUEL•PFUEL	α_{33}	0.0623	0.0104
PFUEL•PLABOR	α_{34}	-0.0489	0.0083
PFUEL•PLOCO	α_{35}	-0.0134	0.0061
PFUEL•K	α_{36}	0.0668	0.1557
PLABOR•PLABOR	α_{44}	0.0860	0.0176
PLABOR•PLOCO	α_{45}	-0.0371	0.0165
PLABOR•K	α_{46}	0.1166	0.2213
PLOCO•PLOCO	α_{55}	0.0505	0.0189
PLOCO•K	α_{56}	-0.1834	0.3739
K•K	α_{66}	2.5447	3.0202

The first-order flow term (Y) is significant and positive. A one percent change in flow will generate a 0.3984 percent increase in short-run variable costs. Thus, $\frac{\partial \ln c^v}{\partial \ln y} = (\frac{y}{c^v})\ (\frac{\partial c^v}{\partial y}) = \frac{MC_y}{AVC_y} = 0.3984$; i.e. short-run marginal costs are below short-run average variable costs at the point of means. Thus at the point of approximation, short-run average variable costs are falling with respect to flow.

Since we view output as reflecting both a physical flow of goods y and a quality measure s (average speed-of-service), simplification of the cost function would result if y and s were jointly separable from the inputs (represented by p and k). The hypothesis that y and s are separable from the inputs was tested (see Daughety and Turnquist (1979), p. B-33 for a discussion of this test) and rejected at the .005 level.

The estimation was performed under the assumption of first-order autocorrelation as discussed earlier and embodied in (2-10). Table 2-3 provides the estimated coefficients, with their standard errors in parentheses. There appears to have been a strong effect on the fuel share equation from all three equations (i.e., the second row coefficients are all significant). Moreover, there is evidence of first-order autocorrelation in the labor share equation from the previous months labor share error term (i.e., ρ_{33} is significant). By allowing for these autocorrelations in the estimation process, we have corrected for their potential effects on the standard errors of the equation coefficients.

2.6 Construction of the Long-Run Cost Function

Let p_k be the price of a mile of effective track (see Section 2.3). At the point of means for all the variables except k we have the following equation for short-run total costs (i.e. variable plus fixed):

$$c(\bar{y}, \bar{s}, \bar{p}; k) = \bar{c}\ \exp(\alpha_{60}\ \ln(k/\bar{k}) + \alpha_{66}(\ln(k/\bar{k}))^2) + p_k k$$

Table 2-3

Autocorrelation Coefficients

	1	2	3
1	-0.3272 (0.1780)	1.2913 (1.1318)	-0.8601 (0.4796)
2	0.0858* (0.0220)	0.7681* (0.1668)	0.1567* (0.0472)
3	0.0393 (.0738)	0.5866 (0.4335)	0.6031* (0.1698)

Table entry: ρ_{ij}

(σ_{ij})

i: 1 Cost Function
2 Fuel Share Function
3 Labor Share Function

* indicates those coefficients that are significant at the 5% level.

where a (-) over the variable represents the mean of the observations for the variable. A one-dimensional search technique (Wilde, 1964) was used to find k^*, the optimal level of the fixed factor. As discussed in Section 2.3, there is considerable uncertainty concerning the proper cost of a mile of effective track. Moreover, the appropriate depreciation rate and cost of capital are also uncertain. We chose to vary the cost of a mile of track from $40,000 to $130,000, the depreciation rate from 3 percent to 9 percent and the cost of capital from 8 percent to 12 percent. The results are very encouraging: k^* is reasonably insensitive to such changes. In what follows we will use the assumed values of track cost of $58,000/effective track-mile, depreciation of 3 percent and cost of capital of 10 percent. These values appear to be reasonable estimates of the appropriate numbers, based both on previous studies and discussions with railroad management.

Under these conditions we find that $k^* = 1.079\ \bar{k}$. At the point of means the optimal plant size (level of fixed factor) is 1.079 times the average for the period studied. Since the configuration is fixed this implies that the firm should increase the quality of the existing track by approximately 8 percent. Using this value of k^*, we can substitute into the short-run cost function and derive the long-run cost which is presented in Table 2-4. Notice that the first order terms have new coefficients, which are:

$$\tilde{\tilde{\alpha}}_{i0} = \alpha_{i0} + \alpha_{i6}\,\ell n(k^*/\bar{k}) \qquad\qquad i = 1,5$$

Moreover, a constant term $\tilde{\tilde{\alpha}}_{00} = \alpha_{60}\ \ell n(k^*/\bar{k}) + \alpha_{66}(\ell n(k^*/\bar{k}))^2$ now appears. It should also be noted that since we are simply substituting k^* into $c^V(y,s,p;k)$, the resulting function is net of fixed costs. This will not affect any of the derivatives of the cost function, and thus Table 2-4 contains all the relevant information associated with the long-run cost function.

Table 2-4

Long-Run Cost Function

Variable	Coefficient	Estimate	Standard Error
Y (flow)	α_{10}	.1793	0.2852
S (speed)	α_{20}	.1611	0.1051
PFUEL	α_{30}	.1953	0.0574
PLABOR	α_{40}	.5342	0.0669
PLOCO	α_{50}	.2705	0.0698
Y•Y	α_{11}	4.1260	1.5776
Y•S	α_{12}	-2.6510	1.3848
Y•PFUEL	α_{13}	-0.0167	0.0069
Y•PLABOR	α_{14}	0.0090	0.0258
Y•PLOCO	α_{15}	0.0077	0.0264
S•S	α_{22}	-0.3681	1.8906
S•PFUEL	α_{23}	0.0404	0.0069
S•PLABOR	α_{24}	0.0067	0.0298
S•PLOCO	α_{25}	-0.0471	0.0306
PFUEL•PFUEL	α_{33}	0.0623	0.0104
PFUEL•PLABOR	α_{34}	-0.0489	0.0083
PFUEL•PLOCO	α_{35}	-0.0134	0.0061
PLABOR•PLABOR	α_{44}	0.0860	0.0176
PLABOR•PLOCO	α_{45}	-0.0371	0.0165
PLOCO•PLOCO	α_{55}	0.0505	0.0189
CONSTANT	α_{00}	-0.0064	0.2283

As indicated in Table 2-4, the first-order coefficient on flow (α_{10}) is insignificant. This appears to be a direct result of the large negative coefficient on the flow/capital cross-term in the short-run function (α_{16} in Table 2-2) coupled with a substantial covariance between α_{16} and the main determinants of k* (α_{60} and α_{66}). Because the estimate of returns-to-scale (or in this case, density) at the point of means depends on this coefficient (see Daughety and Turnquist, 1979; p. B-32), this is a somewhat disappointing result. We will return to this later, after discussing some of the other terms in the function, and will employ an alternative approach to clarify issues of returns-to-density.

The other first-order terms have the expected signs, including speed-of-service which is now significant at approximately the .06 level. The coefficient on the first-order speed variable is positive, as is expected since higher quality service should incur higher costs.

Table 2-5 displays estimates of the factor demand and price elasticities (own and cross). The own and cross-elasticities are computed as follows (δ_{ij} is one if i = j and zero otherwise):

$$\varepsilon_{ij} = \frac{\alpha_{ij}}{\alpha_i} + \alpha_j - \delta_{ij} \qquad i,j = 3,4,5.$$

For example, the own price elasticity of labor is -.3048, meaning that a one percent increase in the wage rate generates a .3048 percent decrease in demand for labor by the firm. On the other hand, a one percent increase in the price of locomotives results in a .397 percent increase in demand for labor: locomotives, labor and fuel are substitutes (at the point of means) for one another, though the degree of substitution appears to be low (indicated by the small magnitudes of the coefficients).

Table 2-5

Own and Cross Factor Demand Price Elasticities

	QUANTITY		
PRICE	Fuel	Labor	Locomotives
Fuel	-.4857	.2838	.2019
Labor	.1038	-.3048	.2011
Locomotives	.1458	.3970	-.5428

Table entry: $\frac{\text{\% change in quantity}}{\text{\% change in price}}$

2.7 The Long-Run Function and Returns-to-Density

As Keeler (1974), Harris (1977) and Sammon (1978) have observed, one can distinguish at least two types of scale economies in the railroad industry. If the size of the firm in terms of geographic points served and configuration of network can adjust, one is measuring economies-of-size (and configuration; see Daughety and Turnquist, 1979; pp. 6-11). If the network configuration is held fixed then economies-of-scale are referred to as economies-of-density. This separation is crucial since having one type of scale economy does not preclude or imply the other. Thus, rail cost models must be structured to measure the two effects separately. Unfortunately, most of the cross-section studies have not done this; railroads reflecting different sizes and configurations are mixed together in the estimations. Mundlak (1961) has shown that time series-cross section studies can be biased if variables are not introduced to control for firm differences. Caves, Christensen and Thetheway (1981), in a study of airlines, found that introduction of dummy variables to distinguish each firm in their sample resulted in findings of returns-to-scale, while deletion of the dummy variables indicated constant returns-to-scale.

While not able to reject conclusively a finding of constant returns-to-scale for railroads, Friedlaender and Spady (1979) indicate that the data tend to support increasing returns-to-scale. Caves, Christensen and Swanson (1980) find constant returns-to-scale. In both cases it is unclear whether these results reflect size/configuration issues or density issues since firm differences are not controlled. Thus the resulting scale economy estimate reflects both types of economies. Friedlaender and Spady do, however, introduce technological variables such as length-of-haul as a proxy for network size, which may account for the difference between their results and those of Caves, Christensen and Swanson.

As indicated above, the standard error on the first-order flow term is very large and thus the usual procedure for examining the cost function for returns-to-scale (specifically returns-to-density, since the size of the firm and configuration of its network is fixed) by computing $1/\alpha_{10}$ seems questionable at best. Consider instead the following procedure. Form the average total short-run cost function (for s and the price vector at their mean values):

$$AC(y, \bar{s}, \bar{p}; k) = \frac{1}{y}[\bar{c} \exp(\alpha_{10} \ln(y/\bar{y}) + \frac{\alpha_{11}}{2}(\ln(y/\bar{y}))^2$$

$$+ \alpha_{60} \ln(k/\bar{k}) + \frac{\alpha_{66}}{2}(\ln(k/\bar{k}))^2) + p_k k]$$

and find values y^* and k^* that simultaneously minimize $AC(y, \bar{s}, \bar{p}; k)$; i.e. find the bottom of the average cost curve. Figure 2-2 displays the result of this exercise (the dotted lines are approximate extrapolations). The horizontal axis represents flow on the fixed configuration in units of .1 MES (minimum efficient scale: where average cost first becomes a minimum). Computations show that $\bar{y}$ is approximately .4 MES. The range over which the average cost function is at its minimum is from MES to 1.1 MES. Extrapolation indicates that for $y > 1.1$ MES, average cost rises rapidly.

Three short-run average cost curves have been drawn in at $k^* = 1.079$, 3.0 and 3.6 $\bar{k}$, respectively. The last two are not overly realistic, since it is hard to imagine the present network being improved to three times its present quality. Rather, the point of the figure is to illustrate the fact that the firm does face significant economies-of-density over a wide range of output; average cost at $y = \bar{y}$ is 1.5 times the average cost at y = MES. This result is consistent with Friedlaender and Spady's work on rail cost functions where they found that exhausting economies-of-scale was not "just around the corner" (Friedlaender and Spady, 1979, p. 263). These results do appear to conflict with those of Caves, Christensen and Swanson (1980), because they found constant returns-to-scale.

A second reason for the difference between this study and the cross-section results mentioned above is the inclusion in this model of the quality-of-service variable, s. Caves, Christensen and Swanson (1980) did not include a quality-of-service variable; Friedlaender and Spady introduce technological variables, but these would at best be surrogates for a quality-of-service measure. This is an advantage of firm-level analysis: such data are generally not available at the aggregate level which cross-section studies must use.

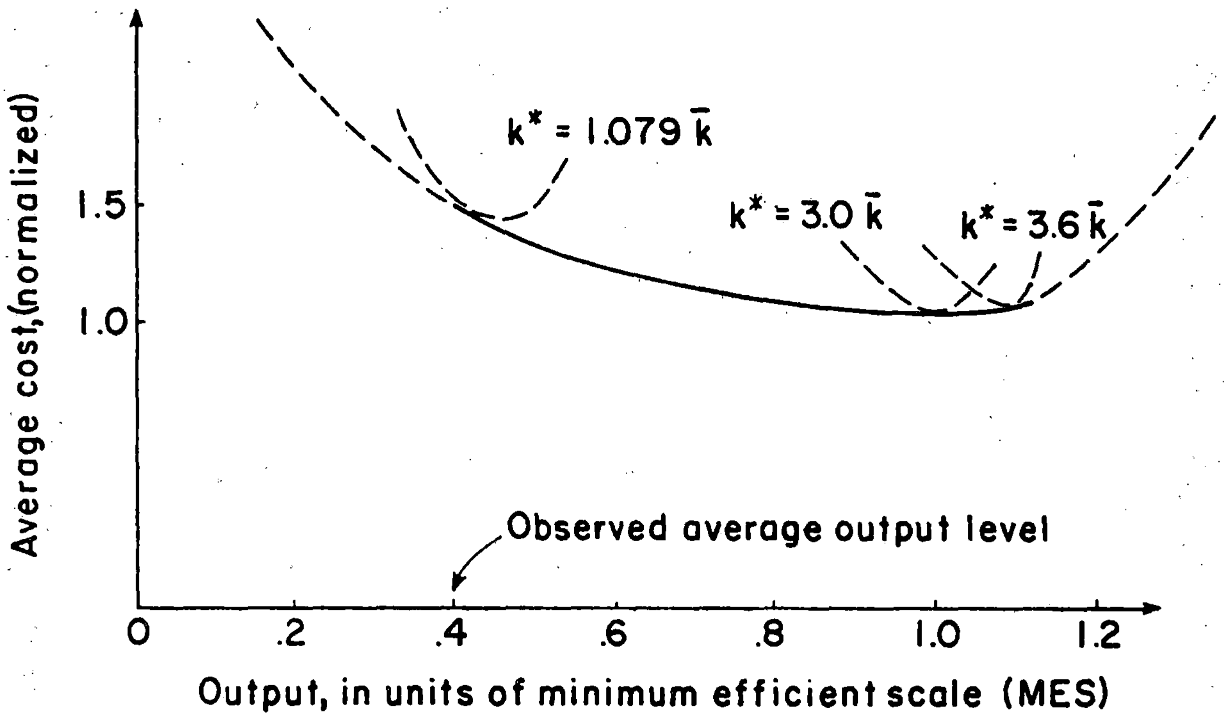

Figure 2-2. The firm's average cost curve.

