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# THE USE OF AUTOMATICALLY ACQUIRED TRACK GEOMETRY DATA IN LONG-RANGE MAINTENANCE PLANNING

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## ABSTRACT

A review of the current status of the use of track geometry data in the preparation of track maintenance schedules. Current research being sponsored by the Federal Railroad Administration to resolve the major problems is discussed in terms of a proposed linear programming algorithm for optimum allocation of maintenance resources.

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# THE USE OF TRACK GEOMETRY DATA IN LONG-RANGE MAINTENANCE PLANNING

## INTRODUCTION

Over the last few years, increasing gross tonnage of railroad traffic and increasing average axle weight of railroad cars have been putting ever-increasing demands on the existing railroad tracks. At the same time, the low profitability of railroad operations has slowed the rate of investment required to upgrade the quality of the track beyond the standards to which it was designed 20 to 50 years ago. The major expenditures which are now being made for tie and rail renewal are confined to that required to bring track back to its original condition or to minimum legislated safety standards.

Under these conditions, the successful use of existing trackage to carry ever-increasing freight loads is dependent on two factors: increasingly vigilant inspection to catch and repair developing local defects in the track structure before they cause derailments, and constantly improving allocation of the resources available for planned, or long-range, maintenance.

The use of special railroad cars to measure track geometry is well established, and continuing efforts are developing ever more useful real-time management reports for use in spot maintenance. The use of this automatically measured track geometry data as a basis for the formulation of long-range maintenance plans has met only limited success--it is considered a useful but not necessary input to the existing planning procedures.

This paper reviews the current status of the efforts to generate useful measures of track condition from the geometry measurements, and investigates the information requirements of the decision-making process which yields the maintenance schedules. Finally, there is a description of some of the work which is being supported by the Federal Railroad Administration to develop techniques for using the available track geometry data as a powerful tool in the optimization of resource allocation and resulting long-range maintenance schedules.

## TEST CARS AND TRACK GEOMETRY DATA

Special test cars are routinely used to measure track geometry on the Japanese National Railroad and many of the European railroads [1, 2, 3], and the use of these test cars is slowly gaining acceptance on a number of U. S. roads. The Federal Railroad Administration has been supporting a program to develop several highly sophisticated test cars for use as research tools on the high-speed passenger tracks in the Northeast Corridor [4]. These cars, which are operated by ENSCO, Inc., under contract from the FRA, have also operated over a number of other cooperating railroads in the U. S. in order to develop a substantial data base of track geometry measurements over various types and classes of railroad track. This data is used as a basis for the development of real-time data processing capabilities, and in the continuing efforts to extend the usefulness of automatically collected track geometry data to the area of long-range maintenance planning.

The track geometry measurements made by the FRA test cars are representative of those made by most of the various test cars in use around the world. Instrumentation on the test car measures the following track geometry parameters:

> Gage--distance between the rails
>
> Profile, for each rail--local vertical deviation as defined by the offset from the middle of a 14.5-foot chord.
>
> Alignment, for each rail--local transverse deviation as defined by the offset from the middle of a 14.5-foot chord.
>
> Superelevation--difference in the height of the surface of the rails.
>
> Crosslevel--high-pass filtered superelevation; this corresponds to the deviations from the designed cant in curves.
>
> Curvature--track central angle contained in a 100-foot chord.

On the FRA test car, these parameters are sampled every 2.4 feet, the values are digitized and stored on magnetic tape, and the values of the various parameters are displayed on an analog strip chart which is driven at a speed proportional to the speed of the test car. The test cars can be used to measure track geometry at speeds up to 150 miles per hour. The primary use of this data is to determine the places where deviations in track geometry exceed thresholds preselected by maintenance-of-way engineers. Geometry defects exceeding a critical threshold will require immediate spot repairs. The real-time data processing system onboard the test cars monitors the data and prints exception reports detailing the location and magnitude of each critical defect. This system immediately generates the paperwork required to initiate spot repairs without requiring additional analysis of the data. Summary reports for management use are prepared later by offline processing of the digital magnetic tape records of the track geometry data. (The data processing program is currently being modified to incorporate exception thresholds with respect to the limits of the FRA Track Standards.)