The effect of dropping this measure is very interesting. As mentioned above, dropping s as a variable is a specification error; the χ^2-value on the hypothesis test of setting the speed-related coefficient to zero was over twice the table value at the .005 level. Proceeding, however, without the speed variable results in a value for k* in excess of the one computed above, slightly higher average costs and a lower estimate of returns-to-density. Thus, failing to include a properly constructed quality-of-service measure alters the estimated cost function in the direction of constant return-to-scale (i.e. density).

Thus, since we are working at the level of an individual firm, our results do not suffer from interfirm biases or specification error due to failing to include quality of service. The railroad under study appears to have very significant returns-to-density; computed in the traditional manner we find them to be $1/\alpha_{10} \approx 5.6$. While the standard error on this number is obviously quite large (since the standard error on α_{10} is large relative to the magnitude of α_{10}), the existence of substantial returns-to-density are also supported by the alternative (optimization) procedure.

The finding of substantial economies-of-density replicates a similar result in our previous study of a smaller railroad (Daughety and Turnquist; 1979; pp. 68-70). It is risky to attempt extrapolation to the entire industry on the basis of two samples, but the fact that we have obtained very similar results from two very different railroads suggests that density economies are not an isolated phenomenon.

2.8 Summary

The analysis described in this Chapter has yielded statistical estimates of both short-run and long-run cost functions for the railroad under study. This type of result is of greatest use to regulators and policy makers, because it focuses attention on matters of general concern, not details. The estimated models provide insight into economies-of-density, elasticities of demand for the factors of production, and elasticities of substitution among input factors. These results are important because they summarize important economic characteristics of the production process.

The analysis performed here differs substantially from more traditional econometric analyses of railroad costs. We have undertaken an analysis of time-series data from a single firm, rather than data for a single time period from many firms. This has allowed us to use data on service quality attributes to improve model specification. It has also allowed us to be precise about the type of scale economies found. These are economies-of-density over a fixed network configuration, and should not be confused with the separate concept of economies-of-size, which relates to the geographic extent of the markets served.

It is also important to recognize that the firm-level data collected to support the analysis can be used in a different way to support a complementary type of analysis. This analysis focuses on the more detailed operating characteristics of the railroad, and gives additional insights into the costs associated with specific origin-destination movements over the railroad's network. These results are more likely to be directly useful to railroad management in determining prices for certain services, in identifying areas where costs are higher than they should be, and in evaluating the effects on costs of changes in operating policies or facilities.

The next Chapter describes a model of operating costs on a network. This model represents another facet of the "hybrid" analysis of costs using both engineering and statistical techniques, which is at the heart of this research.

CHAPTER 3

A NETWORK MODEL OF OPERATING COST

Economic theory specifies that a cost function should be the result of solving a problem which may be stated generally as follows:

(PO) $\min p'x$

$\text{s.t. } f(z,x) = 0$

where: p = vector of input prices

x = vector of input quantities purchased for use

z = vector of outputs produced.

The function $f(z,x)$ provides the information on technological constraints which determine how x is used to produce z. In general, $f(z,x)$ is nonlinear, implying that the cost function is the result of a nonlinear optimization. In this Chapter, we describe a particular formulation of such a nonlinear program which is very useful for studying the nature of operating costs on a railroad network.

3.1 Problem Formulation

The basic structure of the problem may be specified as follows:

inputs: road locomotives
yard locomotives
cars
fuel
road crews
yard crews
maintenance of way

outputs: carloads of all commodities by origin-destination pair.

A fundamental assumption of the model is that the costs of the various inputs can be related to flows of traffic over the links of the rail network. The basic unit of flow is a carload, and links are of two types - linehaul links and yard links.

The flow variables in the model may be defined as follows:

f_{ij}^q = carloads on link ij enroute to destination q

f_{ij} = total carloads on link ij $(= \sum_q f_{ij}^q)$.

The problem may then be formulated as finding a minimum cost assignment of a given set of origin-destination demands to the available network. In creating such a formulation, a change in the structure of the optimization problem is accomplished. Problem (P0) has a linear objective function which is minimized subject to nonlinear constraints. By introducing the link flow variables, the problem becomes one with a nonlinear objective function, and linear constraints. This change is important in achieving an efficient solution procedure.

The problem may be written as follows:

(P1) $$\min G(f) = \sum_i \sum_j c_{ij}(f_{ij}) \cdot f_{ij}$$

$$\text{s.t.} \sum_j f_{ij}^q - \sum_j f_{ji}^q = z_{iq} \qquad \forall\ i,q$$

$$f_{ij} - \sum_q f_{ij}^q = 0 \qquad \forall\ ij$$

$$f_{ij},\ f_{ij}^q \geq 0 \qquad \forall\ ij,\ q$$

where: $c_{ij}(f_{ij})$ = unit cost of flow on link ij, as a function of the volume on that link,

z_{iq} = volume of total demand from origin i to destination q.

The objective function simply indicates that we wish to choose a set of link flows on the network (assign the demands to the network) in such a way as to minimize the total cost incurred. The $c_{ij}(f_{ij})$ cost coefficients include the costs of all the inputs required to cross a particular link.

The first set of constraints state that for each destination, the flow out of any node i minus the flow into i enroute to that destination must equal the flow which originates at i and is destined for q. Thus, these constraints simply ensure conservation of flow in the network.

The second set of constraints defines total carloads on a link (f_{ij}) in terms of the carloads destined for specific destinations (f^q_{ij}), and the last set simply insures that flows on the network are positive.

3.2 Network Representation

The railroad network is represented by a set of nodes connected by links. At a conceptual level, the nodes are yards, terminal facilities and junctions, and the links connecting them are main line and branch line tracks. However, the detailed representation of the system is different from the conceptual model in two ways.

First, the network has been aggregated. Each terminal in the analysis network represents a collection of points in the actual network. The actual system under study has more than a thousand points at which traffic can originate or terminate. In concept, each of these points is a node in the network. For analysis purposes, this set of points has been aggregated into 27 major terminal areas which represent the origin and destination points for traffic in the model. Network links were also aggregated along with the nodes.

Second, the analysis network uses a set of links and nodes to represent each of the 27 terminals. This is because the network flow algorithms used in the analysis require impedance to flow (cost) to occur only on links. Since there are substantial costs associated with the movement of cars through yards in the terminal areas, these terminals cannot be simply represented as nodes.

The general method of representing terminal areas is shown in Figure 3-1. The terminal is represented by three nodes and three one-way links, in addition to the links connecting this terminal to others. Link 1-2 represents the actual yard, and has a positive cost associated with flow through it. Node 3 is used as the actual origin/destination for traffic originating or terminating

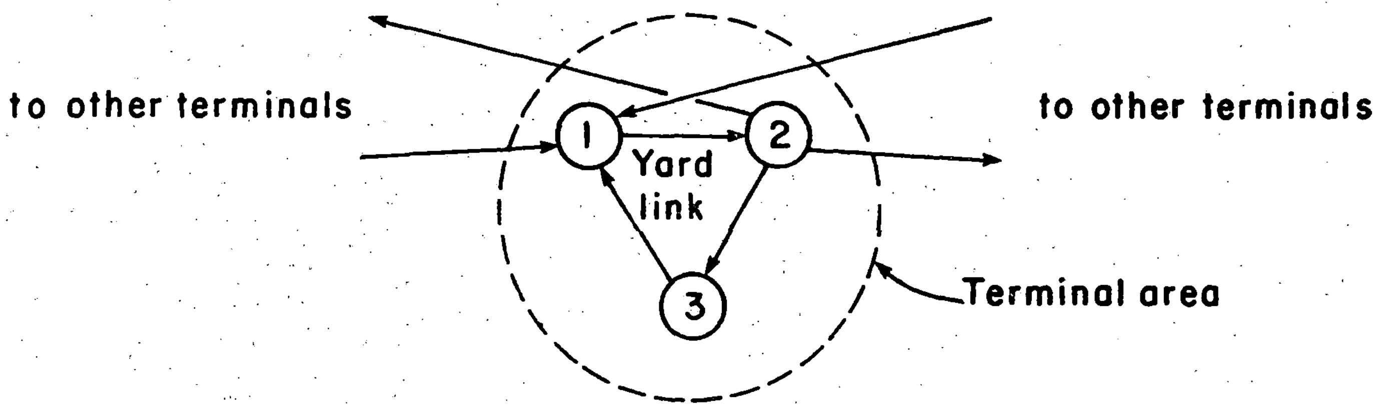

Figure 3-1. Representation of a terminal area in the network.

at this terminal area. Thus, originating traffic must traverse links 3-1 and 1-2 before departing. This forces it through the yard link 1-2. Likewise, terminating traffic must enter at node 1, and traverse links 1-2 and 2-3 before reaching its destination. Through traffic simply crosses link 1-2. As a result, all traffic handled by this terminal area passes through the yard link, and is included in the determination of congestion levels in the yard.

In summary, there are three types of links connecting the various nodes in the network. There are linehaul links which connect the terminal areas; there are yard links representing classification yards; and there are "dummy" links within the terminal areas to connect the actual origin/destination nodes to the rest of the network. The construction of unit cost functions for the linehaul and yard links is described in the following section.

3.3 Costs on Linehaul Links

For each linehaul link in the network a unit cost function $c_{ij}(f_{ij})$, must be computed. This unit cost must include the costs of five basic inputs:

1. train crews
2. fuel
3. locomotive ownership and maintenance
4. car-hire, car ownership and maintenance
5. maintenance of way.

Train crew costs are directly related to train-miles. To establish a unit crew cost on a carload basis, we have used information on average train length and the ratio of empty to loaded car-miles. Train-miles on link ij are computed as follows:

$$TRM_{ij} = \frac{(1 + E)M_{ij}}{ATL} f_{ij} \qquad (3\text{-}1)$$

where: TRM_{ij} = train-miles on link ij

M_{ij} = length of link ij (miles)

E = empty-to-loaded car-mile ratio

ATL = average train length (cars)

f_{ij} = carloads moved on link ij.

Since system-wide average figures for E and ATL are used, some variability among individual links due to specialized operations is lost. For predictions of aggregate system performance from month to month, we have found that this loss of information is not important. However, if the model were to be used for detailed examination of marginal costs of movement on a specific route, appropriate link-specific data could be substituted for the system-wide average figures.

Thus, if we denote train crew wage costs per train-mile as p_w, the wage costs per carload may be expressed as $c_{1ij}(f_{ij})$:

$$c_{1ij}(f_{ij}) = \frac{p_w(1 + E)M_{ij}}{ALT}. \qquad (3\text{-}2)$$

Note that expression (3-2) is independent of f_{ij}. This reflects an assumption that the unit wage costs per train-mile, p_w, are constant, or that total wage costs are linear in volume. Thus, the model does not include detailed information on the effects of changes in the degree of linehaul congestion which could affect the wage cost per train-mile due to changes in running speed. Effectively, the model assumes that the overall level of congestion present in the data used to calibrate p_w will remain unchanged as the model is operated.

A railroad wishing to use this model for a specialized study of a specific movement might decide to relax this assumption, and use more detailed data to estimate p_w as a function of f_{ij}, but for our purposes this was unnecessary. The crew cost p_w was estimated for each month by simply dividing the total train crew costs by total train-miles operated.

Fuel costs may be related directly to car-miles, since fuel consumption is generally proportional to the amount of work performed. Observations on fuel consumption per car-mile over a four-year period (1976-79) on the railroad under study indicated that nearly all the monthly observations were within 10% of the mean. Thus, if we denote the fuel consumption rate per loaded car-mile as r_1, and the price of fuel per gallon as p_F, the fuel cost per carload on link ij is simply:

$$c_{2ij}(f_{ij}) = p_F r_1 M_{ij}. \qquad (3\text{-}3)$$

Locomotive ownership and maintenance costs are computed using locomotive prices, depreciation rates and a utilization rate, expressed as locomotive-months per loaded car-mile. This utilization rate is computed by dividing locomotives owned or leased by total loaded car-miles produced in each month.

The effective price of a locomotive-month is computed using the replacement cost in each month, the current interest rate, and a depreciation rate. The calculation is as follows:

$$p_L^t = (r_t + d)\ U_t/12 \qquad (3\text{-}4)$$

where: p_L^t = ownership cost of a locomotive for month t ($)
r_t = annual interest rate available in month t
d = annual depreciation rate
U_t = price of a new locomotive in month t ($).

We have assumed a normal depreciation rate of 6% per year. This depreciation rate assumes a normal rate of maintenance on the locomotive. Since the price reflects a new locomotive, implicit maintenance costs are included in the total effective cost of a locomotive-month. The price of a locomotive used in the network model for linehaul links reflects only road locomotives, and not yard locomotives. The price used in the econometric estimation in Chapter 2 is a composite price including both types. In the network model, the cost of yard locomotives are included in the yard links, but are separated from the locomotives used in road service.

If we denote the effective rental price of a locomotive-month as p_L, dropping the superscript t, and the utilization rate as u, the locomotive costs per carload are as follows:

$$c_{3ij}(f_{ij}) = p_L u M_{ij}. \qquad (3\text{-}5)$$

To determine car-hire and ownership costs, we must recognize the distinction between system and foreign cars, and must also include two types of car utilization factors. The first of these is the empty-to-loaded car-mile ratio (E) discussed previously. The second relates to the time required to move cars through the system, and can be expressed as the ratio of total car-days-on-line to loaded car-miles. Let us denote this second utilization rate as T.

If a proportion α of the total cars-on-line on an average day are foreign, the daily proportion of the per diem charge for foreign cars is b_0, and the daily ownership and maintenance cost for system cars is p_c, the effective cost per car-day is on average:

$$\alpha b_0 + (1-\alpha)p_c.$$

For foreign cars, there is a mileage charge in addition to the daily charge, b_0. Denote the per-mile cost as b_1. If a proportion β of total car-miles are made by foreign cars, the average cost per loaded car-mile is:

$$(1 + E)\beta b_1.$$

Combining the daily costs and the mileage costs using the utilization rate, T, the total unit car-hire and ownership cost per carload on link ij is:

$$c_{4ij}(f_{ij}) = \{[\alpha b_0 + (1-\alpha)p_c]\, T + (1 + E)\beta b_1\}M_{ij}. \qquad (3\text{-}6)$$

The last category of linehaul costs, maintenance of way, is also included by constructing a unit cost per loaded car-mile. This unit cost is determined by taking total maintenance of way and structures expenditures, less depreciation on non-track structures, and dividing by total loaded car-miles. This has been done on a yearly, rather than a monthly, basis both because the data are more accessible and because maintenance of way expenditures tend to be programmed on an annual basis. Thus, annual values are much more reliable than monthly values. If we define this unit cost to be p_m, the average unit cost of way and structures maintenance is:

$$c_{5ij}(f_{ij}) = p_m M_{ij}. \qquad (3\text{-}7)$$

The total unit cost for linehaul links is then:

$$c_{ij}(f_{ij}) = \sum_{k=1}^{5} c_{kij}(f_{ij}) \qquad (3\text{-}8)$$

where the $c_{kij}(f_{ij})$ terms are given by equations (3-2), (3-3), (3-5), (3-6) and (3-7).

3.4 Costs on Yard Links

Unit costs to be included for the yard links in the network reflect yard locomotive ownership and maintenance costs, fuel, and yard labor costs. Note that the car-hire, ownership and maintenance costs for the time that cars spend in yards have been included implicitly in the linehaul links through the utilization factor T (total car-days on-line per loaded car-mile). Thus, these costs are not included in the yard links to avoid double-counting. The costs of input resources used in the yards are essentially proportional to the number of yard engine-hours operated. As a result, the construction of $c_{ij}(f_{ij})$ functions for the yard links involves two steps. First, the cost per yard engine-hour is determined. This is assumed to be the same for all yards in the system in any given month. Second, the relationship between carloads moving through the yard and the number of yard engine-hours required must be determined. This relationship will depend upon the physical and operating characteristics of each yard, and thus will tend to be yard-specific.

The ownership and maintenance costs for yard locomotives are related to the number of yard engine-hours in a manner similar to that used to relate road locomotive ownership costs to car-miles. The two basic pieces of information are the monthly cost of owning and maintaining a yard locomotive and the utilization rate of these locomotives, expressed as hours per locomotive per month. The locomotive price is determined as described in Section 2.3 but using the replacement purchase cost for yard locomotives only, not the composite cost used in the econometric estimation. Available data on the number of yard engines owned or leased and the total number of yard engine-hours produced in each month have been used to calculate the utilization level for these locomotives. If the price of yard locomotives is denoted p_y and the utilization level (hours/locomotive/month) is denoted γ, the locomotive cost per yard engine-hour is simply p_y/γ.

Fuel costs are also computed quite simply, by computing a fuel consumption rate (gallons/yard engine-hour) and multiplying by the price of fuel. The fuel consumption rate, r_2, has been estimated by dividing gallons of fuel used in yards for each month by the number of yard engine-hours operated. Note that this consumption rate is time-based, rather than distance-based as in the case of road locomotives, because of the different nature of the two operations. The fuel price used is the same as for the road locomotives, p_F.

Yard labor costs per engine-hour are computed by dividing total yard wages paid in each month by the number of engine-hours operated. This includes both engine crews and other yard employees. The result is an effective yard wage rate. This wage rate may be denoted p_v.

The yard operating cost per engine-hour is then:

$$C_y = p_y/\gamma + p_F r_2 + p_v. \tag{3-9}$$

The relationship between yard engine-hours and traffic volume has been determined empirically using monthly data for individual terminals from the period 1976-1979. Seven of the 27 terminals in the network are very large yards, generally handling in excess of 25,000 cars per month, and two of these are hump yards. The other 20 are smaller facilities, generally handling less than 15,000 cars per month.

Figure 3-2 shows a sample of observed data on yard engine-hours and cars handled for three of the large yards. Figure 3-3 shows comparable data for three of the smaller yards. These figures illustrate two basic facts about the data. First, there is substantial variation in the number of yard engine-hours operated at each yard which is not explained by variation in the number of cars handled, especially for some of the smaller yards. Second, the large yards cover a very wide range of volumes, and appear to exhibit some degree of non-linearity, indicating congestion effects.

In an attempt to find an acceptable simple model to relate yard engine-hours to cars handled, a variety of polynomial regression models have been tried. For the large yards, quadratic models of the form:

$$\text{engine-hours} = a + b(\text{cars})^2$$

provided the best results, and a test of the hypothesis that a and b are the same for all yards resulted in rejection. Thus, seven separate quadratic models have been estimated, one for each yard. For the smaller yards, tests of including quadratic terms failed, and the selected model is linear. A single linear model is used for all these yards. Separate models for each individual small yard could have been estimated, but the quality of the data for several of them was suspect, and their overall impact on the network is relatively small. Thus, a single equation was used for all of them.

These simple models form the second component of the yard link-cost functions. When multiplied by the cost per yard engine-hour from (3-9), the result is a total operating cost function for each yard, expressed in terms of traffic volume.

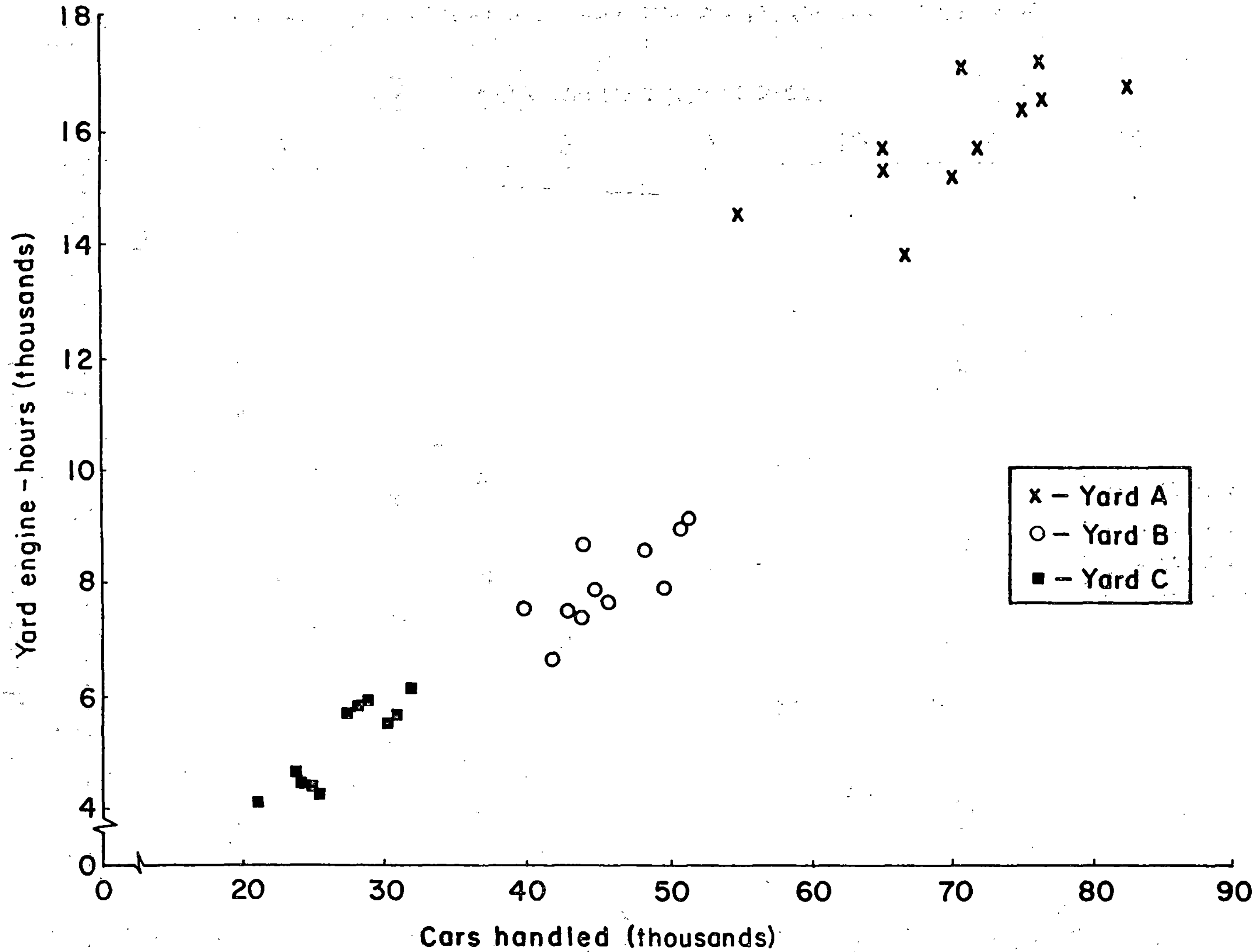

Figure 3-2. Yard engine-hours versus cars handled for three large yards.

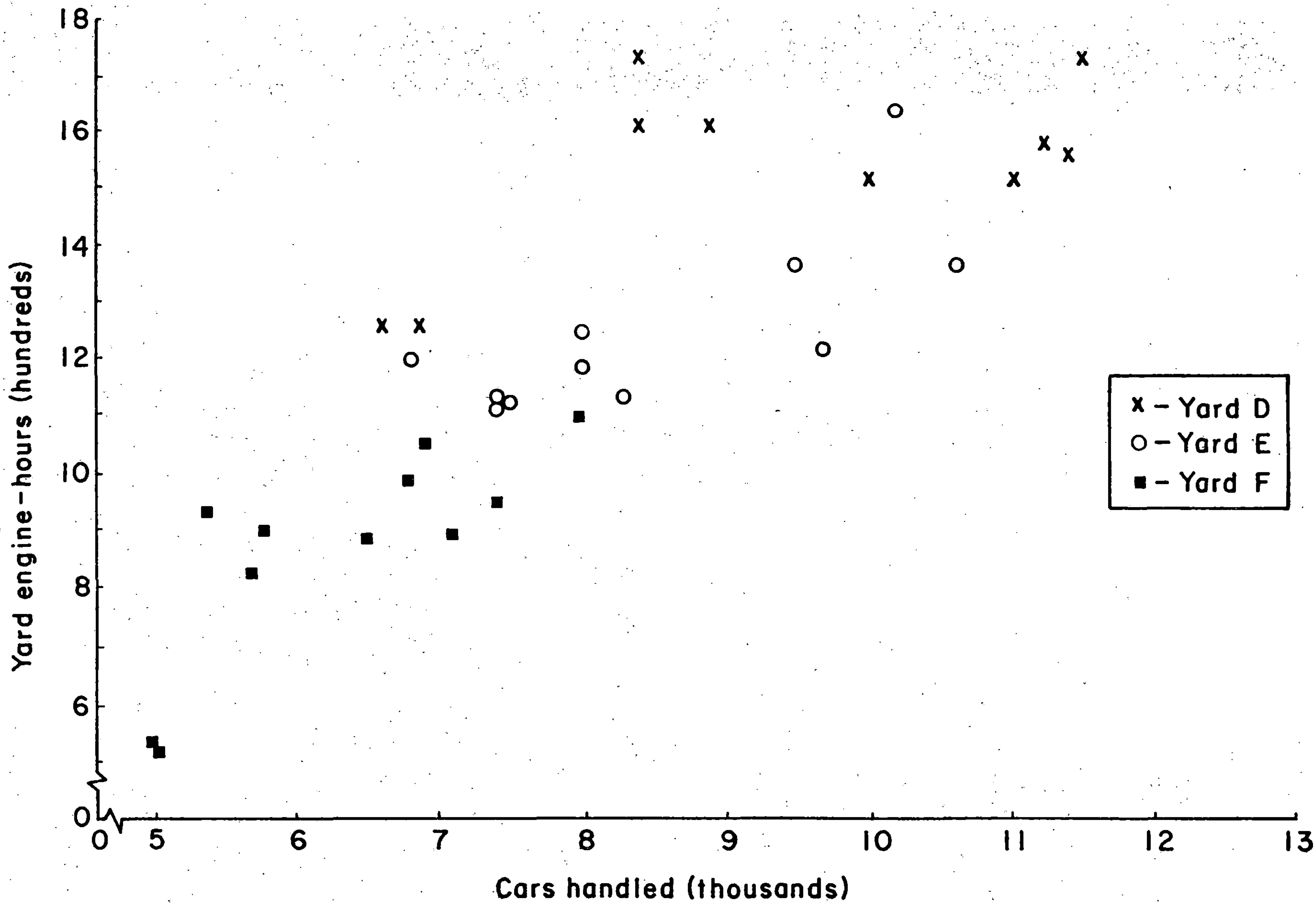

Figure 3-3. Yard engine-hours versus cars handled for three small yards.