## TRACK GEOMETRY DEGRADATION--EVALUATION AND INDICES

The real-time processing of track geometry data to locate critical defects is essentially a microscopic analysis of the track. The geometry measurements do define the track geometry, but the gage, profile, and alignment sensors alone generate nearly 11,000 data points per mile of track, which presents substantial practical problems for any global or average rating of track quality.

Although the correlation of adjacent defects may have substantial influence on ride quality through the development of severe rocking motions in the vehicle, most attempts at generating quality indices for track geometry have treated the data as samples from a stationary random process. This is admittedly a gross simplification of the process which generates defects in track geometry, but the result of this simplification provides a manageable procedure for the generation of statistics which describe the average condition of entire sections of track [3,5].

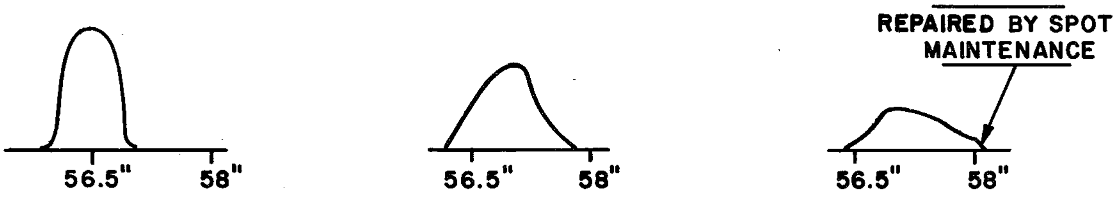
The basic statistical measure of average track geometry is the probability density function of the value of each measured parameter associated with a homogeneous section of track. This probability density function is equivalent to the histogram of all of the measured values of that parameter throughout the section. It has been found that this distribution is approximately Normal Gaussian for new track as the construction errors are essentially random, but that the defects in geometry which develop in service cause the distribution to become distorted as the dispersion of the measured parameters increases. The changes in the distribution function of the various parameters is caused by the non-random generation of changes in the track geometry; the condition of a section of track can be followed through the evolution of the distribution function. The following is a discussion of the changes which have been observed in histograms as the average condition of track has degraded in service over a considerable period of time.

why? So what? [illegible]

GAGE

Nominal track gage in U. S. roads is 56-1/2 inches, although considerable track is purposely laid as much as 1/4 inch narrower on tangent sections, and up to 1/2 inch wider on curves. The histogram of measured gage values is approximately normal and centered on the design value for new track. As the track remains in service, the dynamic forces from the interaction with wheels and the impact forces due to truck hunting all tend to widen the gage. In addition, the magnitude of these forces is apparently related to the extent to which the gage has already widened, so the mode of the distribution not only shifts toward wide gage, but the distribution develops a fatter tail than would be expected for a Poisson distribution. As the track continues to degrade, routine spot maintenance will repair localized areas of wide gage when the gage approaches 57-1/2 to 58 inches. This effectively truncates the tail of the distribution. At some point, areas of wide gage are generated so frequently that major maintenance is performed on the entire section to bring the gage back to its original condition.

Gage Histograms:



NEW TRACK

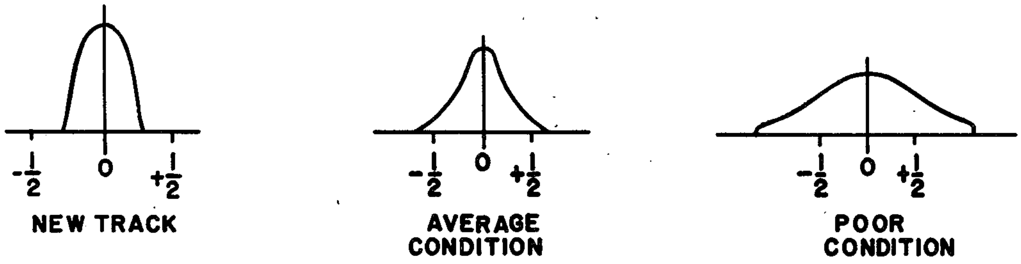
AVERAGE CONDITION

POOR CONDITION

## PROFILE

It would be expected that low spots in the space curve of a rail would be made gradually more severe due to the pounding action of passing trains (similar to the washboard effect on dirt roads). However, the measured profile values determined by mid-ordinate to chord measurements are not a simple function of the space curve, and the histograms of these values have been found to be approximately Gaussian except for long tails which develop on both sides of the mean. As the extreme values are eliminated through spot maintenance of the track, the distribution becomes truncated. Major maintenance is performed when it becomes infeasible to keep up with the rate at which critical defects are generated. The condition of the track is then returned to approximately its original specifications.

Profile Histograms (Mid-ordinate to chord offset data):



**NEW TRACK** **AVERAGE CONDITION** **POOR CONDITION**

## ALIGNMENT

Degradation of the alignment of railroad tracks results in changes in the histogram very similar to those exhibited by histograms of profile. The alignment mid-ordinate to chord measurement is substantially affected by the degree of curvature of the track, so the measured values must be filtered to remove this changing bias, or the histograms must be based on sections having constant curvature throughout. In addition, the lateral force applied to the rails in a curve is generally greater on one rail than on the other, depending on the average train speed and the amount of superelevation. Therefore, the condition of the alignment and rate of degradation of the alignment will usually be significantly different for the two rails through a curve.

## STATISTICAL MEASURES OF AVERAGE TRACK QUALITY

The histograms derived from the track geometry measurements provide valuable information about average track quality in a form that is more easily understood than the raw presentation of data on analog strip charts. The use of statistics to summarize the major features of the histograms further reduces the information to a form that is easily comparable and storable in an automatic data processing system. The commonly used statistics are:

Average value: mean, mode

Dispersion of central part of distribution: standard deviation; mean absolute deviation.

> Dispersion of extreme values of distribution: percent beyond ± 3 standard deviations; percent beyond a set threshold level; fourth central moment of the distribution.

These statistics reduce the data for a section of track to 10 or 12 numbers. This is still too many to use when comparing many different sections of track. Various indices of overall average track geometry have been derived by using arbitrarily weighted sums of the various statistics. While this approach reduces the burden of data manipulation, it also loses much of the information about the causes of the degradation of the track. The unbiased estimates of the average condition of the track geometry still provide useful information for the manual generation of maintenance schedules. Tabulations of track geometry indices are being used by several of the railroads which are the most sophisticated in the use of track geometry measuring cars.

## TRACK QUALITY

The major problem in applying the statistical measures of track geometry to maintenance scheduling is that the quality of the track is a function of many parameters besides the geometry. Track quality is essentially a measure of how well the track is performing its job of guiding vehicles in a safe, smooth manner. Dangerous dynamic conditions may result from unstable vehicle accelerations that are induced by defects in the geometry of the track. The quality of the track is a function of the speed of the vehicle and the mechanical characteristics of the vehicle as well as a function of the magnitude of the random changes in track geometry. The basic FRA Track Safety Standards on track geometry establish limits on the size of individual defects in a section of track as a function of the maximum allowable train speed over the section. These Track Safety Standards are based on the criteria of avoiding derailments, but repeated smaller accelerations will cause equipment wear and damage lading as well as affecting passenger ride comfort.

Because of the complex relationship between track geometry and track quality, track quality measures based on simple considerations of geometry have not been effective as a basis for maintenance planning.

Analysis of the track geometry data collected by ENSCO using the FRA test cars has established that, all other conditions remaining constant, the dispersion of the measured values of any one geometry parameter is a monotonic function of track quality. Recent work by the Southern Railroad has related the relative importance of critical defects of various parameters to the frequency of derailments [5]. What remains to be done is to develop the necessary relationships between track geometry and track quality that will lead to an index whose value is a linear function of the quality of a section of track, and which is directly comparable to the values of the indices for other sections having different kinds of traffic at different speeds. Such an ideal track quality index should be scaled so that it is directly compatible with the decision criteria used in developing long-range maintenance schedules.