In fact, for reasons discussed in detail in the next section, our primary interest is in the marginal cost functions. These marginal cost functions will be linearly increasing functions of volume for the large yards, and constant for the small yards.

As illustrated in Figures 3-2 and 3-3, there is substantial variation in yard engine-hours at each yard not explained by variation in the number of cars handled. Thus, simple statistical models of the type estimated here are by no means a complete picture of yard activities. However, they have proven useful in estimating aggregate yard engine-hours for the system on a monthly basis, and hence total yard operating costs. These results are discussed more fully in Section 3.6. Before proceeding to that, the method for solving the problem formulated in Section 3.1 will be discussed.

3.5 An Algorithm for Solving the Network Problem

The network problem formulated in Section 3.1 is a nonlinear programming problem. The nonlinearity stems from the unit cost functions on some of the yard links, which are functions of the traffic volume handled. When multiplied by the flow volume, as in the objective function of problem (P1), the resulting objective (total cost) is nonlinear. In concept, problem (P1) could be solved by any of several nonlinear programming (NLP) methods, but the size of the problem eliminates most of these from serious consideration. Note, however, that the constraints are linear; the nonlinearity is solely in the objective function. This suggests that a promising approach might be to use successive linear approximations to the objective function because the problem to be solved at each step is then simply a linear programming (LP) problem. An efficient algorithm for large network problems has been developed by LeBlanc *et al*. (1975) based on this idea.

A linear approximation to the objective function can be obtained using a first-order Taylor series expansion. Let f^t be a current set of link flows feasible for problems (P1) (one which satisfies the constraints). The value of the objective function for another set of link flows, y, can be approximated as follows:

$$G(y) \approx G(f^t) + [\nabla G(f)_{f^t}]'(y-f^t) \qquad (3\text{-}10)$$

where $\nabla G(f)_{f^t}$ denotes the gradient of the objective function evaluated at the point f^t.

We are interested in $G(y)$ as a function of y, so the expression for $G(y)$ may be rewritten by grouping the terms differently:

$$G(y) \approx \{G(f^t) - [\nabla G(f)_{f^t}]' \cdot f^t\} + [\nabla G(f)_{f^t}]' \cdot y \,. \qquad (3\text{-}11)$$

The term in braces is a constant, independent of y, so this expression is a linear function of y. The linear approximation of the NLP problem can then be written as follows:

$$\text{(P2)} \qquad \min[\nabla G(f)_{f^t}]' y$$

$$\text{s.t.} \ \sum_j y_{ij}^q - \sum_j y_{ji}^q = z_{iq} \qquad \forall\ i,q$$

$$y_{ij} - \sum_q y_{ij}^q = 0 \qquad \forall\ ij$$

$$y_{ij},\ y_{ij}^q \geq 0 \qquad \forall\ ij,q$$

Since the term in braces is independent of y, it is ignored for the purpose of writing and solving the LP. Once an optimal solution to (P2) is found, we wish to search between f^t and the solution to (P2) for a point which minimizes G. This is a one-dimensional search problem, which can be solved very efficiently by any of a number of methods (Wilde, 1964). This search yields a new feasible solution, f^{t+1}. We can be sure that the new solution is feasible since the feasible region is convex.

It can be shown that the following result holds:

$$\lim_{t \to \infty} f^t = f^*$$

where f^* is the optimal solution to problem (P1). Thus, we can be certain that the iterative procedure converges to the desired solution. This technique of iteratively solving LP problems and one-dimensional searches is known as the Frank-Wolfe algorithm (Frank and Wolfe, 1956).

The attractiveness of this method for solving problem (P1) stems from the fact that the LP problem which must be solved at each step has a very special structure. It is simply a multicommodity transhipment problem, which can be solved using a shortest-path algorithm. Thus, while the problem is quite large, very fast and efficient methods are available to solve it.

At each iteration, we evaluate the marginal link cost functions (find the gradient of the objective function) and use these values as link cost coefficients for a shortest-path problem for each origin-destination pair. This is the reason for the interest in the marginal link cost functions expressed in the previous section. Note that for the linehaul links and for the small yards, the total cost functions are linear in volume. Thus, the marginal costs are constant. It is only for the major yards that the marginal costs are a function of volume, reflecting congestion.

The Appendix to this report describes in more detail how to use the model developed for solving the network flow problems. The next section describes the results of testing this model using data from a sample of months in the 1976-1979 period.

3.6 Testing the Model

Tests of the model have been conducted using a sample of 12 months from the 1976-1979 period. The sample of months allowed us to test the model and illustrate its use. The sample months were selected to span as wide a range of variation as possible. As a summary of the results of these tests, we wish to focus on three important measures; loaded car-miles, yard engine-hours, and total operating cost. For each of these measures, observed data from the actual operations are available for comparison to the model predictions.

Loaded car-miles measures linehaul activity, and the degree to which the model reflects total traffic flows on the network. Yard engine-hours measures yard activity, and tests the degree to which the simple statistical models described in Section 3.4 reflect actual resource requirements. Finally, total operating cost is the measure we are most interested in predicting.

Figure 3-4 illustrates the test results for loaded car-miles (normalized to protect proprietary data). These results appear to be quite satisfactory. In general, the model underpredicts, but in most cases by less than 10%. This underprediction was expected, since the model produces an "optimal" solution, given all the input data for the entire month. This should be better than the solution achievable by managers in the actual system, who must respond to conditions on a day-to-day basis with only partial information available.

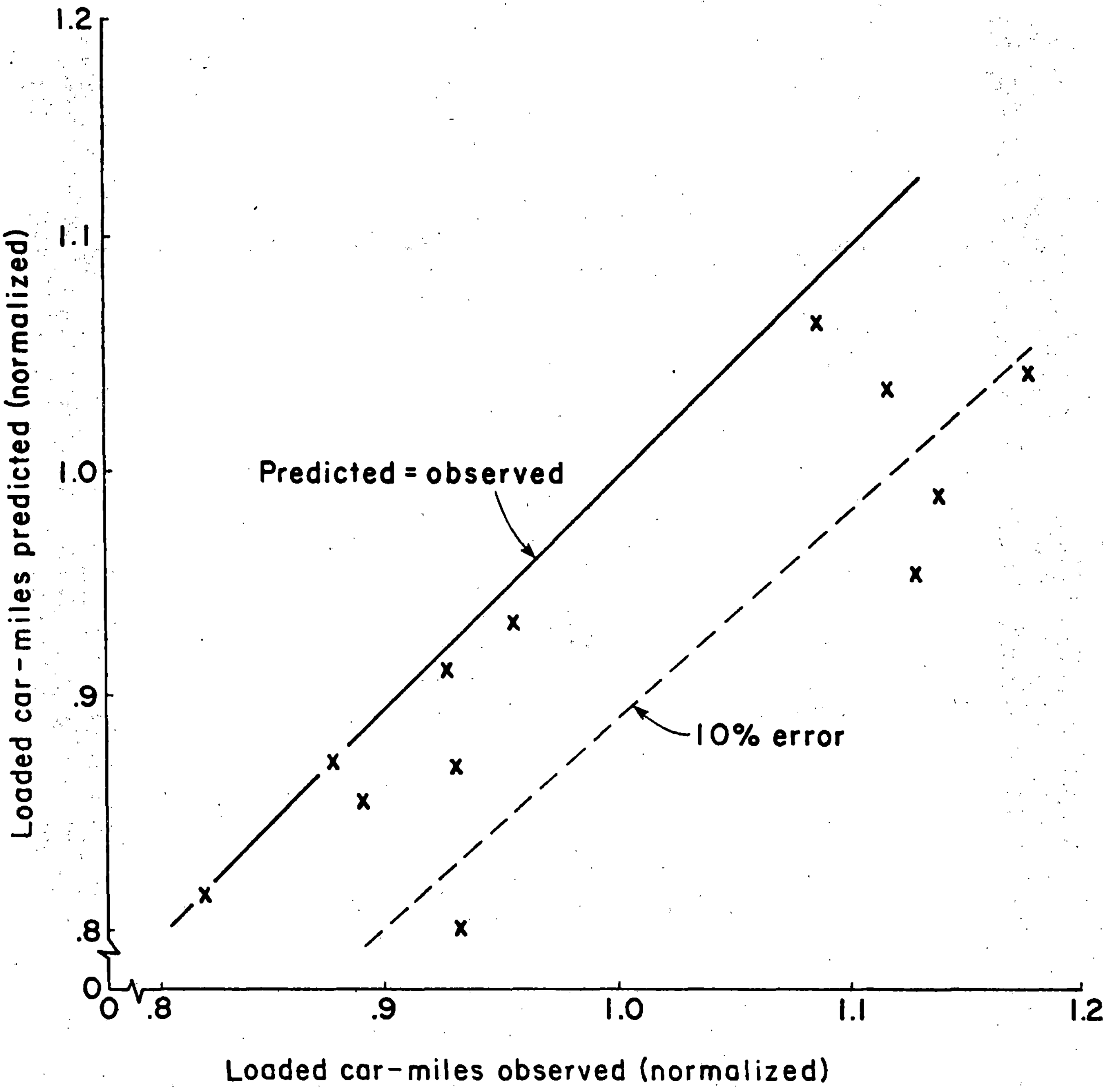

Figure 3-4. Predicted versus observed loaded car-miles.

Figure 3-5 shows the predicted versus observed results for yard engine-hours. In general, these results also appear to be satisfactory. Despite the simple form of the estimated relationships between cars handled and yard engine-hours for each yard, in the aggregate the predictions are typically within 10% of the observed values. The tendency of the model to overpredict the number of yard engine-hours by a small amount is probably due to the aggregation of yards that has been done in the network. The actual system has many more than 27 yards, but these have been combined in the representation of the network used in the model. As a result, the flow through some of the yard nodes is much greater than the individual yards experience in the real system. If the relationship between flow and yard engine-hours were linear, this aggregation of yards would not matter; however, since the relationship for at least some of the yards is nonlinear, the predicted number of yard engine-hours for the aggregated flow is greater than the sum of the engine-hours for the disaggregated flows would be. The overall effect of this aggregation error, however, appears to be minor.

Finally, Figure 3-6 shows predicted versus reported operating expenses (again, normalized). These results are quite satisfactory. The model underpredicts, generally in the range of about 5-15%. the reason for this is the same as for the tendency to underpredict loaded car-miles. The model provides an "optimal" solution. This should always be better than the observed results. However, the closeness of the model predictions to the observed results indicates that the model is a useful predictive tool.

In summary, the model results appear quite acceptable. It has a tendency to underpredict slightly the level of activity (and hence costs) on the line-haul portion of the network, and to overpredict activity (and costs) in the yards. Errors, however, are typically less than 10% in the aggregate measures, and the predictions of total operating costs are generally within 15% of the observed figures.

As a result, this network model provides a reasonable basis for the estimation of a marginal operating cost function. Such a function serves to summarize the information in the network model relating to the sensitivity of operating costs to various input prices and traffic levels. The estimation of this type of marginal cost function is described in Chapter 4.

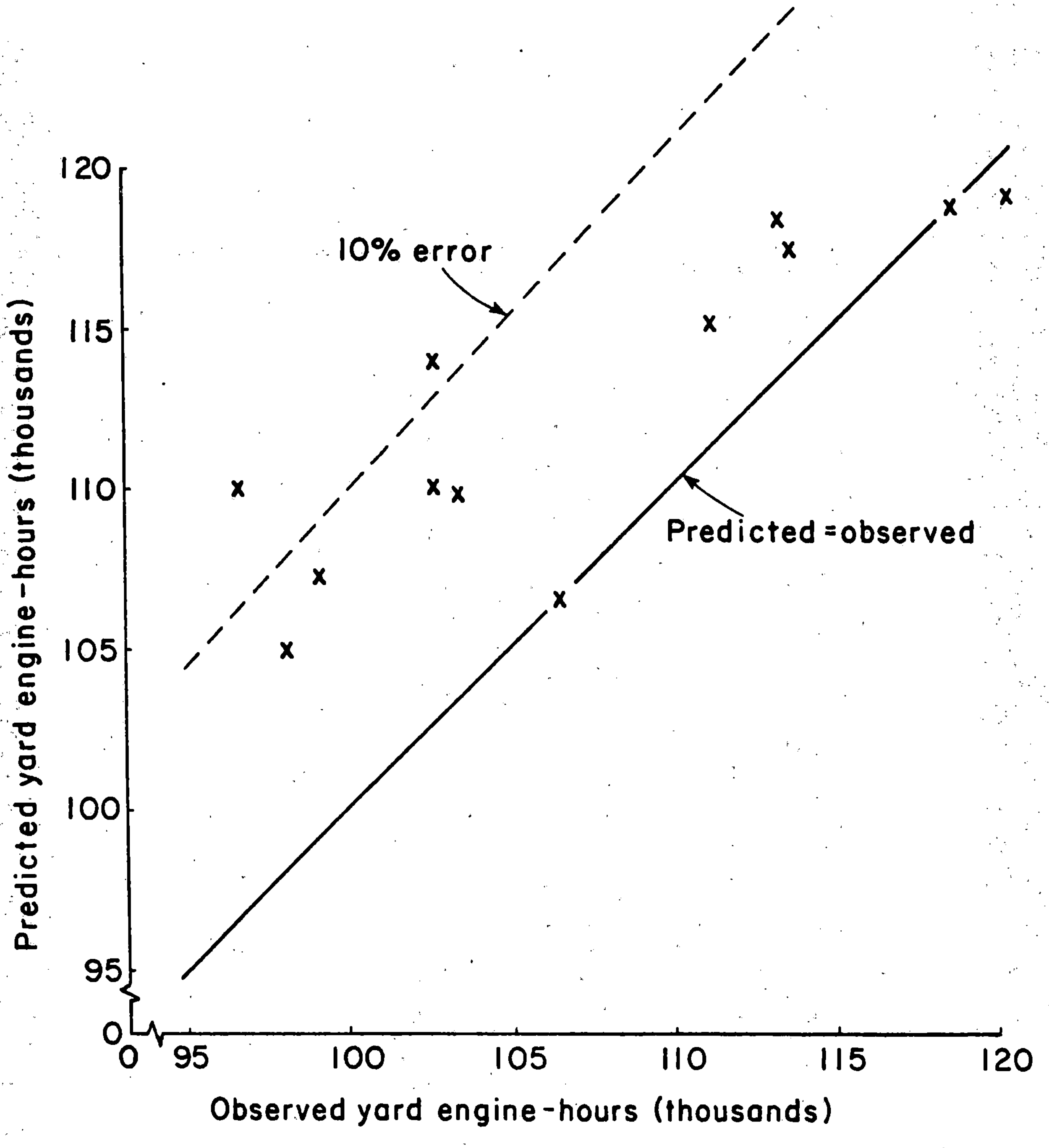

Figure 3-5. Predicted versus observed yard engine-hours.

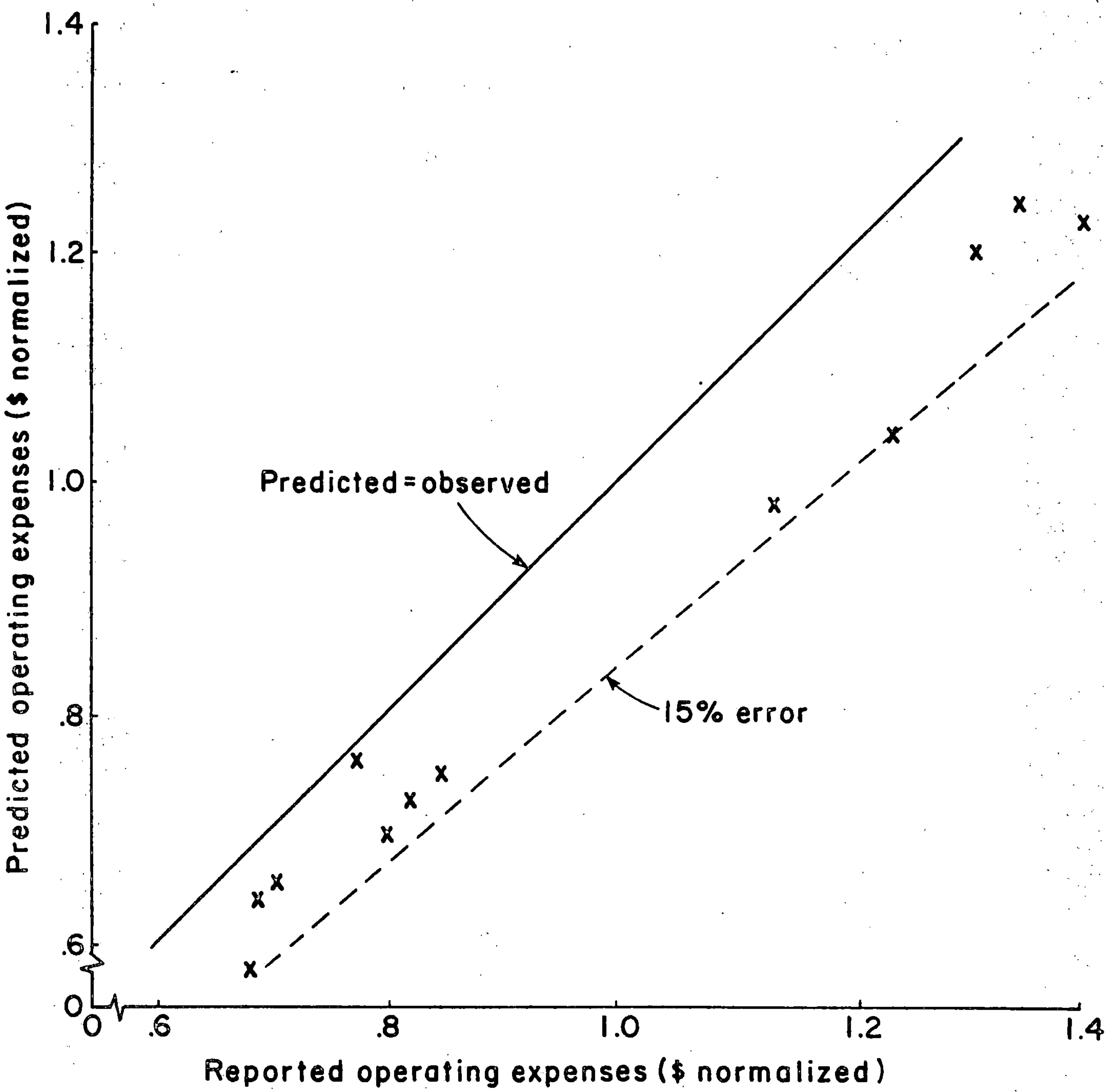

Figure 3-6. Predicted versus reported operating expenses.

CHAPTER 4

ESTIMATION OF AN ORIGIN-DESTINATION SHORT-RUN MARGINAL OPERATING COST FUNCTION

In this Chapter we extend the discussion of Chapter 3 to provide an equation for predicting marginal operating costs, by origin-destination (O-D) pair, as a function of traffic volume, input prices and fixed factors such as levels of capital utilization. The marginal operating cost function is estimated and its use for predicting O-D marginal operating costs is discussed.

4.1 Procedure

The operations model in Chapter 3 can be summarized as the non-linear program (P1):

$$\text{(P1)} \qquad \min_{f} G(f, p; k)$$

$$\text{s.t. } Af = z$$

$$Bf = 0$$

$$f \geq 0$$

where f is the vector of flows within the network, p a vector of prices (such as yard crew wages, per diem rates, fuel price, switching locomotive prices, etc.), k is a vector of fixed factors (capital utilization rates such as loaded car-miles/car-day, empty-to-loaded car-miles ratio, etc.), z is the vector of flows from origins to destinations, and the matrices A and B describe the network.

For a given network (i.e. fixed A and B matrices) the optimal value of (P1) is a short-run variable operating cost. If p and z are varied, we trace out the short-run variable operations cost function $c^o(z, p; k)$:

$$c^o(z, p; k) = \{\min_{f} G(f, p; k) \quad Af = z, Bf = 0, f \geq 0\}$$

Because z is a vector of O-D flows, the gradient of c^o with respect to the vector z is the vector of short-run marginal operating costs; i.e., if z_{ij}(an element of z) is the flow from origin i to destination j then the marginal operating cost for the O-D pair (i,j) is:

$$MC^{ij} = \frac{\partial c^{o}(z,\ p;\ k)}{\partial z_{ij}} \ .$$

We know, however, that MC^{ij} is the shadow price on the constraint in (P1) associated with right-hand-side z_j . Thus, if we run (P1) for month t, using p^t, z^t and k^t as data, (P1) provides an "observation" on $(c^o)^t$ and on the vector of O-D marginal costs $(MC^{OD})^t$. If we do this for N months (t = 1,...,N), we have a sequence of observations $\{(c^o,\ MC^{OD},\ p,\ z,\ k)^t\}_{t=1}^{N}$ on the short-run variable cost function, the marginal cost function and the relevant variables.

Note that the number of observations is N times the number of O-D pairs. Thus, if there are 25 nodes that can be origins or destinations, the potential number of "observations" is 600N, since z would be of size 600 (25^2-25). Of course, this means that the z vector in c^o is also very large (in the example, it is of size 600). Let r be the vector of O-D distances and define aggregate output as the scalar y (loaded car-miles) with

$$y = z'\ r\ . \tag{4-1}$$

Now let us express the short-run variable operating cost function as a function of y instead of z; i.e., as $c^o(y,\ p;\ k)$. To estimate this function we assume (as in Chapter 2) that the function form is a translog:

$$C^O = \sum_{q=1}^{n} \alpha_{q0} X_q + \frac{1}{2} \sum_{q=1}^{n} \sum_{\ell=1}^{n} \alpha_{q\ell} X_q X_\ell \tag{4-2}$$

where, for variable x_q, $X_q = \ell n(x_q/\bar{x}_q)$ and $C^0 = \ell n(c^o/\bar{c}^o)$. Let $x_1 = y$; i.e., X_1 is the transformed aggregate loaded car-miles of flow on the network. Differentiating (4-2) we have:

$$\frac{\partial C^0}{\partial X_1} = \frac{y}{c^o}\frac{\partial c^o}{\partial y} = \alpha_{10} + \sum_{\ell=1}^{n} \alpha_{1\ell} X_\ell \ .$$

Since $y = z'r$, then $dy = (dz)'r$. In particular, if we associate the increment to flow with the O-D pair (i,j) and set the rest of the dz vector to 0, then:

$$dy = (dz_{ij})r_{ij}$$

and hence:

$$\frac{\partial C^0}{\partial X_1} = \frac{y}{c^o} \cdot \frac{\partial c^o/\partial z_{ij}}{r_{ij}} = \frac{y}{c^o} \cdot \frac{MC^{ij}}{r_{ij}} \; . \qquad (4\text{-}3)$$

Thus, the substitution of y for the vector z maintains the O-D aspect of the marginal costs. The marginal cost function (or more precisely flow-elasticity-of-cost function) for the O-D pair (i,j) is:

$$\frac{y}{c^o} \cdot \frac{MC^{ij}}{r_{ij}} = \alpha_{10} + \sum_{\ell=1}^{n} \alpha_{1\ell} X_\ell \; . \qquad (4\text{-}4)$$

We can estimate (4-4) using our sequence of observations. Since some of the x_i are prices, we can augment (4-4) with factor share equations. If we assume that x_2 to x_{n-1} are price terms (i.e. there are n-2 prices, one output and one fixed factor making up n variables) then the factor share equations to add (n-3 in total) are the following:

$$m_q = \alpha_{q0} + \sum_{\ell=1}^{n} \alpha_{q\ell} X_\ell \qquad q = 2,\ldots,n-2 \qquad (4\text{-}5)$$

where m_q is the share of optimal cost represented by factor q (q = 2,...,n-2). As before, we assume symmetry:

$$\alpha_{q\ell} = \alpha_{\ell q} \qquad q,\ell = 1,\ldots,n \qquad (4\text{-}6)$$

Finally, since a cost function should be homogenous in prices, so should the marginal cost function. Thus, we restrict the estimation to satisfy

$$\sum_{q=2}^{n-1} \alpha_{q0} = 1$$

$$\sum_{\ell=2}^{n-1} {}_{q\ell} = 0 \qquad q = 1,\ldots,n \qquad (4\text{-}7)$$

Unfortunately, (4-5) and (4-7) cannot both be satisfied directly. There are n-3 equations in (4-5) from the n-2 price terms. (Adding the last factor share equation is redundant, since $\sum_{q=2}^{n-1} m_q = 1$.) On the other hand, (4-7) calls for restrictions over <u>all</u> the price-related coefficients, even those in the <u>dropped</u> factor share equation. There are two alternative ways to resolve this problem.

1. Estimate (4-4), (4-5) and the cost function jointly, since all the price coefficients appear in the cost function.
2. Estimate (4-4) and (4-5) with all prices expressed relative to the last price, i.e. X_{n-1}. Thus, for example, the factor share equations become:

$$m_q = \alpha_{q0} + \alpha_{q1}X_1 + \sum_{\ell=2}^{n-2} \alpha_{q\ell}(X_\ell - X_{n-1}) + \alpha_{qn}X_n ,$$

since $\ell n[(x_\ell/\bar{x}_\ell)/(x_{n-1}/\bar{x}_{n-1})] = X_\ell - X_{n-1}$.

The second procedure above incorporates the homogeneity restrictions. Each equation has one fewer variable and the restrictions (4-7) must be solved after the estimation to calculate the coefficients for X_{n-1}.

Thus, the system we will estimate is as follows:

$$\left(\frac{y}{C^0} \cdot \frac{MC^{ij}}{r_{ij}}\right) = \alpha_{10} + \alpha_{11}X_1^t + \sum_{\ell=2}^{n-2} \alpha_{1\ell}(X_\ell^t - X_{n-1}^t) + \alpha_{1n}X_n^t + \varepsilon_1^t \qquad (4\text{-}8)$$

$$m_q^t = \alpha_{q0} + \alpha_{11}X_1^t + \sum_{\ell=2}^{n-2} \alpha_{q\ell}(X_\ell^t - X_{n-1}^t) + \alpha_{qn}X_n^t + \varepsilon_q^t \qquad q = 2,\ldots,n-2$$

subject to the symmetry conditions (4-6) and any appropriate autocorrelation structure assumptions, such as (2-10).

Note that the second approach (the relative price approach) estimates a system of (n-1) variables by (n-2) equations. The first approach in which one estimates the full translog function with (4-4) and (4-5) results in a much larger system since the cost function has all the second-order terms, as well as first-order terms. For example, if there are 3 prices, one output and one fixed factor, alternative 1 results in 5 equations and 29 variable, while alternative 2 results in 4 equations and 15 variables.