## REQUIREMENTS OF TRACK QUALITY INDICES

Any index which is used to describe track quality must meet two criteria:

(1) Be directly usable in terms of the decision process;

(2) Be directly related to measurable parameters of track geometry.

Arbitrarily defined indices based solely on statistical measures of track geometry could not be expected to be directly usable as a basis for maintenance decisions. Work in progress at ENSCO is aimed at providing insight into the relationships of track geometry to the economic problems confronting railroad operations and maintenance. This work will provide a basis for directly usable indices of track quality. The following sections describe some of the topics which are under investigation.

## TRACK QUALITY PARAMETERS

Since the basic goal of the maintenance schedule is to minimize track-related costs, the quality of each section of track must be defined in economic terms. Ideal track would not cause any vibration associated wear of the equipment or lading, and would not cause any accidents. Real track is less than perfect and the poorer the track, the higher the costs associated with accidents and wear and tear of equipment, lading, and passengers. Since these costs are a function of the number of vehicles which use the track, the costs should be normalized for the intensity of traffic.

The total cost associated with track quality is obviously a function of many parameters. For track of a given geometry quality, the wear and lading damage will be a function of the magnitude of accelerations that are forced by the track, and these are a direct function of vehicle speed. The probability of a derailment occurring due to a track defect of a given size is again a function of speed. The expected loss in event of a derailment is a function of the type of traffic (passenger, hazardous material, normal freight), loss potential of the surrounding area (urban vs. rural), and the speed of the train. This total expected cost can be developed through straight-forward application of risk analysis procedures [6].

If the total cost due to defects in track geometry had a linear relationship to changes in each of the parameters used to describe the severity of the defects, a portion of the expected loss could be assigned to each parameter according to its measured deviation from the idealized perfect track. The investigation of the actual form of this relationship is now in its primary phases, and current research will demonstrate the relative economic importance of various types of defects in track geometry. Second-order interactions among the geometry parameters are probably quite important, but this relationship has not yet been clarified. The form of the statement of this relationship will depend on the way it will be used.

Formal models are being developed, not as a final answer to the scheduling problem, but as a guideline for research efforts into the functional relationships which exist between track geometry and the economic criteria of railroad operations. An example of a single period Linear Programming model is presented in the Appendix. Further extensions of this model could allow for several different maintenance procedures to be performed on each section of track during the planning period. Multi-period horizons are also easily handled. However, the model in its present form is already too simplified to give realistic answers and too complex to be easily implemented.

The lack of reality in this model should not be of major concern. Maintenance schedules are being made now without formal algorithms, and with often unstated assumptions. A formal model, such as this Linear Program, is valuable in that it provides a theoretical structure for the analysis of the effects of alternative maintenance and economic criteria. The Linear Programming model describes explicitly the minimum information and policy guidelines which are required to achieve an optimum allocation of maintenance resources, and the same information in much the same form is required by the presently used implicit models.

The FRA test cars are being used to collect track geometry data on a number of different railroads. Correlation of this data with independent rankings of ride quality made by the railroads and with their current maintenance plans will clarify the relationship that exists between the present statistical measures of track geometry and the economic measures of track quality.

## TRACK QUALITY DEGRADATION

Track maintenance schedules must be based not only on the present condition of the track, but on the future condition of the track as affected by maintenance procedures and continued usage. It is therefore important to be able to estimate the future quality of each section of track from measurements made of its present condition and knowledge of the mechanism and expected rate of degradation of the quality of the track.

Test runs using the FRA test cars have been made over certain tracks at six-month or annual intervals for several years. Comparison of the track geometry statistics calculated from the various runs is an indication of the track degradation which has occurred. The FRA is supporting a continuing program of planned tests to evaluate the rate of change of track geometry as a function of a number of parameters such as traffic density, speed, and track structure. These tests are being performed both on the standard track of participating railroads and on the specially constructed FRA test track near El Dorado, Kansas.