4.2 Estimation Results

System (4-8), subject to (4-6), has been estimated for a three price, one fixed factor, one aggregate output (i.e. n = 5) model using data from the twelve month sample mentioned in Chapter 3: February 1976, December 1976, February 1977, July 1977, December 1977, February 1978, April 1978, February 1979, March 1979, May 1979, August 1979 and November 1979. The months were selected to maximize the variation in observed prices and flows. The independent variables are the following:

x_1: aggregate loaded car-miles

x_2: fuel price in dollars/gallon

x_3: price for locomotives in dollars/locomotive-month

x_4: price for yard labor in dollars/engine-hour

x_5: loaded car-miles/total car-days on-line (= 1/T).

These variables constitute the "best" subset of the inputs to the operations model in the sense that all the other input prices, etc., are highly correlated with one of the above variables.

Variable X_5 represents a car utilization rate which is not easily adjusted and has been used as a fixed factor. Note that X_5 is different from the speed variable used in Chapter 2 as an output measure. While the units of the variables are the same, X_5 includes implicitly the empty-to-loaded car-miles ratio, which is clearly a capital utilization measure. This ratio is also difficult to adjust, at least in the short-run. Thus X_5 appears to be a reasonable choice for a fixed factor to represent the nature of operations.

Tables 4-1, 4-2 and 4-3 provide the estimation results for the marginal operating cost function, the fuel share equation and the labor share equation. Since wages for yard and road labor are very highly correlated, we have used only the price of yard labor, but the share reflects both yard and road labor. Table 4-4 provides the equipment share equation, which is computed via the homogeneity restrictions. Standard errors are in parentheses below each estimated coefficient in each table.

The constant term in each equation is the first-order term in the underlying cost function. Thus, for example, α_{10} in Table 4-1 is the first-order output term in the underlying cost function.

Using the results from Table 4-1 we can express the marginal operating cost function as:

$$MC^{ij} = r_{ij} \frac{c^o}{y} \cdot \{.9398 + 0.152 \ \ell n(y/\bar{y}) + 0.0061 \ \ell n(p_F/\bar{p}_F) + 0.0361 \ \ell n(p_{LA}/\bar{p}_{LA}) - 0.0422 \ \ell n(p_{LO}/\bar{p}_{LO}) - 0.0217 \ \ell n(k/\bar{k}) \ . \qquad (4\text{-}9)$$

Thus, marginal operating cost for any given O-D pair (i,j) is simply the distance between i and j (r_{ij}) times a function of y, c^o, the prices and the fixed factor. At the point of means:

$$MC^{ij} = r_{ij} \frac{\bar{c}^o}{\bar{y}} (.9398). \qquad (4\text{-}10)$$

The coefficients on output, input prices and the fixed factor which appear in Table 4-1 (and equation (4-9)) are the coefficients on second-order terms involving output in the original cost function. After differentiating with respect to output to construct the marginal cost function, they become linear terms. Note that the sum of the terms on input prices ($\alpha_{12} + \alpha_{13} + \alpha_{14}$) is zero. This is as specified by constraint (4-7). The fact that the coefficient on locomotive price (p_{LO}) is negative does not mean that marginal costs will fall as locomotive prices rise, because a change in locomotive price will also affect c^o which is also in the equation.

Table 4-1
Estimation Results
Marginal Operating Cost (Share)

Variable	Coefficient	Estimate	Standard Error
Y	α_{11}	0.1520	0.0052
PFUEL	α_{12}	0.0061	0.0008
PLABOR	α_{13}	0.0361	0.0011
PLOCO	α_{14}	-0.0422	0.0016
K	α_{15}	-0.0217	0.0026
CONSTANT	α_{10}	0.9398	0.0004

Table 4-2
Estimation Results
Fuel (Share)

Variable	Coefficient	Estimate	Standard Error
Y	α_{21}	0.0061	0.0008
PFUEL	α_{22}	0.1113	0.0004
PLABOR	α_{23}	-0.0417	0.0004
PLOCO	α_{24}	-0.0696	0.0003
K	α_{25}	0.0459	0.0004
CONSTANT	α_{20}	0.1116	0.0001

Table 4-3
Estimation Results
Labor (Share)

Variable	Coefficient	Estimate	Standard Error
Y	α_{31}	0.0361	0.0011
PFUEL	α_{32}	-0.0417	0.0004
PLABOR	α_{33}	0.1109	0.0006
PLOCO	α_{34}	-0.0692	0.0001
K	α_{35}	0.0289	0.0005
CONSTANT	α_{30}	0.2590	0.0001

Table 4-4*
Estimation Results
Equipment (Share)

Variable	Coefficient	Estimate	Standard Error
Y	α_{41}	-0.0422	0.0011
PFUEL	α_{42}	-0.0696	0.0007
PLABOR	α_{43}	-0.0692	0.0010
PLOCO	α_{44}	0.1388	0.0011
K	α_{45}	-0.0748	0.0001
CONSTANT	α_{40}	0.6294	0.0001

* All coefficient estimates and standard errors in this table are computed via the homogeneity restrictions. For example, $\alpha_{40} = 1 - \alpha_{20} - \alpha_{30}$.

For example, suppose initially all variables are at the point of means, so that the marginal cost for O-D pair ij is given by (4-10). Now suppose that rising interest rates cause the price of equipment (represented by p_{LO}) to increase by 10%. What is the effect on marginal costs?

To answer this, we must refer to equation (4-2) to recompute c^0. Because all variables are initially assumed to be at the point of means, and only p changes, all X_ℓ except X_4 will be zero. Thus, we have

$$\ln(c^0/\bar{c}^0) = \alpha_{40} \ln(p_{LO}/\bar{p}_{LO}) + \alpha_{44}[\ln(p_{LO}/\bar{p}_{LO})]^2.$$

From Table 4-4, we find that $\alpha_{40} = .6294$, and $\alpha_{44} = .1388$. In this case $p_{LO} = 1.1\ \bar{p}_{LO}$, so we have

$$\begin{aligned}\ln(c^0/\bar{c}^0) &= .6294 \ln(1.1) + .1388[\ln(1.1)]^2 \\ &= .06124.\end{aligned}$$

This implies that:

$$c^0 = (e^{.06124})\ \bar{c}^0 = 1.063\ \bar{c}^0.$$

Returning to (4-9), all the terms inside the brackets except the constant and the p_{LO} term are zero, so we have

$$MC^{ij'} = r_{ij} \frac{1.063\bar{c}^0}{\bar{y}} \{.9398 - .0422 \ln(1.1)\}$$

$$= r_{ij} \frac{\bar{c}^0}{\bar{y}} (.9948)$$

Thus, $MC^{ij'}/MC^{ij} = .9948/.9398 = 1.058$, so marginal cost has risen about 5.8% as a result of the 10% increase in p_{LO}. Similar analyses can be done to examine the effects of other changes in prices or output level.

The constant terms in Tables 4-2, 4-3 and 4-4 represent the first-order terms in the cost function, and thus sum to one as a result of the first constraint in (4-7). At the point of means in the data, all terms in the factor share equations except these constants go to zero, and hence at that point the shares of total operating cost attributable to fuel, labor and equipment are about 11%, 26% and 63%, respectively. Because the point of means average conditions over a period during which fuel prices especially were rising rapidly, these shares are not very reflective of current conditions. To evaluate current conditions more carefully, the other terms in the factor share equations (with coefficients indicated in Tables 4-2, 4-3 and 4-4) would have to be used also.

As a final comment regarding interpretation of these results, some caution should be exercised in applying them to very short or very specialized movements. Very short movements may require relatively more switching and local train activity than normal, and thus the linear relationship to distance may be inaccurate. At the other extreme, the costs unit train movements may not be reflected well because they have not been separated from other train operations in the analysis, even though their operating costs may be relatively different.

4.3 The Contribution of Economic Theory to Operational Analysis

It is worth noting that without the above structure, derived from economic theory, the estimation of the marginal cost function would have been much poorer. To see this, suppose that we had simply estimated (4-4) without the factor share equation (4-5), the symmetry restriction (4-6) or the homogeneity restriction (4-7). The result of such an estimation is shown below; with standard errors in parentheses:

$$\begin{aligned}
\frac{y}{c^0}\frac{MC^{ij}}{r_{ij}} = {} & \underset{(0.0005)}{0.4029} + \underset{(0.0090)}{0.0688}\,\ln(y/\bar{y}) - \underset{(0.0039)}{0.0158}\,\ln(p_F/\bar{p}_F) \\
& + \underset{(0.0036)}{0.0315}\,\ln(p_{LA}/\bar{p}_{LA}) + \underset{(0.0091)}{0.0428}\,\ln(p_{LO}/\bar{p}_{LO}) \\
& - \underset{(0.0033)}{0.0155}\,\ln(k/\bar{k})\,. \qquad (4\text{-}11)
\end{aligned}$$

Comparing (4-11) to Table 4-1 is quite revealing. The estimated coefficient on $\bar{y}$ and the estimated intercept in (4-11) are both less than half of the corresponding estimates in Table 4-1. Both the price of fuel and the price of locomotives have experienced sign changes. We also observe that (4-11) does not reflect homogeneity in prices; the price coefficients should add to zero. Since we know that the results in Table 4-1 are theoretically justifiable, we can see that dropping the theoretical restrictions (which yields (4-11)) can lead to seriously flawed and inferior results.

4.4 Summary and Implications

We have described a procedure for predicting marginal operating costs for specific origin-destination prices in a rail firm. Such estimates reflect an optimization model, and thus they provide lower bounds on actual marginal costs. Obviously, the more accurate the network representation and cost data the better the estimates will be. In general, however, the procedure outlined above provides a systematic and defensible way to compute O-D specific marginal operating costs and would be useful both in pricing decisions and in operations analysis studies. As an example of the former, it is clear that prices should be above the marginal operating cost; this procedure provides a floor and its justification. An an example of its role in operations analysis studies, one could use such a procedure to examine the impact of network changes on the sensitivity of marginal operating costs to input prices (such as fuel) by estimating MC^{ij} functions for each network specification and then comparing the functions.

CHAPTER 5

CONCLUSIONS AND IMPLICATIONS

5.1 Conclusions and Implications from the Cost Model of the Firm

Probably the most striking conclusion of the results presented in Chapter 2 is the clear evidence of returns-to-density for the railroad under study. When coupled with the results of Phase I (returns-to-density for the small railroad), the implication is that returns-to-density is not an isolated phenomenon.

This finding can be contrasted with those cross-section studies which have ignored density issues and have either found constant returns-to-scale or slightly increasing returns-to-scale. In general such studies have not controlled for differences between firms within samples. As discussed in Chapter 2, this can lead to biases in estimates. Without such control, economies-of-size (i.e. changes in configuration and size of the firm) and economies-of-density (configuration and size held fixed) are confused.

A second reason for the difference is inclusion of a properly constructed quality-of-service measure. In general the cross-section studies have either not included quality measures or they have had to use surrogates. The test on inclusion of the speed term clearly indicates that dropping the variable leads to a misspecification error. It also leads to a lower estimate of economies-of-density. Thus, the lack of a properly constructed quality-of-service variable in earlier studies may also be partially responsible for the difference between our results and the cross-section results.

We also found that the most variable factors of production (fuel, labor and equipment) do not appear to substitute readily for one another, and all the factor demand functions are inelastic, with demand for labor most inelastic. Labor is the major share of short-run variable costs (53%).

It is also interesting to examine the share of fuel in costs (19%) and compare it to the result from the smaller railroad studied in Phase I (6%). While part of this difference may be due to the difference in the sizes of the firms, part may also reflect the fact that the estimate for the larger railroad reflects conditions in the mid-to-late seventies, while the estimate for the small firm reflects the early-to-mid seventies. Both firms were inelastic with respect to fuel prices and, given the enormous fuel price change experienced, one might expect fuel's share of cost to increase.

5.2 Conclusions and Implications from the Operations Model

One principal conclusion from the network model described in Chapter 3 is that it is quite feasible to produce reasonable operating cost estimates from a relatively simple network model. This is true even for network operations as complex as those on the railroad under study, which is a major Class I system. The predictions of two major measures of activity - loaded car-miles and yard engine-hours - were typically within 10% of observed numbers for a sample of months over a four-year study period. The resulting operating cost estimates were all within 15% of the reported operating expenses for those months.

In general, the model underestimates operating costs by a small amount. This is to be expected, because it is an optimization model, and predicts the "optimal" (minimum cost) pattern of car flows given all the information on movements for the month. In reality, decisions made on a day-to-day basis from incomplete information are unlikely to be optimal in the same sense, even though they may be the best decisions possible under the given circumstances. Thus, in some ways, the model provides a standard against which observed results can be compared. This presents a potential use of the model for management evaluation and control of operations.

The purposes of this network model should be made clear. In contrast to some other railroad network models, it is not intended to address detailed operational questions involving blocking, train dispatching, power utilization, etc. Its purpose is to provide estimates of overall operating costs and origin/destination-specific marginal costs which are sensitive to changes in: 1) the level of traffic on the system, 2) the prices of cars, locomotives, labor, fuel, maintenance, etc., and 3) aggregate characteristics of operations on the network such as average train length, empty-to-loaded car-miles, loaded car-miles per car-day, locomotive utilization and use of system vs. foreign cars. This objective is quite different from that of many previous railroad network models which have focused on routing, scheduling and blocking policies, or on detailed operations of specific elements of the network.

The model is inexpensive to use, both in terms of analyst time and computer time. Setting up for a run from "scratch" requires a few hours of analyst time to compute input prices and the various utilization factors. Subsequent runs to explore changes in various inputs can be made very quickly. A preprocessor program could make this setup even easier and faster if the model were to be used regularly, but such a program has not been developed in this project. The computer costs have been less than $10 per run on Cornell's IBM 370/168 system.