## EFFECT OF TRACK MAINTENANCE PROCEDURES

Maintenance scheduling involves a choice among the various maintenance procedures which may be applied to each section of track. The value of the procedure depends on the degree of improvement which is achieved in the track geometry and the rate of degradation of track geometry following the maintenance. Both the resulting quality and stability of the track are important parameters in evaluating the benefit achieved from various types of planned maintenance.

No! also a function of the quality of the components

The FRA test cars are being used to evaluate the effects of various maintenance procedures on track geometry. Tests are made before and after maintenance is performed on a section of track to define the changes caused by the maintenance. Continued observation of the track allows estimates to be made of the effect of the maintenance activity on the ensuing rate of degradation of the track geometry.

## SUMMARY

Automatically acquired track geometry data will become a valuable input to the maintenance scheduling activity only after it becomes possible to relate this data to the economic criteria of track quality. Further understanding of the mechanism of track degradation and the effects of various maintenance procedures will contribute to better allocation of maintenance activities.

Research being sponsored by the FRA is directly concerned with determining these various functional relationships. As this research progresses, it will become possible to report the information from track geometry measurements in a form directly usable by the staff which prepares the maintenance schedule. Presentation of automatically collected track geometry data in a form which is easily retrievable and directly comparable will greatly increase the value of the information obtained from the test cars regardless of whether or not a formal algorithm is used to generate the final long-range maintenance schedule.

# APPENDIX

## THE APPLICATION OF TRACK QUALITY INDICES IN A LINEAR PROGRAMMING MODEL FOR MAINTENANCE SCHEDULING

Assume that a maintenance schedule must be developed for the next period (for instance, the following year) which will best allocate the available resources over N sections of homogeneous track within certain manpower, equipment, and budget restrictions. We will define certain parameters and their assumed interrelationships which will lead to the development of a linear programming model. This model can be used to optimize the allocation of maintenance resources.

### TRACK AND MAINTENANCE PARAMETERS

$N$ ≡ Number of different sections of homogeneous track

$M$ ≡ Number of different maintenance and operating alternatives (surface lift; replace x% of ties and regage; tamp; reduce speed limit; spot work; do no maintenance; etc.)

$D_k$ ≡ Length of section k, miles

$$\sum_{k=1}^{N} D_k = \text{total length of track considered for maintenance}$$

$d_{ik}$ ≡ Length of section k to be maintained using procedure i

$$\sum_{i=1}^{M} d_{ik} = D_k \quad k = 1, 2, \ldots N$$

$G_k \equiv$ Amount of traffic which will pass over section k during the next period, tons.

$L_i \equiv$ Total miles of track to which maintenance alternative i can be applied. (Obviously, $L_i = \Sigma D_k$ for alternatives such as no work or slow orders, but would be a function of equipment capacity for certain other alternatives.)

## TRACK QUALITY PARAMETERS

Perfect track will not impart any accelerations into a vehicle other than those resulting from planned curves and hills. The resulting perfectly smooth ride will minimize wear of equipment, damage of lading, and derailments. As the track degrades through the accumulation of random errors in the track geometry, the train operating costs associated with wear and damage will increase.

It is reasonable to consider the quality of a section of track to be a function of the train-related costs due to errors in the track geometry. The absolute quality would then be the costs which would result from operating trains on the track at standard specified conditions.

Let $Q^*_k \equiv$ Total train operating costs due to the operation of $G^*$ tons of traffic over one mile of section k at a specified speed, assuming a standard consist, lading, and hazard potential. This cost level is a measure of the average quality of section k at a specific point in time, and is not a function of any track maintenance cost associated with the operation of the track. ($G^*$ can be one million gross tons or any other specified normalizing quantity of traffic.)

New track laid to the best engineering practice will have certain deviations in geometry due to construction errors. The cost associated with standard train operation over this best achievable condition of track geometry is a measure of the best achievable track quality, which we will denote by $Q^*$.

Degradation of track quality in section k, denoted by ${Q'^*}_k$, is a measure of the increased costs due to accumulated errors in excess of those expected on new track. The units of $Q_k^*$, $Q^*$, and ${Q'^*}_k$ are \$/mile/$G^*$ tons of standard traffic due to track degradation of section k. Therefore:

$$Q^*_k = Q^* + {Q'^*}_k \tag{A1}$$

The actual train-related costs associated with the operation of $G^*$ tons of traffic over section k will be a function of the actual operating speed, susceptibility of lading to damage, and hazard potential.

Let $Q_k \equiv$ actual train-related costs over section k, (\$/mile/ $G^*$ tons)

Then $Q_k = f(Q^*_k$, speed, type of traffic, hazard potential) (A2)

The severity of the operating conditions over section k is defined as the ratio of expected costs at actual conditions to expected costs at standard train operating conditions

$$S_k = Q_k/Q^*_k \tag{A3}$$

Track can degrade due to the accumulation of errors of a number of different geometry parameters such as gage, profile, cross-level, etc.

Let $J \equiv$ Number of parameters associated with track degradation.

The total cost associated with train operation is some function of the quality associated with these parameters. Assuming that the components of cost associated with the various parameters are independent, we have for standard operating conditions:

how?

$$Q^*_k = \sum_{j=1}^{J} q^*_{jk} \tag{A4}$$

where $q^*_{jk} \equiv$ cost component associated with j-th parameter.

Let $q^*_j \equiv$ minimum value of $q^*_{jk}$, i.e., best achievable quality for j-th parameter.

Then $Q^*$ is simply $Q^* = \sum_{j=1}^{J} q^*_j$ (A5)

$q'^*_{jk} \equiv$ \$/mile/G* tons due to degradation of parameter j.

Then

$$Q'^*_k = \sum_{j=1}^{J} q'^*_{jk} \tag{A6}$$

Substituting (A5) and (A6) into (A1) yields:

$$Q^*_k = \sum_{j=1}^{J} (q^*_j + q'^*_{jk}). \qquad (A7)$$

Similarly, for actual operating conditions on section k, the best condition of parameter j, denoted by $q_j$, is given by

$$q_j = S_k q^*_j \qquad (A8)$$

and the degradation of parameter j on track section k, denoted by $q'_{jk}$, is given by

$$q'_{jk} = S_k\, q'^*_{jk}. \qquad (A9)$$

Utilizing equations (A7), (A8), and (A9) in (A3) results in

$$Q_k = \sum_{j=1}^{J} (q_j + q'_{jk}). \qquad (A10)$$

## TRACK MAINTENANCE PROCEDURES

Normal track maintenance procedures such as regaging, surfacing, etc., cannot return the quality of a section of track to like-new condition. At best, the maintenance can remove only a portion of the accumulated degradation of each geometric parameter.

Assume that maintenance procedure i will correct a certain percentage of the accumulated degradation of each geometry parameter:

Let $E^{m}_{ij} \equiv$ (1-fractional reduction in $q'_{jk}$ due to maintenance procedure i)

Then

$$Q_k = S_k \sum_{j=1}^{J} (q^*_j + q'^*_{jk}) \tag{A10}$$

before maintenance is performed; and

$$\hat{Q}_k = S_k \sum_{j=1}^{J} (q^*_j + E^m_{ij} q'^*_{jk}) \tag{A11}$$

where $\hat{Q}_k$ is the quality of section k after the performance of maintenance procedure i.

Also to be considered is the cost associated with each maintenance procedure.

Let $C^m_i \equiv$ Cost per mile of maintenance procedure i applied to a mile of track.

Note that $C^m_i = 0$ for "procedure" of no maintenance work or changes in operating policies.