A primary potential use of this model is investigation of the marginal costs of moving traffic on specific O-D pairs for pricing purposes. Because the model is an optimization model, it produces a lower bound, or floor, under the marginal costs of providing various services. The rates charged should be at least this great. The capability to evaluate such marginal costs is of great use to railroads, in both justifying rates for regulatory proceedings and in negotiating contracts with shippers.

The statistical estimates of marginal cost described in Chapter 4 provide a convenient method for summarizing a large amount of information from the network model. The functional form expresses the marginal cost as a function of distance, input prices, total volume being moved on the system and a network parameter (used as a fixed factor). This model is an excellent example of the power of the "hybrid" approach to cost analysis which has been the cornerstone of this project. The means for generating individual observations on marginal costs for specific movements is an engineering model; the means for summarizing this data in a meaningful way makes substantial use of economic theory. It should be emphasized that without the economic theory to provide a structure within which to do the statistical analysis in Chapter 4, the results were much less satisfactory. Together, the economic theory and the engineering models provide a powerful tool for the analysis of railroad costs.

The resulting statistical model in Chapter 4 is a convenient form in which to express the general effects of changes in traffic volume or various input prices on marginal costs. This can be extremely useful, for example, in examining the potential impacts of fuel price increases on marginal costs, and hence the potential of such price changes to affect rates. It also provides a means for examining the potential impact of improvements in car utilization, as measured by changes in the number of loaded car-miles per car-day on-line, to reduce marginal costs. Information on such issues is very important to both railroad management and to Federal policy-makers and analysts.

While the operating analysis as it stands is useful, there are also some enhancements which might improve it further. First, somewhat more detailed analysis of yard operations could lead to improved specification of the relationship between yard engine-hours and the number of cars handled in yards. This, in turn, would improve the ability of the network model to reflect changes in yard operation (which would affect costs) more accurately. Second, it could be desirable to incorporate different types of train operation (e.g. local, run-through, unit trains, etc.) in the network specification. This would require definition of various classes of links in the network. Such modifications would improve the sensitivity of the network model to more detailed operational changes, but would make the solution of the optimization problem more expensive. Further investigation is required to determine whether the benefits would be worth the costs. Finally, better specification of the output variable in the statistical cost function is desirable. Currently, the variable used is total flow on the system. This reflects the fact that flows other than that on the O-D pair in question affect the marginal costs for any given O-D pair. However, it is also true that not all the flows on the network affect every other flow. It would be desirable to reflect only those flows which directly interact with the flow on a given O-D pair. This presents some difficult measurement problems, but is a useful area for further investigation.

5.3 Use of the Research Products

Figure 5-1 summarizes the potential uses of this research. On the left-hand side of the diagram are the three major research products - the network model, the short-run marginal operating cost function, and the firm-level cost functions (short-run and long-run). In the middle of the diagram are the principal outputs of these models. On the right-hand side are three major kinds of activities undertaken by railroad firms and/or government policy makers or regulators. The arrows indicate how each of the activities depend on the results produced by the three models.

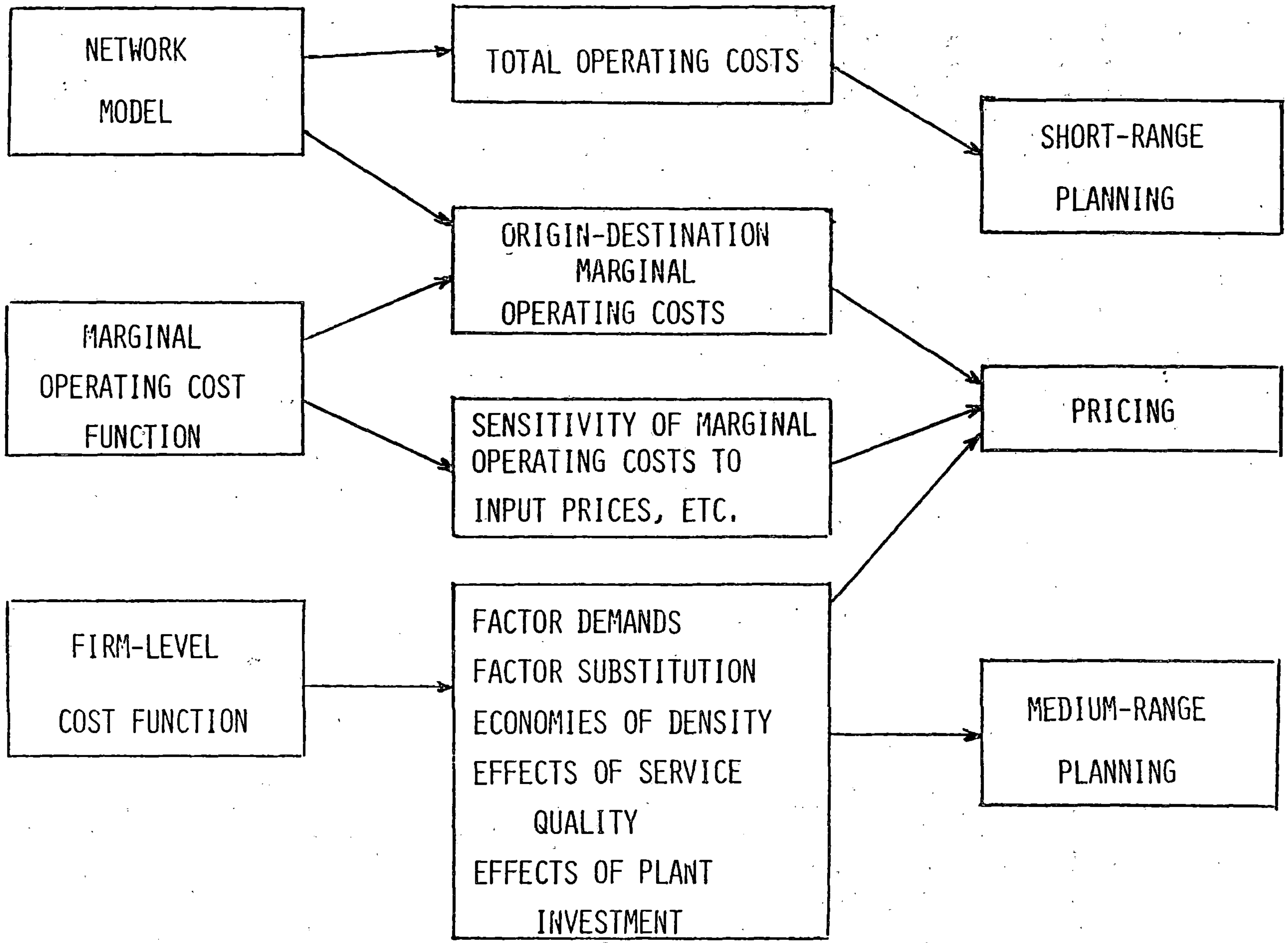

Figure 5-1. Use of the research products.

For example, short-range planning activities of the railroad would use the total operating cost predictions from the network model, but probably not the information on factor substitution, for example, from the firm-level cost function. On the other hand, pricing decisions rely on information from the firm-level model as well as marginal operating cost information. Medium-range to long-range planning that might be done either by railroads or government would not rely much on details of network operation, but would make use of the more general information in the firm-level analysis.

In closing, we wish to emphasize that the form of cost analysis described in this report differs markedly from earlier studies. We have demonstrated that cross-sectional econometric studies using aggregate data are prone to misinterpretation because of model misspecification. On the other hand, we have also demonstrated that economic theory is very important in making sense of marginal cost estimates produced by engineering models. Together, the econometric and engineering techniques complement each other very effectively. Joint use of these methods can lead to more reliable estimates of cost than can be obtained from either by itself. This provides the basis for improved public policy decisions and for improved ability of railroad management to understand the costs of providing services.

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APPENDIX

USING THE NETWORK OPERATING COST MODEL

RAILNET is a FORTRAN program designed to predict operating costs on railroad networks by solving the network flow problem described in Chapter 3. The current version has been designed for the IBM VM/370 operating system, using the Conversational Monitor System (CMS). The program includes four major modules: (1) a control module which handles much of the interaction with the user and invokes other modules in response to the user's instructions, (2) a data input module, (3) an analysis module which performs the actual computations, and (4) an output module. The nature and capabilities of each of these modules are described in the individual sections which follow. Detailed information on the use of the various options within each module is provided in Section A.5.

A.1 Program Control

To initiate a session with RAILNET, the user logs on at a terminal, attaches a data set (if one is to be used) and invokes the program. S/he is then communicating with the program control module. The program will ask the user whether or not an existing data set is to be read, what kind of terminal is being used, and then give the user a menu of activities from which s/he may select what to do next. This menu includes: examining (and possibly changing) the data, writing a file to save the current data including changes made during this session, conducting an assignment of traffic to the network, restarting the program, or ending the session. Additional details on these various options are described in Section A.7.

When the user responds, the control module may ask additional questions to clarify exactly what the user wishes to do. When the instructions are completely specified, the control module invokes one of the other modules to begin the requested activity. Each activity includes a set of potential tasks. In some cases, the sequence of tasks in an activity is relatively fixed. In other cases, there is considerable room for user intervention. At the completion of each task, the control module may request further instructions from the user if the next task in the sequence is not predetermined.

At the completion of an activity, the control module repeats the menu, and the user may select another activity. This process repeats until the user terminates the session.

A.2 Data Input

Data input may be from a permanent file constructed prior to the session or directly from the terminal. In either case, the data include three sets of information, pertaining to system parameters, links and traffic volumes. The parameter data specify values which pertain to all links, such as wage rates, fuel prices, equipment prices, etc. The link data describe each link in the network including origin node, destination node, length, capacity, and a set of parameters for use in computing the link cost as a function of volume. The volume data provide an origin-destination (O-D) flow table. Data input formats are described in Section A.5.2.

An important element of the program is the capability to modify the data file during a terminal session. This makes the analysis truly interactive, and is of great use for exploring the effects of changes in the network quickly and easily. Any element of the data file can be modified or deleted, and new nodes or links can be defined.

If the user makes several changes in the data file (or creates a new data file) at the terminal, this new data may be stored as a permanent file. This is done using the "WRITE" option in the activity menu, as described in Section A.7. The program than writes the data to an output file in a format suitable for rereading at a later time.

A.3 Computation

The RAILNET program has the ability to assign traffic to links in the rail network using the algorithm summarized in Chapter 3 (pp. 47-49). The user must supply a FORTRAN subroutine to compute the unit cost of carload movements over each network link. This allows the user to specify the form of marginal cost function used, and to use different functions for different classes of links. The specifications for this subroutine are described in Section A.5.1.

A.4 Output

The output module of the program allows the user to obtain detailed link-by-link output in tabular form at the end of an assignment. It also produces summary statistics on total volume moved, total loaded car-miles, and total cost.

A.5 Detailed Instructions for Use of the Program

This section provides detailed instructions for developing the subroutine to calculate link impedances, preparing data input, and using the major options within the program. These topics are discussed individually in the following subsections.

A.5.1 The Link Cost Routine

The user must provide a subroutine which will calculate the unit cost of a carload movement over each link in the network. This user-provided routine enhances the capability of RAILNET because several different link types may be designated and the link cost calculated for each type using a different formula. The subroutine may use values of up to 5 parameters for each link, which are input as arguments to the subroutine. The subroutine must be written in FORTRAN and named LKCOST. The format which should be followed is:

```
SUBROUTINE LKCOST (LT,CAP,DIST,ARG,VOL,COST)
DIMENSION ARG(5)
.
.
.
RETURN
END
```

where:
- LT = link type (integer-valued)
- CAP = link capacity (cars/time period) (real-valued)
- DIST = length of the link (miles) (real-valued)
- ARG = vector of link parameters (real-valued)
- VOL = link volume (real-valued)
- COST = resulting cost (real-valued)

LT, CAP, DIST, ARG and VOL are inputs to LKCOST. COST is the output. Link type (LT) may be used as an indicator to determine how to use the parameter vector to compute the cost.

As a simple example of a possible LKCOST subroutine, suppose we desire to calculate costs using the following function:

$$c = \{a + bv + dv^2\}m$$

where:
- c = cost ($/carload)
- v = volume (carloads/unit time)
- m = length of the link (miles)
- a,b,d = parameters.

This cost function has four link-specific parameters: the length and values of a, b and d. The link length is passed to subroutine LKCOST explicitly, but a, b and d would have to be passed as arguments in the ARG array.

Under the assumption that ARG(1), ARG(2) and ARG(3) give values for a, b and d, respectively, for a given link, the following subroutine would perform the required computations:

```
SUBROUTINE LKCOST (LT,CAP,DIST,ARG,VOL,COST)
DIMENSION ARG(5)
COST = (ARG(1) + ARG(2)*VOL + ARG(3)*VOL*VOL)*DIST
RETURN
END
```

Note that the link type parameter, LT, is not used in this routine. If some links in the network were to have costs calculated according to this formula, and others use a different cost function, the LT parameter could be used to distinguish among them and branch to the appropriate section of the LKCOST routine.