## CHANGES IN TRAIN OPERATING POLICIES

The actual cost associated with operating on section k is a function of both the absolute quality of the track geometry and the severity of the operating service. This severity can be changed through changes in operating policies such as reduction of the speed limit, elimination of passenger service, embargo of hazardous materials, etc. Any such change in operating policy will also result in increased costs of train operation due to higher labor costs, lost business, etc.

Let $E^o_i \equiv$ 1-fractional reduction in the severity of service due to operational policy change i.

Then

$$Q_k = S_k \sum_{j=1}^{J} (q^*_j + q'^*_{jk}) \text{ before change} \quad (A10)$$

and

$$Q_k = E^o_i S_k \sum_{j=1}^{J} (q^*_j + q'^*_{jk}) \quad (A12)$$

after operating policy change i.

Let $C^o_i$ = increased business costs per G* tons per mile of track due to operating policy change i.

Note that $C^o_i = 0$ if no change in operating policy.

## RATE OF TRACK DEGRADATION

The degradation of railroad track is the result of rail wear and the accumulation of permanent deformations of the track structure caused by the cyclic stresses imposed by train operation. These stresses are a function of both the static load of the wheels and the dynamic forces caused by vehicle accelerations which are largely imparted by defects in the track geometry. Therefore, it is reasonable to assume that the overall rate of track degradation can be expressed as the sum of the rates of degradation resulting from the two different mechanisms:

1. Constant rate of degradation of parameter j due to stresses imposed by traffic: $\Delta q^*_j$ per G* tons at standard operating conditions.

2. An exponential rate of degradation of parameter j due to impact loads caused by accumulated errors in track parameter j.

   Let $B_j$ = rate of degradation of parameter j equal to the fractional increase in parameter j per G* tons.

The total rate of degradation with respect to the amount of traffic G passing over track section k is expressed by the equation:

$$\frac{d(q_j + q'_{jk})}{d(\frac{G}{G^*})} = S_k \Delta q^*_j + B_j (q_j + q'_{jk}) \qquad \text{(A13)}$$

This is a simple, first-order differential equation. Defining $Q_k$ as the initial quality of section k and $Q'_k$ as the quality of the section after exposure to G tons of traffic, the following relationship is found by substituting equation A10 into A13 and solving for the initial condition $Q'_k = Q_k$ when G = 0.

$$Q'_k = \sum_{j=1}^{J} [(q_j + q'_{jk} + \frac{S_k \Delta q^*_j}{B_j}) e^{\frac{GB_j}{G^*}} - \frac{S_k \Delta q^*_j}{B_j}] \qquad \text{(A14)}$$

## MULTI-PERIOD ALLOCATION OF MAINTENANCE COSTS

Once the quality of a section of track is improved by the performance of a maintenance procedure, additional maintenance will not be required for several planning periods. Since the benefit derived from the maintenance is received during several periods, it is reasonable to allocate the cost of the maintenance over this same number of periods. This will allow a direct cost-benefit analysis to be made of the effect of various maintenance procedures.

There are many different criteria which could be used for this cost allocation. If one assumes that overall track condition remains constant for the entire system, then a section of track will be maintained every time it degrades to a certain condition. Based on this assumption, the economic life of maintenance procedure i applied to section k would be the length of time that the post-maintenance quality $\hat{Q}_k$ takes to degrade to the pre-maintenance quality $Q_k$.

Let $t_{ik} \equiv$ economic life of procedure i applied to section k

$$Q_k = \sum_{j=1}^{J} (q_j + q'_{jk}) \quad \text{initially} \tag{A10}$$

Since $E^o_{ij} = 1$ for track maintenance procedures,

$$\hat{Q}_k = \sum_{j=1}^{J} (q_j + E^m_{ij}\, q'_{jk}) \text{ after maintenance procedure i.} \tag{A11}$$

Since the condition of the improved track degrades back to the initial quality $Q_k$ after $t_{ik}$ periods (that is, after exposure to a total amount of traffic equal to $t_{ik}G_k$ tons), the economic life of the maintenance may be found by solving the following equation for the value of $t_{ik}$:

$$Q_k = \sum_{j=1}^{J} [(q_j + E^m_{ij} q_{jk} + \frac{S_k \Delta q^*_j}{B_j}) e^{\frac{t_{ik} G_k B_j}{G^*}} - \frac{S_k \Delta q^*_j}{B_j}] \qquad \text{(A15)}$$

The cost of the maintenance is then prorated over the effective life of the maintenance $t_{ik}$.

$\overline{C}^m_{ik}$ = cost of maintenance procedure i applied to section k and allocated to planning period.

$$\overline{C}^m_{ik} = C^m_i / t_{ik}$$

AVERAGE TRACK QUALITY DURING NEXT PERIOD

$\overline{Q}_{ik}$ = the cost of operating over section k during the next period under conditions of maintenance procedure i (assuming for simplicity that all maintenance is performed at the beginning of the period.)

$\overline{Q}_k$ is derived from equations A11, A12 and A14.

$$\overline{Q}_k = \int_0^{G_k/G^*} \sum_{j=1}^{J} [(E^o_i[q_i + E^m_{ij} q'_{jk}] + \frac{S_k \Delta q^*_j}{B_j}) e^{\frac{GB_j}{G^*}} - \frac{S_k \Delta q_j}{B_j}] \, d(\frac{G}{G^*})$$

$$= \sum_{j=1}^{J} [(E^o_i[q_i + E^m_{ij} q'_{jk}] + \frac{S_k \Delta q^*_j}{B_j}) \frac{e^{\frac{G_k B_j}{G^*}}}{B_j}$$

$$- \frac{G_k S_k \Delta q_j}{G^* B_j} - \frac{1}{B_j} (E^o_i[q_i + E^m_{ij} q'_{jk}] + \frac{S_k \Delta q^*_j}{B_j})] \qquad \text{(A16)}$$

## LINEAR PROGRAM FOR MAINTENANCE ALLOCATION

In previous sections we defined track quality as a measure of those costs of train operation caused by defects in the track. Each track maintenance procedure would result in a reduction of operating costs, but would result in maintenance costs which are allocated over a specific economic lifetime. Changes in operating policies can also reduce the track related train operating costs, but will also result in increases in other costs.

The optimum maintenance schedule is one which allocates the available maintenance resources and makes the necessary changes in operating policy in such a way that the total costs are minimized during the following period. Such an optimum schedule can be formulated using a Linear Programming model. This algorithm guarantees an optimum allocation of resources. The dual of the linear program establishes equivalent values of the maintenance resources, which provides a method for evaluating the economic effect of additional maintenance equipment, overtime operations, or a larger maintenance budget.

Using the symbols previously defined, the single period linear programming model can be stated in the following form:

$$\text{minimize} \sum_{i=1}^{M} \sum_{k=1}^{N} d_{ik}\left(\bar{C}^{m}_{ik} + \frac{G_k C^{o}_{i}}{G^*} + \bar{Q}_{ik}\right)$$

where $\bar{C}^{m}_{ik}$ = that portion of the cost of maintenance procedure i performed on one mile of section k which is allocated to the following period.

$C^o_i$ = Increased business cost per G* tons per mile of track due to train operating policy change i.

$\overline{Q}_{ik}$ = Total train operating costs during the following period which are due to defects in the track geometry

This minimization is calculated subject to the following constraints:

$\sum_{i=1}^{M} d_{ik} = D_k$ for all k (all track has some maintenance procedure assigned, even if it is the dummy "no-work")

$\sum_{k=1}^{N} d_{ik} \leq L_i$ for all i, where $L_i$ is the equipment or manpower availability constraint, in terms of the maximum miles that can be maintained using procedure i.

$\sum_{i=1}^{M} \sum_{k=1}^{N} C^m_i d_{ik} \leq C^*$ where $C^m_i$ is a maintenance procedure and C* is the maintenance department expenditure budget.

$d_{ik} \geq 0$ for all ik (non-negativity constraint on length of sections treated.)

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