This sort of branching is shown more clearly in a second example, which implements the cost calculations described in Chapter 3 of this report. The listing of LKCOST is shown in Figure A-1. The first executable statement of the subroutine tests for actual usability of the link whose cost is to be calculated. A link can be "removed" from the network by setting its capacity to zero. In that case, LKCOST will return a very large unit cost for that link ($\$10^8$) so that no traffic will be assigned to it.

```
      SUBROUTINE LKCOST (LT, CAP, DIST, ARG, VOL, COST)
       DIMENSION ARG(5)
C
C  COMMON BLOCK /PARM/ CONTAINS ALL THE SYSTEM PARAMETERS
C
       COMMON /PARM/ RCWAGE, YCWAGE, FUELP, RLFRAT, YLFRAT,
     X          RLOWN, RLUTIL, YLOWN, YLUTIL, COWN, PDD,
     X          PDM, ATL, ELCMR, FCOL, FCM, CDLCM, WSCLCM
C
C*****************************************************************
C
C  RCWAGE = ROAD CREW WAGE RATE
C  YCWAGE = YARD CREW WAGE RATE
C  FUELP = FUEL PRICE
C  RLFRAT = ROAD LOCOMOTIVE FUEL CONSUMPTION RATE
C  YLFRAT = YARD LOCOMOTIVE FUEL CONSUMPTION RATE
C  RLOWN = ROAD LOCOMOTIVE OWNERSHIP COST
C  RLUTIL = ROAD LOCOMOTIVE UTILIZATION RATE
C  YLOWN = YARD LOCOMOTIVE OWNERSHIP COST
C  YLUTIL = YARD LOCOMOTIVE UTILIZATION RATE
C  COWN = SYSTEM CAR OWNERSHIP COST
C  PDD = DAILY PER DIEM CHARGE ON FOREIGN CARS
C  PDM = MILEAGE PER DIEM CHARGE ON FOREIGN CARS
C  ATL = AVERAGE TRAIN LENGTH
C  ELCMR = EMPTY-TO-LOADED CAR-MILE RATIO
C  FCOL = PROPORTION OF CARS ON LINE WHICH ARE FOREIGN
C  FCM = PROPORTION OF TOTAL CAR-MILES MADE BY FOREIGN CARS
C  CDLCM = CAR-DAYS PER LOADED CAR-MILE
C  WSCLCM = WAY AND STRUCTURES MAINTENANCE COST PER LOADED
C                 CAR-MILE
C
C******************************************************************
C  FIRST CHECK FOR EXISTENCE OF LINK
C
       IF (CAP .GT. 1) GO TO 10
C  IF NO LINK, SET VERY HIGH COST AND RETURN
       COST = 1.0E8
       RETURN
C
C  TEST FOR LINK TYPE 1 (LINEHAUL LINKS)
C
   10  IF (LT .NE. 1) GO TO 20
C  COMPUTE ROAD CREW COST PER CAR-MILE
       C1 = RCWAGE * (1. + ELCMR)/ATL
C  COMPUTE FUEL COST PER CAR-MILE
       C2 = FUELP * RLFRAT
C  COMPUTE LOCOMOTIVE OWNERSHIP COST PER CAR-MILE
       C3 = RLOWN * RLUTIL
C  COMPUTE CAR OWNERSHIP COSTS PER CAR-MILE
       C4 = (FCOL * PDD + (1. - FCOL) * COWN) * CDLCM
    X           + (1. + ELCMR) * FCM * PDM
C  GET TOTAL COST BY ADDING C1 - C4, PLUS W&S MAINTENANCE,
C    AND MULTIPLYING BY THE LENGTH
       COST = (C1 + C2 + C3 + C4 + WSCLCM) * DIST
       RETURN
C
```

Figure A-1. Listing of LKCOST subroutine to implement the formulae given in Chapter 3.

```
C   TEST FOR LINK TYPE 2 (YARD LINKS)
C
   20    IF (LT .NE. 2) GO TO 30
C   COMPUTE COST PER YARD ENGINE HOUR
         CY = YLOWN/YLUTIL + FUELP*YLFRAT + YCWAGE
C   COMPUTE MARGINAL YARD ENGINE HOURS PER CAR
         MEH = 2. * ARG(1) * VOL/(CAP*CAP)
C   THEN GET MARGINAL COST
         COST = CY * MEH
         RETURN
C
C   TEST FOR LINK TYPE 3 (LOCAL CONNECTORS)
C
   30    IF (LT .NE. 3) GO TO 40
C   COST IS LOCAL AREA SPECIFIC ($/MILE * DISTANCE)
         COST = ARG(1) * DIST
         RETURN
C
C   IF LINK TYPE NOT 1, 2 OR 3, THERE IS AN ERROR IN DATA
C
   40    WRITE(6,1000) LT
 1000    FORMAT(' *** ERROR IN LINK TYPE ***  LT = ',I5)
         RETURN
         END
```

Figure A-1. Continued.

If the link exists, the test at statement 10 checks for a link of type 1 (linehaul). If LT is 1, the next five statements are executed, representing equations (3-2), (3-3), (3-5), (3-6) and (3-8) from Chapter 3. Equation (3-7) is embedded implicitly in the calculation of COST, and the mileage, DIST, is included at the end, rather than in each separate computation.

If LT is not 1, the test at statement 10 causes a jump to statement 20, which checks for a link of type 2 (yards). If LT is 2, the cost/yard-engine hour (CY) is computed, representing equation (3-9) from Chapter 3. The marginal engine-hours per car (MEH) is then computed. Note that the user may set a specific ARG(1) value for each yard, to represent different operating characteristics among the yards. COST is then computed as the product.

If LT is not 2, the test at statement 20 causes a jump to statement 30, which checks for a link of type 3 (dummy connector, as described in section 3.2). The cost on such a link may be used to represent local switching and movement costs to and from shipper docks. Again, ARG(1) is used to specify a link-specific parameter.

If LT is some number other than 1, 2 or 3, an error message is written, and no computation is done.

A.5.2 Preparing Data Input

A data file for RAILNET includes three separate sets of information, describing system parameters, links and traffic volumes. A data file may be prepared and stored on a disk so that the data can be used at a later time. Data may also be entered within the program or changed to consider the results of an altered network.

To prepare a data file, the structure shown in Figure A-2 must be followed. The first record in the data file must contain the word "PARAMS" in columns 1-6. The next six records (lines) contain the system parameters, as follows:

Figure A-2. Example input data set.

```
PARAMS
5.00    42.00
0.85    0.2     11.5
250.    5.0E-7  170.    18.3
15.00   7.00    0.04
67.2    0.78    0.60    0.52    0.04
0.21
LINKS
10
1, 2, 1, 57.0, 3000, 3000, 0., 0., 0., 0., 0.
1, 3, 1, 85.0, 3000, 0, 0., 0., 0., 0., 0.
2, 3, 1, 72.0, 3000, 0, 0., 0., 0., 0., 0.
3, 4, 2,  0.0, 1800, 0, 622.5, 0., 0., 0, 0.
4, 1, 1, 85.0, 3000, 0, 0., 0., 0., 0., 0.
4, 2, 1, 72.0, 3000, 0, 0., 0., 0., 0., 0.
4, 5, 3, 20.0, 2000, 0, 0.83, 0., 0., 0, 0.
4, 6, 1, 93.0, 3000, 0, 0., 0., 0., 0., 0.
5, 3, 3, 20.0, 2000, 0, 1.03, 0., 0., 0., 0.
6, 3, 1, 93.0, 3000, 0, 0., 0., 0., 0., 0.
END LINKS
O-D DATA
5
1, 2, 34, 12
1, 5, 52, 73
2, 5, 90, 81
2, 6, 112, 17
5, 6, 42, 15
END DATA
```

Record #	Data Items
2	Road crew wage rate ($/train-mile)
	Yard crew wage rate ($/yard engine-hour)
3	Fuel price ($/gal)
	Road locomotive fuel consumption rate (gals/loaded car-mile)
	Yard locomotive fuel consumption rate (gals/yard engine-hour)
4	Road locomotive ownership cost ($/time unit)
	Road locomotive utilization rate (loco-time units/loaded car-mile)
	Yard locomotive ownership cost ($/time unit)
	Yard locomotive utilization rate (hours/locomotive/time unit)
5	Car ownership cost ($/car/time unit)
	Daily per diem charge, avg ($/car)
	Per diem mileage charge ($/car-mile)
6	Average train length (cars)
	Empty-to-loaded car-mile ratio
	Proportion of total cars-on-line which are foreign, average
	Proportion of total car-miles made by foreign cars, average
	Total car-days on line/loaded car-mile
7	Way and structures maintenance cost factor ($/loaded car-mile)

Within each record the items are not limited to specific columns, but must be input in the order indicated. All data are read as floating point numbers, with decimal points.

Immediately after the last parameter input record, a record with "LINKS" in columns 1-5 must be included. The next record specifies the number of links in the network (free-format). For purposes of data input, a two-way link between a pair of nodes is considered as one link.

For each link, the following pieces of information must be input in order (free-format, separated by commas): node number 1, node number 2, link type, link length (miles), capacity in the 1 to 2 direction (cars/unit time), capacity in the 2 to 1 direction (cars/unit time), and the five link parameters which are user-specified. Note that because free-format input is used, all five parameters must be specified for each link, even if they are not used. Specifying a zero capacity in one direction can be used to make a link one-way; this is done for many of the links in Figure A-2.

Immediately after the last link record, a record with "END LINKS" in columns 1-9 must be included, followed by a record with "O-D DATA" in columns 1-8. The next record specifies the number of interchange records included, and is free-format.

The format of the interchange records is yard r, yard s, carloads/unit time from r to s, and from s to r (free-format, separated by commas). Thus, one record specifies traffic volumes in both directions for a single pair of points. This is important in determining the number of interchange records to be input. Note that there is no need to input data for interchanges where there are zero carloads in both directions. However, if one direction is non-zero and the other is zero, the zero must be entered to complete the record.

The format for entering network data from the terminal within the program is identical to that just described for establishing a file. the LOOK option is used and the program will prompt the user for each line of data input (including format instructions). The LOOK option also enables the user to change network characteristics for analysis without changing the network data maintained in the file. The program prompts the user for the particular changes desired.

A.6 Keywords in RAILNET

RAILNET is an interactive system. As such, it pauses at certain points in the program when input is required from the user. Prompting messages are displayed which tell what appropriate user responses are. The use of keywords allows a short response to signify the desires of the user. Prompting occurs at delays for observation of results, for input of data, and for selection of program options.

Four keywords are common throughout the program and have meanings which are relatively obvious. <YES> signifies an affirmative response to the question being displayed and correspondingly, <NO> signifies a negative response. The keyword <CONT> is used to instruct the program to continue processing after a pause in which some result is displayed. The use of <EXIT> will cause processing in one phase to end and the program will move to the next sequence, return to the main program, or terminate as appropriate.

Additional keywords are used to select particular program options. The display will show appropriate selections and provide a brief description of each choice.

A.7 The Menu of Activities in RAILNET

After beginning execution of the program and inputting data, RAILNET presents the user with a menu of activity options to determine what actions will occur next. These options are as follows.

WRITE causes the current data set and results to be written on a disk file for future use and comparison of results. This write operation occurs to FORTRAN logical unit 2, so the file will normally be called FT02F001. It is the responsibility of the user to rename and catalog this file as appropriate if it is to be retained after the terminal session ends.

LOOK gives the user access to the data files. S/he can then examine the files, add new data, delete data, or make changes to the existing file.

RESTART causes the program to start at the beginning, including rereading of the data set. Note that the program option RESTART should be used between traffic assignments performed in sequence. The ASSIGN option, to be explained next, uses the last result as a starting point. In order to preclude erroneous results, it is advisable to use RESTART when you have examined one assignment action and wish to begin another.

ASSIGN is the principal option of RAILNET. Selection of this keyword starts the assignment process. Immediately after selecting ASSIGN, the program will prompt for the number of iterations to be performed. Zero iterations results in an all-or-nothing assignment. Since equilibrium results of sufficient accuracy for most problems can be obtained with 10 or fewer iterations, if the user specifies more than 10 iterations, the program requests confirmation before proceeding. This is done to avoid excessive computation as a result of a simple typing error.

A.8 Output from RAILNET

The primary output from RAILNET is two tables. The first is a link-by-link report of volume (carloads) and marginal cost for crossing that link, together with summary statistics on carloads moved, loaded car-miles and total operating cost. An example of such output is shown in Table A-1. The values in this table are the results of using the example input data in Figure A-2 together with the LKCOST subroutine shown in Figure A-1.

The second output table is origin-to-destination marginal costs, for each pair of points between which shipments occur. An example of this output is shown in Table A-2, again corresponding to the sample problem setup using the data in Figure A-2 and the LKCOST subroutine in Figure A-1.

A.9 Obtaining the Software

Copies of the RAILNET software are available either from the U.S. Department of Transportation or from the authors of this report. The addresses are shown below:

Mr. Joel Palley, RRP-32
Federal Railroad Administration
U.S. Department of Transportation
400 Seventh Street, S.W.
Washington, D.C. 20590

Professor Mark A. Turnquist
Hollister Hall
Cornell University
Ithaca, NY 14853

Questions regarding the capabilities of the model, computer system requirements, etc., should be addressed to Professor Turnquist.

ORIGIN NODE	DEST. NODE	VOLUME	MARGINAL COST ($)
1	2	34	54.56
1	3	52	81.36
2	1	12	54.56
2	3	202	68.91
3	4	482	11.31
4	1	73	81.36
4	2	98	68.91
4	5	157	16.60
4	6	154	89.01
5	3	196	20.60
6	3	32	89.01

TOTAL CARLOADS MOVED = 528

TOTAL LOADED CAR-MILES = 59205

TOTAL OPERATING COST = $59278.

Table A-1. Example of link-by-link output from RAILNET.

ORIGIN NODE	DEST. NODE	CARLOADS	MARGINAL COST ($)
1	2	34	54.56
1	5	52	109.27
2	1	12	54.56
2	5	90	96.82
2	6	112	169.23
5	1	73	113.27
5	2	81	100.82
5	6	42	120.92
6	2	17	169.23
6	5	15	116.92

Table A-2. Example of origin-destination marginal operating costs from RAILNET.

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